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Batch settling curve registration via image data modeling

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Abstract

To this day, obtaining reliable characterization of sludge settling properties remains a challenging and time-consuming task. Without such assessments however, optimal design and operation of secondary settling tanks is challenging and conservative approaches will remain necessary. With this study, we show that automated sludge blanket height registration and zone settling velocity estimation is possible thanks to analysis of images taken during batch settling experiments. The experimental setup is particularly interesting for practical applications as it consists of off-the-shelf components only, no moving parts are required, and the software is released publicly. Furthermore, the proposed multivariate shape constrained spline model for image analysis appears to be a promising method for reliable sludge blanket height profile registration.

Keywords: secondary clarifiers, separation processes, shape constrained splines, sludge blanket height, sludge settling, wastewater treatment

List of acronyms

Acronyms	Full expression
AO	Automated, on-line
QR	Qualitative representation
SBH	Sludge blanket height
SCS	Shape constrained splines
VO	Visual, off-line
VS, VS1, VS2	Visual, simultaneous
ZSV	Zone settling velocity

List of Symbols.

Symbol	Definition
β	All spline function coefficients
β_k	Coefficients of the k th spline function
θ, θ_k	Transitions (for function k)
Θ	Feasible set for θ
Ω	Feasible set for β
$\underline{b}, \bar{b}, \underline{b}_e, \bar{b}_e$	Interval boundaries (for episode e)
d	Derivative index
e	Episode index
f_k	k th function
$f_k^{(d)}$	d th derivative of the k th function
g	Objective function
j	Pixel index
$\hat{h}, \hat{h}_{AO}, \hat{h}_{VO}, \hat{h}_{VS1}, \hat{h}_{VS2}$	Sludge blanket height (SBH) estimates (AO/VO/VS1/VS2: see list of acronyms)
h_L, h_U	Physical height corresponding to the top/bottom (U/L) pixel
i	Image index
k	Data series index, color channel index
r_v	Parameter of the Vesilind equation
q	Flux
$s_{e,d+1}$	Sign for the d th derivative in the e th episode
u	Integration variable
v	Settling velocity

v_0	Parameter of the Vesilind equation
x, x_h	Pixel position (for the h th row in $\tilde{\mathbf{Y}}$)
$\mathbf{x}$	Independent data vector
$z, z_{AO}, z_{VO}, z_{VS}$	Sludge blanket height (SBH) sampling times (AO/VO/VS1/VS2: see list of acronyms)
$\mathbf{B}_k$	Basis matrix for f_k
D_k	Maximum considered derivative degree for the shape constraints applied to f_k
E	Number of episodes
I	Number of images in an experiment
J	Number of functions to fit
H	Number of rows in $\tilde{\mathbf{Y}}$
K	Number of data series to approximate
$\mathbf{S}$	Matrix describing the qualitative sequence, i.e. series of primitives
T	Number of transitions
$\mathbf{Y}$	Model estimates
$\tilde{\mathbf{Y}}$	Measurement matrix

12 1. Introduction

13 Settling is one of the key processes in activated sludge wastewater treat-
 14 ment plants (WWTPs, Ekama et al., 1997). Its primary function is the
 15 clarification of mixed liquor, thereby preventing wasting organic material
 16 and nutrients into the water bodies that receive the treated wastewater. In
 17 addition, settling results in the thickening of sludge thereby increasing the
 18 efficiency of biological conversion processes occurring in the reactor tanks
 19 of the WWTPs. Accurate design of settlers requires a proper characteriza-
 20 tion of the sludge settling properties. In addition, WWTP operation can be
 21 improved by avoiding overloaded settler conditions or increasing the reactor
 22 efficiency by *(i)* manipulation of the recycle flow rate (Balslev et al., 1994;
 23 Chen and Beck, 2001; Mines Jr et al., 2001), *(ii)* step feed (also: step aera-
 24 tion) and step sludge control (Chen and Beck, 2001), or *(iii)* short-term use
 25 of the reactor tanks for sludge sedimentation and storage. Settling also plays
 26 an essential role during the primary clarification process. Inefficient removal
 27 of suspended solids prior the biological treatment results in higher oxygen
 28 demand (for oxidizing organic pollution) and lower biogas production. For
 29 innovative technologies, such as granular sludge processes, settling governs
 30 the separation of the slow and fast settling biomass, i.e., the selection of the
 31 granules and the selective removal of flocs through excess sludge removal. A
 32 proper characterization of the settling properties of the sludge is thus nec-
 33 essary for a variety of separation processes that can be found on small and
 34 large WWTPs.

