

Reply to “Discussion of “The relation between dilatancy, effective stress and dispersive pressure in granular avalanches”

by P. Bartelt and O. Buser (DOI: 10.1007/s11440-016-0463-7)”

by Richard Iverson and David L. George (DOI: 10.1007/s11440-016-0502-4)

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Abstract Iverson and George largely agree with our mathematical description of the dispersive pressure and dilatancy for a dry granular avalanche; however, they disagree with our views of effective stress and pore-fluid pressure in a debris flow containing solid granular material fully or partially submerged in a muddy fluid. Here we counter their concerns by deriving time-dependent relations for pore-fluid pressure in a two-component debris flow in which the solid phase is undergoing dilations and contractions. This analysis was not contained in our original paper. We explicitly show how the excess fluid pressure arises from dispersive *accelerations* associated with changes in configuration of the solid material. Additional contributions to the pore-fluid pressure are associated with the solid–fluid drag and buoyancy. In our analysis, we find that pore-fluid pressures can only be calculated (1) by modelling their time-dependent source, the frictional work rate, and (2) by accounting for the time-dependent inertial forces associated with the solid configuration’s center of mass. This leads to an alternative physical description of pore-fluid pressure, especially when the debris flow is far from equilibrium.

Keywords Buoyancy · Debris flow · Dilatancy · Dispersive pressure · Effective stress · Fluid-pore pressure · Free fluid · Hydrostatic fluid pressure · Interstitial drag · Jerk

1 Introduction

Iverson and George are correct to maintain that our recent paper [1] primarily concerns the mechanics of dry granular avalanches. They question the application of our theory to fluid-saturated debris flows, especially our criticism of effective stress concepts. Indeed, our criticism, and it is harsh criticism, is directed toward the snow avalanche community which still invokes effective pressure concepts to explain dry avalanche fluidization. In our recent paper we show that a consequential application of Newton’s law implies that any imbalance of forces must be associated with an inertial acceleration. It is therefore highly questionable to model avalanches and debris flows by introducing out-of-balance forces (such as “excess pressures”) and then assume the system is in equilibrium.

We therefore restate the fundamental purpose and idea of our paper: The mechanics of avalanches and debris flows requires an understanding of transient, time-dependent states in which the flow is not in equilibrium. The mechanics of avalanches and debris flows requires a theory explaining how a flow could reach an equilibrium state from another equilibrium state when it is perturbed by changing boundary conditions (slope angle, roughness, etc). Dilations and contractions are fundamental features of this transition and therefore define the time-dependent behavior, especially the basal friction. Only by considering the transient behavior can we model flow mobility to assess hazard. We had no intention of criticizing D-CLAW which readily provides a solution to this fundamental problem [4, 5].

The immediate, and perhaps hidden, question posed by Iverson and George is: Can an inertia-based theory of dispersive pressure and dilations be applied to model a

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fluid-saturated flow? Clearly drag and buoyancy effects must be included in the mathematical formulation. What does such a formulation imply for the calculation of the effective stress?

Here we answer these questions by deriving a time-dependent equation for pore-fluid pressure in a two-component granular debris flow. We show, in agreement with Iverson and George, that the excess pore pressure is directly related to granular dilatancy. However, unlike Iverson and George (and the model applied in D-CLAW), our pore-fluid equation requires the calculation of the inertial forces; that is, the location, velocity, and acceleration of the center of mass of the solid granular material. We come to the conclusion that dilatancy in a debris flow can not be well represented by the solid fraction *content*, rather the solid fraction *configuration*.

The derivation is based on a simple application of Newton's law. We will avoid any use of “granular minutiae,” thus separating constitutive formulations from the underlying physics. Once again, as in the original paper, before we can discuss dilations and contractions, we must carefully define mass and volume in a fluid-solid flow.

2 Solid configuration in a fully or partially saturated granular debris flow

For the derivation we invoke the concept of a representative element volume of the debris flow, extending from the base of the flow to the top free surface, see Fig. 1. These volumes are sometimes called flow “columns” with basal area A . In our debris flow model the areas are fixed and mass flows in and out of the volume.