35 Different parameters are available to characterize the sludge settling prop-
 36 erties (van Loosdrecht et al., 2016): the sludge volume index (SVI), the di-

luted sludge volume index (DSVI), the stirred specific volume index at 3.5 min (SSVI3.5), etc. Unfortunately, such measures provide an incomplete description of the sludge settling properties (van Loosdrecht et al., 2016). A more detailed characterization of the sludge settleability is provided by sludge blanket height (SBH) profiles (van Loosdrecht et al., 2016). However, measuring such profiles is significantly more time-consuming than measuring sludge settling properties (SVI, etc.). Therefore, several empirical correlations were thus proposed to link the settling model parameters to the sludge settling measures that are easy to obtain. Such empirical correlations form the basis of today's practice, including control systems (e.g., Traoré et al., 2006) and despite known limits reported in several studies (Ozinsky and Ekama, 1995a,b; Bye and Dold, 1998).

A dynamic settling model commonly used in today's practice is the exponential model first proposed in Vesilind (1968). Still, several more detailed models have been built and studied (e.g., Cacossa and Vaccari, 1994; Plósz et al., 2007; Ramin et al., 2014; Li and Stenstrom, 2016). Critical to any study of settling behavior is the collection of SBH profiles registered during batch settling experiments as discussed above. All of the aforementioned studies rely on SBH measurements obtained by visual inspection of a settling column during the batch settling experiments. Typical use of SBH profiles may provide only one data point per experiment, e.g. when the zone settling velocity (ZSV) measurement corresponding to a single total suspended solids concentration is of interest only. As a result, collecting sufficient data to empirically describe the zone settling velocity and flux curves is a cumbersome and time-intensive task that can be afforded within research projects

62 but is rarely executed routinely on WWTPs, as already reported in Daig-
 63 ger and Roper Jr (1985). The potential of the developed settling models
 64 for optimization or control of WWTP operation is likely only realized if a
 65 routinely applicable yet inexpensive method for SBH registration is avail-
 66 able. The lack of an easy, quick, and reliable method for the measurement
 67 of batch settling curves is still one of the main limitations for both research
 68 and practice (Li and Stenstrom, 2014). We therefore focus on the problem
 69 of batch settling curve registration yet also demonstrate how the resulting
 70 batch settling profiles can be used for dynamic modeling.

71 Devices for automated SBH registration are available today (e.g., Van-
 72 rolleghem et al., 2006). However, they are likely too expensive to obtain
 73 and maintain for routine monitoring purposes. Methods to automate and/or
 74 advance SBH registration include (i) light intensity scanning (Vanrolleghem
 75 et al., 1996), (ii) measurement of a radioactive tracer (De Clercq et al., 2005),
 76 (iii) use of an ultrasonic transducer (Locatelli et al., 2015), and (iv) high-
 77 speed camera imaging (Mancell-Egala et al., 2016). The applicability of such
 78 techniques may remain limited unless (i) on-site use is feasible to avoid ef-
 79 fects of sample deterioration and (ii) the devices are easy to maintain by
 80 technical staff on typical wastewater treatment plants.

81 The main objective of our work is to produce, demonstrate, and validate
 82 a novel method for automated image-based SBH registration. Automated
 83 sample preparation is considered for future study. The proposed method for
 84 batch settling curve registration consists of (i) using an inexpensive off-the-
 85 shelf camera to collect images during multiple batch settling experiments
 86 and (ii) fitting a shape constrained spline (SCS) model extended for the

87 purpose of image analysis. The main advantage of our method is that the
 88 experimental method is accessible to researchers and practitioners as only
 89 inexpensive off-the-shelf equipment is used. A side benefit of applying the
 90 SCS model is that it enables use of multi-channel data present in the collected
 91 images and avoids differentiation of the noisy data during image analysis, in
 92 contrast to existing methods (e.g., Kim et al., 2011).

93 The proposed method exploits knowledge about the expected shape of
 94 the light intensity profile along the vertical dimension of the sludge column.
 95 Using shape information to characterize settling behavior is not new however.
 96 For example, it is known that the SBH profile obtained with conventional
 97 batch settling experiments with ideal suspensions is described as a convex
 98 profile, corresponding to a convex section of the solids flux curve section that
 99 governs such experiments. Similarly, recently proposed batch experiments
 100 deliver concave height profiles governed by concave sections of the solids
 101 flux curve. Knowledge about the shape is exploited in Diehl (2007); Bürger
 102 and Diehl (2013); Diehl (2015). Our work is different from these historical
 103 approaches in two ways. First, our method allows fitting functions which
 104 have a changing shape along their domain, as opposed to previous work. In
 105 our particular study, functions consisting of a concave segment followed by a
 106 convex segment are estimated. Secondly, the convex-concave shape enables
 107 explicit accounting of non-ideal behavior during experiments. Indeed, we
 108 study batch settling experiments where the effects of turbulence at the start
 109 of the experiments cannot be ignored. In what follows, the most important
 110 aspects of our method and the most significant results are explained.

111 2. Materials and Methods

112 2.1. Batch Settling Experiments

113 Nine batch settling experiments have been executed. Each of the settling
 114 experiments is executed for a dilution of a granular sludge sample obtained
 115 from a column sequencing batch reactor located at Eawag and fed with with
 116 low-strength municipal wastewater. Two such samples were taken directly
 117 from the reactor as source material for diluted sludge sample preparation.
 118 The sludge sample index and the applied dilutions for each of the experiments
 119 are given in Table 1. The total suspended solids concentration on the day
 120 of experimentation was 2.4 g/L. Throughout experimentation, a 2 L vertical
 121 glass cylinder is used. Each batch settling experiment was started by filling
 122 the glass cylinder with the (diluted) sludge and by ensuring homogeneity at
 123 the start of the experiment by means of manual stirring with a glass rod.

124 A scheme of the experimental set up can be seen in Fig. 1. During each
 125 experiment, images are taken by means of a digital camera (Camera 1, Canon
 126 PowerShot G9) equipped with a continuous power supply and modified with
 127 the Canon Hack Development Kit (CHDK, 2016) software to continuously
 128 capture images at intervals of 15s. Camera 1 was positioned so that (i)
 129 the height of the camera corresponds to the top of the sludge column, (ii)
 130 the central line of sight of the camera is directed at the top of the sludge
 131 column and (iii) the complete column is visible in the image. Within each
 132 experiment, the images are indexed with i ($i = 1, \dots, I$).

133 In experiments 5 and 6, two experimenters (experimenter 1 and 2) regis-
 134 tered the SBH by means of visual inspection of the settling sludge column at
 135 time intervals of 30s as conventional experimentation (van Loosdrecht et al.,

2016) and as close as possible to every 2nd image registration by camera 1. Results obtained by the experimenters are indicated with the subscripts v_{S1} and v_{S2} (visual, simultaneous). In addition, a second camera (Camera 2; Samsung Galaxy S5, Model No.: SM-G800F, OS: Android 4.4.2) was used to collect simultaneous close-up images of the sludge blanket at time intervals of 30s by means of frame lapse recording software (Framelapse by Neximo Labs, v2.1.1). Camera 2 was moved manually during each batch experiment by a third experimenter so to match the visually recognized SBH as close as possible. Results obtained with the close-up images are referred to by the subscript v_O (visual, off-line).

2.2. Shape Constrained Spline Function Fitting

The proposed image-based sludge blanket registration method consists of an extension of the pre-existing SCS method reported in Villez et al. (2013). Whereas the original method only allows analysis of univariate signals, the extended method permits simultaneous fitting of multiple SCS functions to multivariate data series. Each of fitted functions is however subject to the same shape constraints. Each part of the method SCS method is introduced generally followed by a discussion of details pertaining to the analysis of image data.

2.2.1. Modeled Data

General treatment. The measurements are given as an $J \times K$ matrix $\tilde{\mathbf{Y}}$ of which $y_{j,k}$ is the element in the j th row and k th column and $\tilde{\mathbf{Y}}_{:,k}$ is the k th column vector. Each row vector $\tilde{\mathbf{Y}}_{j,:}$ is associated with the j th element of $\mathbf{x}$ which is the $J \times 1$ vector containing the values of a single independent

160 variable.

161 *Application.* The analyzed data sets correspond to rectangular selections
 162 of red-blue-green images (see the *Supplementary Information*, Fig. S.1). A
 163 rectangular section of the image is selected so that the horizontal dimension
 164 of the selection covers the center of the photographed column and has the 200
 165 mL and 1000 mL marks on the column as limits in the vertical dimension.
 166 The width of the section is arbitrarily set to 51 pixels for all experiments.
 167 The heights of the image sections changed slightly across experiments and
 168 are reported in Table 1. All color channels (3) are included for analysis.

169 The data in each image are initially organized as a 3-D array with dimen-
 170 sions corresponding to the vertical image dimension, the horizontal image
 171 dimension, and the color channel. For the purpose of analysis, we consider
 172 each set of 51 light intensities corresponding to the same vertical position
 173 and color channel as repeated measurements of the same light intensity. To
 174 generate a 2-D matrix of the form $\tilde{\mathbf{Y}}$ the following *unfolding* procedure is
 175 executed. One first retrieves the matrix of light intensity values in the top
 176 row of pixels in the image and defines this matrix as $\tilde{\mathbf{Y}}$. One continues by
 177 selecting the same matrix for the second row and places this matrix below
 178 the previously obtained matrix. This concatenation process is continued un-
 179 til the bottom row pixels are reached and added. At this stage, the matrix
 180 $\tilde{\mathbf{Y}}$ is completed and has dimensions 51×3 . The corresponding vector $\mathbf{x}$ con-
 181 tains the row pixel index for each row in $\tilde{\mathbf{Y}}$. In the case of experiment 6,
 182 the dimensions of the matrix $\tilde{\mathbf{Y}}$ are $J = 1227 \times 51 = 62577$ and $K = 3$.
 183 The independent data vector ($\mathbf{x}$) contains the corresponding pixel positions,
 184 meaning that each of the 1227 vertical pixel positions appears 51 times within

185 $\mathbf{x}$.