We place no restriction on the distribution of the solid granular mass within the fluid. The volumes can be “over-saturated” with most of the solid at the bottom of the column, or the solid can be evenly distributed throughout the fluid (Fig. 1a); or the volumes can be “undersaturated” (Fig. 1b), etc. To track the changes in configuration, some reference coordinate system is required. We define the volumes of solid material (V_s^0) and muddy fluid (V_f^0) to be the “separated” component volumes. We define the total mass of each component,

$$M_s = \rho_s^0 V_s^0, \quad (1)$$

$$M_f = \rho_f^0 V_f^0 \quad (2)$$

where ρ_s^0 and ρ_f^0 are the material densities of the solid and fluid materials. The total mass of the column is simply

$$M = M_s + M_f \quad (3)$$

The bulk density of the mixture is

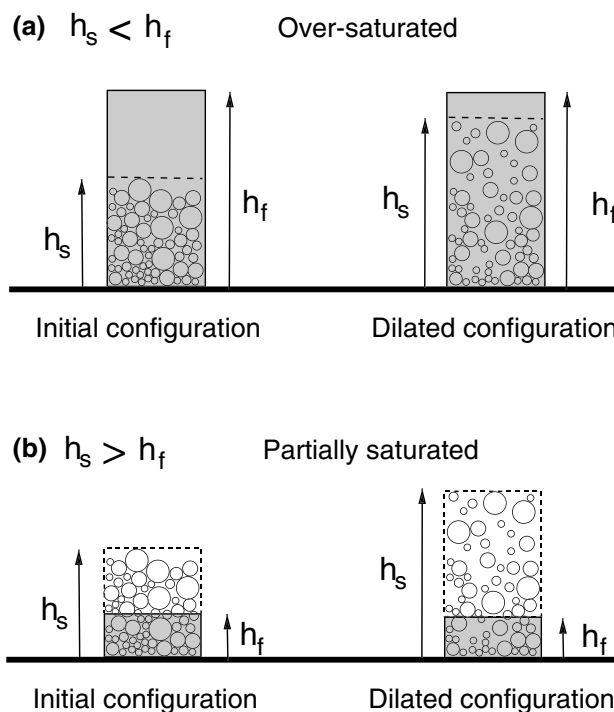


Fig. 1 A two-component debris flow column contains solid and fluid mass. The height of the solid material is h_s ; the height of the fluid h_f . Different mass configurations are possible, including over-saturated and partially saturated volumes

$$\rho = m\rho_s^0 + (1 - m)\rho_f^0 \quad (4)$$

where m is the solid fraction, defined with respect to the total volume $V^0 = V_s^0 + V_f^0$,

$$m = \frac{V_s^0}{V^0}. \quad (5)$$

These simple definitions of mass, volume and solid content need to be extended to provide the required mathematical foundation to track dilations and contractions. The total solid volume content m does not consider how the solid mass is distributed (or “configured”) within the fluid.

To this end, we define two heights. The height h_s is the height of the solid material in the fluid and the height h_f is the total height of the debris and the fluid (see Fig. 1). We consider dilatancy to be connected to the expansion of the solid material within the fluid. Because the mass is constant, we have

$$M_s = \rho_s^0 V_s^0 = \rho_s V_s \quad (6)$$

with $V_s = h_s A$. Moreover, the void space of the granular matrix can increase, leaving more volume to filled by the fluid. The “dry” density of the solid decreases. When the solid is submerged, the height of the fluid will not change, even if the solid matrix is dilating or contracting. The height of the solid h_s is changing in time, allowing different solid configurations. For example, for the submerged solid