186 2.2.2. Data Model and Definition of Optimality

187 *General treatment.* The multivariate data series are modeled by means of
 188 K spline functions, $f_k(\boldsymbol{\beta}, \mathbf{x})$ ($k = 1, \dots, K$) (Ramsay and Silverman, 2005).
 189 The choice for spline functions is especially motivated by the fact that shape
 190 constraints applied to non-empty intervals of a spline function domain can
 191 be formulated in a finite number of equality and inequality constraints (see
 192 e.g., Papp and Alizadeh, 2014; Villegz et al., 2013; Villegz and Habermacher,
 193 2016). The degrees of the spline functions are given as D_k . Internal spline
 194 knots determine where one polynomial segment ends and the next one starts.
 195 They can be placed in arbitrary locations within $[x_1, x_J]$. Because of our
 196 function choice, each function is linear in the spline coefficients (function
 197 parameters). More specifically, the spline function model generates estimates
 198 of the measured variables ($\mathbf{Y}$) given function parameters ($\boldsymbol{\beta}$) as follows:

$$\mathbf{Y}(\boldsymbol{\beta}) = \mathbf{f}(\boldsymbol{\beta}, \mathbf{x}) \quad (1)$$

199 with:

$$\begin{aligned} \mathbf{f}(\boldsymbol{\beta}, \mathbf{x}) &= \begin{bmatrix} f_1(\boldsymbol{\beta}_1, \mathbf{x}) & \dots & f_k(\boldsymbol{\beta}_k, \mathbf{x}) & \dots & f_K(\boldsymbol{\beta}_K, \mathbf{x}) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{B}_1(\mathbf{x}) \boldsymbol{\beta}_1 & \dots & \mathbf{B}_k(\mathbf{x}) \boldsymbol{\beta}_k & \dots & \mathbf{B}_K(\mathbf{x}) \boldsymbol{\beta}_K \end{bmatrix} \end{aligned} \quad (2)$$

$$\boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{\beta}_1^T & \boldsymbol{\beta}_2^T & \dots & \boldsymbol{\beta}_k^T & \dots & \boldsymbol{\beta}_K^T \end{bmatrix}^T \quad (3)$$

200 In the above, the matrices $\mathbf{B}_k(\mathbf{x})$ correspond to the evaluation of the
 201 spline basis of the k th function in the arguments $\mathbf{x}$.

202 The model is fitted to the data by minimizing the following least-squares
203 lack-of-fit:

$$g(\boldsymbol{\beta}) = \sum_{j=1}^J \sum_{k=1}^K \left(\tilde{\mathbf{Y}}_{j,k} - \mathbf{Y}_{j,k}(\boldsymbol{\beta}) \right)^2 \quad (4)$$

204 For primers on spline models, we refer to (Hastie et al., 2001) and (Ramsay
205 and Silverman, 2005).

206 *Application.* In the present study, three natural cubic ($K = 3$) B-spline
207 functions are fitted to three column vectors of $\mathbf{Y}$. Internal spline knots are
208 placed at every 8th pixel following the first pixel for all functions (i.e., x_9 ,
209 $x_{17}, \dots$). This knot placement was found to deliver sufficient flexibility to
210 the fitted functions while ensuring a reasonably short computational effort.
211 As both the knot locations and independent data vectors are the same for
212 each function, the matrices $\mathbf{B}_k(\mathbf{x})$ are the same for every function (i.e., $\mathbf{B} =$
213 $\mathbf{B}(\mathbf{x}) = \mathbf{B}_k(\mathbf{x}), k = 1, \dots, K$).

214 2.2.3. Shape Constraints

215 *General treatment.* During model fitting, the spline functions are con-
216 strained to have a predefined shape. The assumed shape can be derived from
217 expert knowledge or based on rigorous qualitative simulation (Kuipers, 1994;
218 Shaich et al., 2001; Bredeweg et al., 2009). In either case, the shape is defined
219 as a sequence of E episodes ($e = 1, \dots, E$). These episodes are contiguous
220 intervals of the function domain within which a number of the function's
221 derivatives do not change sign. Such a sequence is known as a *qualitative se-*
222 *quence*. It is defined mathematically as a matrix $\mathbf{S}$ with $\mathbf{S}(e, d + 1) = s_{e,d+1}$
223 specifying the sign of the d th derivative in the e th episode. The elements of $\mathbf{S}$

can taken on the integer values +1, 0, and -1 to indicate positive, zero, and negative signs of the derivatives. When the sign is unspecified, a question mark (?) is used instead. The matrix $\mathbf{S}$ is specified a priori. The episodes themselves are defined by $T = E - 1$ transitions, $\boldsymbol{\theta}$ ($\theta_t = \boldsymbol{\theta}(t)$; $t = 1, \dots, T$), which are the function argument values where the episodes meet and which need to be estimated. The complete description of the shape of a function by means of $\mathbf{S}$ and $\boldsymbol{\theta}$ is known as a *qualitative representation* (QR).

Application. The fitted functions are constrained to have a shape defined by two episodes. The first episode has a concave shape, i.e. a negative sign for the second derivative. The second episode has a convex shape and decreasing. Consequentially, one writes $\mathbf{S}$ as a matrix with two rows, one for each episode:

$$\mathbf{S} = \begin{bmatrix} ? & ? & -1 & ? \\ ? & -1 & +1 & ? \end{bmatrix}. \quad (5)$$

The corresponding QR thus exhibits a single transition which corresponds to the location of the inflection point between the two episodes: $\boldsymbol{\theta} = \theta = \theta_1$.

2.2.4. Optimization

General treatment. With the above definitions, the least-squares SCS function fitting problem is written mathematically as follows:

$$\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\beta}, \boldsymbol{\theta}} g(\boldsymbol{\beta}, \boldsymbol{\theta}) = \sum_{j=1}^J \sum_{k=1}^K \left(|Y_{j,k} - \mathbf{Y}(\boldsymbol{\beta})_{j,k}|^2 \right) \quad (6)$$

$$\text{s.t. } \boldsymbol{\beta} \in \Omega(\boldsymbol{\theta}, \mathbf{S}) \quad (7)$$

$$\boldsymbol{\theta} \in \Theta \quad (8)$$

236 and subject to the linear constraints Eq. 1–3. In the above, Θ is the
 237 set containing all feasible values for θ and $\Omega(\theta, \mathbf{S})$ is the set containing all
 238 values for β satisfying the shape constraints. Θ is defined mathematically as
 239 follows:

$$\begin{aligned}
 & \beta \in \Omega(\theta, \mathbf{S}) \\
 & \Updownarrow \\
 & \forall d = 0, \dots, D_k, \forall k = 1, \dots, K, \forall x \in [x_1, x_J] : \\
 & f_k^{(d)}(\beta_k, x) \begin{cases} \geq 0, & \text{if } \underline{b}_e \leq x \leq \overline{b}_e \wedge \mathbf{S}(e, d+1) = +1 \\ = 0, & \text{if } \underline{b}_e \leq x \leq \overline{b}_e \wedge \mathbf{S}(e, d+1) = 0 \\ \leq 0, & \text{if } \underline{b}_e \leq x \leq \overline{b}_e \wedge \mathbf{S}(e, d+1) = -1 \end{cases} \\
 & \underline{\mathbf{b}} = \begin{bmatrix} \underline{b}_1 & \underline{b}_2 & \dots & \underline{b}_e & \dots & \underline{b}_{E-1} & \underline{b}_E \end{bmatrix} \\
 & = \begin{bmatrix} x_1 & \theta_1 & \dots & \theta_{t-1} & \dots & \theta_{T-1} & \theta_T \end{bmatrix} \\
 & \overline{\mathbf{b}} = \begin{bmatrix} \overline{b}_1 & \overline{b}_2 & \dots & \overline{b}_e & \dots & \overline{b}_{E-1} & \overline{b}_E \end{bmatrix} \\
 & = \begin{bmatrix} \theta_1 & \theta_2 & \dots & \theta_t & \dots & \theta_T & x_J \end{bmatrix}.
 \end{aligned} \tag{9}$$

240 with $f_k^{(d)}(\cdot, u)$ the d th derivative of $f_k(\cdot, u)$ with respect to u .