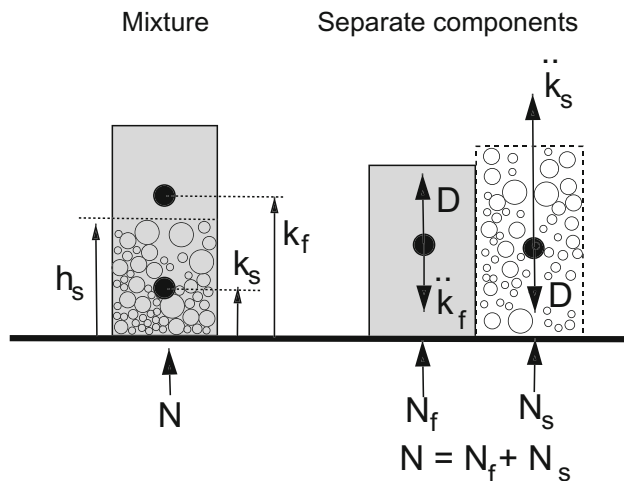


Fig. 2 The total normal force N is associated with the total mixture; the normal forces N_f and N_s are associated with the fluid and solid components, respectively. The effect of drag and buoyancy is to change the overall acceleration of the total center of mass k . Drag and inertial effects cannot be separated with independent measurements of fluid and solid components

case, the height of the fluid h_f can be found from the fluid volume by noting that the fluid fills in the void space,

$$h_f = h_s + \frac{V_s^0 V_f^0}{V_s A}. \quad (7)$$

The definition of h_s and h_f imply that the distribution of mass in the volume may not be homogeneous. Different mass configurations are possible; the solid mass can rest at the bottom of the flow, leading to an “overflow” of the fluid and possible dewatering of the solid. The solid material can also be distributed evenly over the entire flow height, representing more “buoyant” flow states with suspended particles.

To track the changes in solid configuration, we follow the center of mass of the solid and fluid components, k_s and k_f , respectively (Fig. 2). The center of mass of the solid stands in some relation to the height h_s . For simplicity, we take a homogeneous solid distribution

$$k_s = \frac{h_s}{2}. \quad (8)$$

Because the solid and fluid are complementary configurations, it is possible to define the fluid center of mass k_f as well as the center of mass of the entire volume, k (Fig. 3). More importantly, the component center of masses are not independent, but related by the assumption of the mass distribution.

3 Normal forces and pore-fluid pressure

The different solid configurations are associated with different frictional behavior. In equilibrium the force measured on the bottom of the debris flow will be equal to the

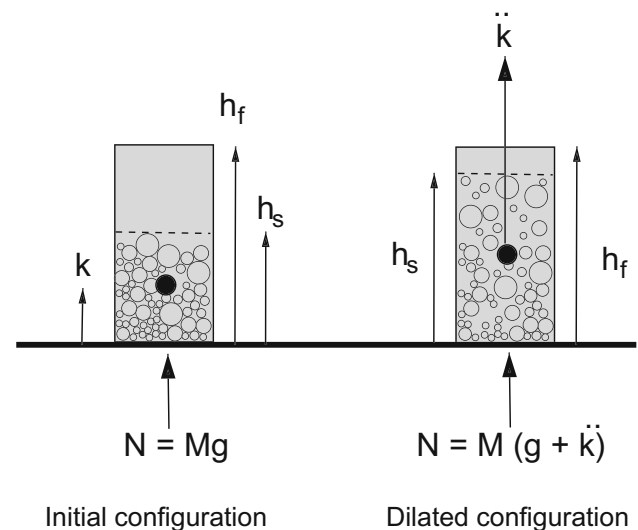


Fig. 3 Submerged particles in the volume V are accelerated upwards. The center of mass of the total volume (fluid and solid) is located at k . The acceleration of the center of mass is \ddot{k} . The normal force during the dilation is $N = M(g + \ddot{k})$

total weight of volume. When the configuration changes, the normal force will differ from the weight.

The total force N is composed of solid N_s and fluid N_f parts,

$$N = N_s + N_f. \quad (9)$$

We derive our force balances by simple application of Newton’s law. A force balance for the solid part consists of the weight $M_s g_z$ and the inertial force $M_s \ddot{k}_s$ as well as the drag D opposing the movement,

$$N_s = M_s(g_z + \ddot{k}_s) + D - \rho_f V_s^0(g_z + \ddot{k}_s). \quad (10)$$

A subtle detail of this analysis is that we define g_z positive downwards by convention, whereas the z -axis is positive upwards, also by convention. This gives the desired result that an upward acceleration increases the normal force on the bottom. The last term in Eq. 10 is of particular importance. We define N_s to be the buoyant weight and therefore must subtract the volume of displaced fluid. The drag force $+D$ acting on the solid increase the reaction N_s at the bottom. The fluid normal force N_f is similar,