241 The objective function (Eq. 7) is quadratic in β . The shape constraints
 242 (Eq. 9) are convex in β . In the case of univariate spline functions, as in this
 243 study, they can be formulated as a finite number of necessary and sufficient
 244 inequality constraints (Papp and Alizadeh, 2014). As a result, the problem
 245 has a single optimum and can be solved efficiently to global optimality given
 246 values for θ . Consequentially, the complete optimization problem can be
 247 solved as a nested optimization problem where the values for β are repeat-

edly obtained for considered candidate values for θ . The above problem is however non-convex and possibly multi-modal in θ . Still, globally optimal estimates for θ can be found by means of the branch-and-bound algorithm as used in Villez et al. (2013); Villez and Habermacher (2016). The bounding procedures and their proofs are similar to those presented in Villez et al. (2013); Villez and Habermacher (2016) and are given in the *Supplementary Information*. Importantly, the bounding gap does not converge to zero in the multivariate case, in contrast to results obtained for the univariate case ($K = 1$) studied in Villez et al. (2013). This situation is however similar to the case studied in Villez and Habermacher (2016), where the presence of discontinuous trends in univariate data series was explicitly accounted for. For more details we refer to the *Supplementary Information*.

Application. Cubic spline functions are chosen to fit to the data. The feasible set for the transition is the function domain, i.e. $\Theta := [x_1, x_J]$. Optimization of θ is continued until a tolerance of 1/8 of a pixel is achieved for the optimal position of the inflection point. This optimization is repeated for each image registered with camera 1 in each of the experiments. For a given experiment, the obtained value ($\hat{\theta}$) for image i is given as $\hat{\theta}_{\text{SCS},i}$.

2.3. Sludge Blanket Height Registration Methods

SBH estimates are obtained with four distinct methods. The first two methods are automated. A third method is based on off-line visual inspection of the close-up images of the sludge blanket. The fourth method consists of registering the SBH visually during the batch experiments by two experienced experimenters. More details follow next.

272 *2.3.1. Automated Sludge Blanket Height Registration with the Shape Con-*
 273 *strained Splines Method*

274 Following optimization as described above, the location of the inflection
 275 points (θ) are given as a vertical pixel position (direction: top-down) within
 276 the analyzed segment of the images. To obtain the SBH for image i in a given
 277 experiment, measured from the bottom of the glass column, the following
 278 linear expression is used:

$$\hat{h}_{\text{SCS},i} = h_L + (h_U - h_L) \cdot \left(1 - \frac{\hat{\theta}_{\text{SCS},i} - x_J}{x_1 - x_H} \right), \quad i = 1, \dots, I \quad (10)$$

279 with previously undefined parameters given in Table 1. The complete
 280 time series of SBH estimates is given as the vector $\hat{\mathbf{h}}_{\text{SCS}}$ and the corresponding
 281 sampling time vector as $\mathbf{z}_{\text{SCS}}$.

282 *2.3.2. Automated Sludge Blanket Height Registration with the Maximum Slope*
 283 *Method*

284 In Kim et al. (2011) an image analysis method for sludge blanket registra-
 285 tion is proposed and positively evaluated. For every pixel along the vertical
 286 column dimension, one computes a slope parameter, S_j , as the difference be-
 287 tween the light intensity at the considered pixel and the light intensity at the
 288 lowest pixel divided by the absolute distance between the considered pixel
 289 and the lowest pixel:

$$S_j = \frac{y_j - y_J}{|x_j - x_J|} \quad (11)$$

with y_j and y_J light intensities for a single colour channel obtained within a single column of pixels. The pixel j corresponding to a maximal slope is referred to as the knee in the light intensity profile and is identified as the sludge blanket height. Note the definition of such a knee is different from the definition of an inflection point. In Kim et al. (2011) this is executed with only one column of pixels in the recorded images and with the red color channel only. Following pixel location, the sludge blanket height is computed by linear interpolation as above ((10)). The maximum slope (MS) method is implemented as in Kim et al. (2011) except for the following modifications:

1. Instead of selecting one column of pixels, the light intensity data are averaged along the horizontal dimension prior to analysis. The pixel selection is the same as for the shape constrained spline method.
2. Instead of computing the slope for every pixel, the slope is only computed for the first 1250 pixels. Doing so, helps to avoid errors due to noise as will be demonstrated below.

The sludge blanket heights obtained with the MS method are reported as $\hat{h}_{\text{MS}}$ and the corresponding time instants as z_{MS} .

2.3.3. Visual Sludge Blanket Height Registration via Close-up Inspection after Experimentation

The third method to establish SBH estimates is based on a visual inspection of close-up images after the experiment is finished. The close-up images are used as a reference in what follows. The obtained SBHs are given as the vector $\hat{h}_{\text{VO}}$. The corresponding time instants are given as z_{VO} .

313 2.3.4. Conventional Sludge Blanket Height Registration during Experimenta- 314 tion

315 A 4th and 5th SBH estimate is obtained by means of a visual inspection
316 of the glass column during the batch experiment. These SBH estimates are
317 referred to as $\hat{h}_{VS1}$ and $\hat{h}_{VS2}$. The times of registration are the same for both
318 estimates and are given as z_{VS} .

319 2.4. Zone Settling Velocity Estimation

320 Each of the obtained SBH profiles can be used to model hindered and
321 compressed settling in detail as described in Torfs et al. (2016) and as also dis-
322 cussed in the introduction. Given our focus on SBH registration, we demon-
323 strate the utility of the method by computing the ZSV, which reflects on the
324 hindered settling only and is conceptually simpler compared to compressed
325 settling model identification procedures. The ZSV is computed by means of
326 locating the inflection point with negative tangent slope in the considered
327 SBH profile (e.g., Vanderhasselt and Vanrolleghem, 2000). Indeed, the shape
328 of the profile is known to consist of a downward concave episode followed by
329 a downward convex episode with the transition corresponding to the SBH.
330 The sign matrix $\mathbf{S}$ thus is:

$$\mathbf{S} = \begin{bmatrix} ? & - & - & ? \\ ? & - & + & ? \end{bmatrix} \quad (12)$$

331 To obtain the ZSV from the SBH estimates obtained via SCS-based im-
332 age analysis, the optimization problem described above (Eq. 7–9) is modified
333 as follows. The data model is changed so that the univariate vector contain-

334 ing the SBH profile are approximated with a single univariate cubic spline
 335 function with knots at every sampling time:

$$K = 1 \quad (13)$$

$$\tilde{\mathbf{Y}} = \tilde{\mathbf{Y}}_{:,1} = \tilde{\mathbf{y}}_k = \hat{\mathbf{h}}_{\text{SCS}} \quad (14)$$

$$\mathbf{x} = \mathbf{z}_{\text{SCS}} \quad (15)$$

336 All other settings are kept the same so that a least-squares fit of a SCS
 337 function to the SBH profile is obtained. Importantly, the modified optimiza-
 338 tion problem reduces to the univariate case studied in Villez et al. (2013).
 339 As a result, the best location of the inflection point can be determined with
 340 absolute precision and global optimality. Upon fitting the SCS function, the
 341 ZSV is obtained by computing the first derivative (tangent slope) in the in-
 342 flection point. The absolute tangent slope is reported as the ZSV. This is
 343 executed for every experiment.

344 The above computation of the ZSV is also executed for the SBH esti-
 345 mates obtained with off-line and simultaneous visual inspection by replacing
 346 the dependent data vector $\tilde{\mathbf{y}}_k$ with the data series containing the SBH es-
 347 timates ($\hat{\mathbf{h}}_{\text{VO}}$, $\hat{\mathbf{h}}_{\text{VS1}}$, $\hat{\mathbf{h}}_{\text{VS2}}$, $\hat{\mathbf{h}}_{\text{MS}}$) and the independent data vector with the
 348 corresponding image and SBH registration times. The ZSVs are obtained for
 349 experiments 5 and 6.

350 2.5. Data and Software

351 All data and software required to reproduce the results of this study are
 352 released publicly with a GPL v3 license and are added to the *Supplementary*
 353 *Information*.

3. Results

3.1. Demonstration of the Shape Constrained Splines Method for Automated Sludge Blanket Height Registration

Fig. S.1 shows a section of a single image obtained during experiment 5. The pixels selected for further analysis are indicated with yellow lines. Fig. 2 shows the corresponding light intensity measurement as a function of the pixel index for the three channels. While the data series exhibit considerable levels of noise, one clearly observes the sludge blanket as an inflection point in the data series. The inflection point corresponds to the discontinuity in the sludge concentration better known as the SBH. Solving the SBH estimation problem (Eq. 7–9) delivers three optimized SCS functions -one for each color channel- with the same concave-convex shape and the same location for the inflection point. The SCS functions and the pixel index corresponding to the identified sludge blanket (1056) are also shown in Fig. 2.

3.2. Demonstration of the Maximum Slope Method for Automated Sludge Blanket Height Registration

In the top panel of Fig. 3 one can see the average light intensity for the red color channel as a function of the pixel index for image considered above. The slope values are given in the bottom panel. One can see that selecting the pixel with the MS method results in the selection of the second last pixel at the bottom the image. This is caused by substantial noise amplification of the slope computation especially close to the reference pixel at the bottom of the image. The modified method considering the top 1225 pixels selects pixel 977, which is a more sensible choice. However, a human observer may

378 instead select pixel 1035 as the knee. This difference is again explained by
 379 noise amplification of the slope computation. However, even pixel 1035 is
 380 higher in the image compared to the shape constrained spline method result
 381 (1056). This is explained by the fact that the maximal slope method selects
 382 a location for a knee point rather than an inflection point.