$$N_f = M_f(g_z + \ddot{k}_f) - D + \rho_f V_s^0(g_z + \ddot{k}_s). \quad (11)$$

It contains the hydrostatic pressure $p_h = M_f g_z$, the inertial force $M_f \ddot{k}_f$ as well as the equal and opposite drag force $-D$. Thus, if the solid phase is moving upwards, the drag on the particles will cause a reduction of N_f . Conversely, the displaced volume of water must be added to the normal force of the fluid. Simply, the submerged particles cause

the height of the fluid to rise. It is now possible to define the “excess” pore-fluid pressure p_e . Since

$$N_f = p_h + p_e \quad (12)$$

and

$$p_h = M_f g_z \quad (13)$$

then

$$p_e = M_f \ddot{k}_f - D + \rho_f V_s^0 (g_z + \ddot{k}_s). \quad (14)$$

At present we make no constitutive assumptions (like Darcy’s Law or others) regarding D . An important conclusion is that the pore-fluid pressure depends directly on the acceleration of the fluid, which is driven by the acceleration and changing configuration of the solid. Thus, to know how the excess fluid pressure is changing, we must understand the evolution of the solid configuration. This is the major task which we will come to directly.

We note that both \ddot{k}_s and \ddot{k}_f are defined positive upward which implies an increase in the reaction at the bottom. Because we conserve mass during the change in configuration we will certainly have that if \ddot{k}_s is an acceleration upwards, then \ddot{k}_f is the corresponding acceleration downwards, and vice versa. It is unlikely that the void space left by the moving solid will be filled instantaneously, meaning that in general $\ddot{k}_s \neq \ddot{k}_f$. The imbalance between accelerations is a measure of cavitation effects.

In the above derivation fluid-pore pressure is defined exclusively by bulk properties—the acceleration of center of mass. We have avoided any use of granular minutia (tortuosity, fluid pathways, grain sizes, grain size distributions, grain contacts, etc.) or constitutive assumptions.

By summing the forces N_s and N_f

$$N = N_s + N_f = M_s(g_z + \ddot{k}_s) + M_f(g_z + \ddot{k}_f) \quad (15)$$

we see that the drag and buoyancy forces cancel, whereas the inertial forces remain. By definition of the center of mass of the entire mixture,

$$Mk = M_s k_s + M_f k_f. \quad (16)$$

$$M\ddot{k} = M_s \ddot{k}_s + M_f \ddot{k}_f. \quad (17)$$

Therefore, by simple substitution

$$N = N_s + N_f = M(g_z + \ddot{k}). \quad (18)$$

We have thus written the total normal stress in terms of the acceleration of the total center of mass k . Clearly, this equation could have been stated directly, without considering the individual components.

The summation reveals a result of great practical importance for debris flow experiments: Measurements of the total normal force N will not provide any information

on drag or buoyant effects. These can only be captured by measuring the fluid-pore pressure [6]. A clear interpretation of fluid-pore pressures will be hampered by the fact that different physical processes are at play: drag, inertia, and buoyancy (solid fraction). The different effects cannot be identified in a single measurement.

The effective stress N_e is N_s as

$$N_e = N_s = N - N_f \quad (19)$$

In a steady flow, the effective stress is simply the buoyant weight of the solid. In a non-steady flow the solid stress includes the changing configuration \ddot{k}_s .

We are criticized by Iverson and George because we do not acknowledge the transient nature of their excess pore pressure equation. Clearly, we both agree on this point: the pore pressure evolves in time. The question, however, is what is the mechanism driving the transient behavior. In our approach the fluid-pore pressure evolves according to the change in configuration of the solid that excites both drag and inertial contributions. Because the inertial terms are a function of both the solid and the fluid accelerations, the excess fluid pressure will thus depend on the specific mass distribution. In Iverson and George, the excess pore pressure contains no dependency on the configurational inertia.

4 The transient nature of dilatancy

Energy in the form of frictional work is required to suspend the granular solid in the fluid. The buoyant effect of the fluid can be identified by rewriting the equation of the solid normal force N_s (Eq. 10) in a more convenient form,

$$N_s = (\rho_s^0 - \rho_f) V_s^0 (g_z + \ddot{k}_s) + D. \quad (20)$$

The advantage of this form is that it reveals the three primary processes at work: buoyancy, represented by the density difference $(\rho_s^0 - \rho_f)$; inertia, given by the acceleration \ddot{k}_s , and drag D .

The inertial term arises from the *energy input* from shearing. Shear work is either dissipated to heat (molecular random energy) or used to create velocity fluctuations (granular random energy). It is the interaction of the granular fluctuations with the hard basal boundary that leads to the acceleration of the solid center of mass and the dilation of the volume. This can be calculated by equating the work done to dilate the volume with the rate of energy input, which we denoted \dot{P}_V in our original paper,

$$\dot{N}_s + N_s \frac{\dot{k}_s}{k_s} = 2\dot{P}_V. \quad (21)$$

Because

$$\dot{N}_s = (\rho_s^0 - \rho_f)V_s^0(\ddot{k}) + \dot{D} \quad (22)$$

we see the result of the frictional working: It creates a jerk in the solid material, but, in comparison with a dry material, it is easier to suspend the solid because of the buoyancy. Of course, the granular fluctuations will be eventually dissipated to heat as well.

An inertia-based theory of debris flow dilatancy provides a simple explanation for solid surges and bulking. The larger the weight (N_s) in the volume, the more energy is required to dilate it, especially if the solid material is only partially submerged and buoyant effects can be neglected. This effect is apparent in Eq. 22: The contribution involving N_s reduces the dilatant acceleration; more energy \dot{P}_V is required to fluidize “heavier” volumes.

In the original paper, we have relied on thermodynamic partitioning arguments to constrain \dot{P}_V . Qualitatively, the source of dilatancy will always be some fraction of the frictional work rate. In the case of a debris flow, it should be a fraction of the frictional work rate in the solid. We would assume that the shear work in the fluid is dissipated entirely to heat. In this case there would be no velocity fluctuations in the fluid.

Finally, we emphasize that the inertial forces causing the dilatancy must be advected with the flow. The advection of the solid acceleration will cause transient excess fluid-pore pressures (and therefore effective stresses).

5 Conclusion

Because effective stress is used to describe the frictional resistance of debris flow motion, it is a key physical component of debris flow hazard modelling and mitigation. The application of effective stress concepts requires a method to calculate the evolution of fluid-pore pressure. Iverson and George base their calculation of the excess pore pressure on drag and diffusion processes arising during the dilation and contraction of the debris flow body; we base our calculation of the excess pore pressure on the production and advection of inertial accelerations arising from the frictional shear rate. These accelerations change the configuration of the solid mass. The configuration is geometrically represented by the center of mass and physically represented by the potential energy of the granular ensemble.

The location, velocity, and acceleration of the center of mass cannot be described by a collection of independent constitutive relations. Rather, precise kinematic relations exist that define how the solid configuration changes in

response to a given energy input from the frictional work rate. The change in the location of the center of mass of the solid is facilitated by buoyancy, but is resisted by internal drag forces. In this sense, pore-fluid pressure is analogous to dispersive pressure because it is the reaction to the dispersive pressure in the fluid. In a fluid-saturated debris flow, however, drag and buoyancy terms must be included in the time-dependent force balance that exists between the solid and fluid components.

A continual input of energy is required to maintain the particles in suspension; equilibrium states are given by the balance of energy input and dissipation, including drag and inelastic collisions between solid granules. Different equilibrium states exist for different roughness, slope angle, and solid-fluid properties.

We agree with Iverson and George that dispersive pressure is not a rheological property of a grain flow. We define it as the inertial force induced by shearing and resulting in dilation or contraction of the volume under consideration. As we have shown, the concept can be applied to both dry and fluid-saturated flows. The concept has been helpful to understand density variations in flowing avalanches [3] and the formation of powder snow and ice avalanches [2]. Whether these ideas will have practical significance in the modelling and mitigation of debris flows remains an open and yet unanswered question.

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