383 3.3. Sludge Blanket Height Profiles

384 Fig. 4 shows a composite image obtained by collating the analyzed seg-
 385 ments of the images taken from the 127 consecutive images collected dur-
 386 ing experiment 5. The image is presented here without any modification,
 387 mainly to visualize the rather low contrast in the collected images. The pixel
 388 heights corresponding to the SCS-based inflection points ($\hat{\theta}_{\text{SCS}}$) and the MS
 389 knee ($\hat{\theta}_{\text{MS}}$) are also indicated in the collated image. Close inspection reveals
 390 that the identified inflection points correspond to the SBH at the front of
 391 the cylindrical column. One can see a semi-dark area above the identified
 392 inflection points. The top of the semi-dark area correspond to the back of
 393 the cylindrical column. The fact that the front and back side of the sludge
 394 blanket can be distinguished is a consequence of the applied position and
 395 angle of the camera. The MS knee pixel cannot be tied easily to any of the
 396 two sludge blanket features in the images. In addition, the MS knee profile
 397 appears to be more erratic than the profile of the inflection
 398 points. The collated images with and without SBH estimates obtained for
 399 all experiments are available in the *Supplementary Information*.

400 In experiments 5 and 6, all considered methods for SBH registration were
 401 applied. In Fig. 5 one can see the obtained SBH estimates. The SBH esti-
 402 mates obtained with close-up visual inspection and by simultaneous visual

inspection ($\hat{h}_{VO}$, $\hat{h}_{VS1}$, and $\hat{h}_{VS2}$) appear fairly close to each other. The SCS-based SBH estimates ($\hat{h}_{SCS}$) are about 70 mL higher in the concave episode (hindered and compressed settling) for experiment 5 and about 30 mL lower for experiment 6. For the MS method, the offset is +100 mL for experiment 5 and -5 mL for experiment 6.

Each of the obtained SBH profiles shown in Fig. 4 and Fig. 5 can be described as a decreasing trend that is composed of a concave episode followed by a convex episode. As in previous studies (e.g., Diehl, 2015), the concave episode is explained as a result from turbulence stemming from the stirring and possibly flocculation before the start of the experiment. Such behavior is generally considered non-ideal as the first data points do not provide information about the settling process. To account for this, one can model the effect of turbulence explicitly (e.g., Diehl, 2015) or manipulate the data to remove the concave episode entirely (e.g., De Clercq, 2006). In this work, we fit a shape constrained spline function to the SBH profiles with the desired concave-convex shape. The resulting functions for experiments 5 and 6 are shown in Fig. 5. The standard error for each SBH measurement profiles, obtained by taking the fitted curve with the close-up SBH profile ($\hat{h}_{VO}$) as a reference, are reported in Table 2. The reference curve fits the close-up SBH profile best, as expected since $\hat{h}_{VO}$ were used to fit the reference curve. The worst standard error is obtained by the MS method in both experiments ($\hat{h}_{MS}$). The best standard error, apart from the result with close-up data, is obtained with the data one human experimenter, which is however different in each experiment ($\hat{h}_{VS1}$, $\hat{h}_{VS2}$). In each experiment, the SCS-based SBH data ($\hat{h}_{SCS}$) leads to a standard error that is smaller than one of the standard

428 errors obtained by the human experimenters.

429 The curve-fitting is also executed for the other experiments by using the
 430 SBH profiles obtained the MS and SCS methods. Table 3 lists the obtained
 431 coefficients of determination (R^2). One can see that R^2 is above 0.995 in all
 432 cases except for the MS method, which delivers R^2 values as low as 0.235.
 433 This means that the SBH measurement profiles satisfy the expected concave-
 434 convex shape very well, except for the profiles obtained with the MS method.
 435 The corresponding SBH profiles ($\hat{h}_{MS}$) and the fitted curves are displayed in
 436 the *Supplementary Information*. Visual inspection allows to conclude that
 437 the MS method remains extremely sensitive to noise. Indeed, the MS method
 438 frequently identifies pixels that are close to the bottom of the image due
 439 to the high noise in the computed slopes. Several tests were executed to
 440 evaluate whether the considered set of pixels could be expanded or reduced
 441 (above/below 1225 pixels). However, in the 8th and 9th experiment the
 442 sludge blanket at the end of the experiment is located close to the 1225th
 443 pixel so that further decreases are difficult to motivate. At the same time,
 444 the results for experiment 1 show sensitivity to noise at the start of the
 445 experiment. Expanding the considered pixel selection makes things even
 446 worse. It is therefore impossible to define a single set of top-most pixels
 447 to be considered in the MS method in such a way that low sludge blanket
 448 levels can be identified while also avoiding errors due to noise amplification
 449 for pixels close to reference pixel at the bottom of the images. Given such
 450 poor performance, further analysis excludes results on the basis of the MS
 451 method.

452 3.4. Zone Settling Velocity Estimation

As indicated above, computing the ZSV is one way to usefully interpret the obtained SBH profiles. The curve fitting described above identifies the location of the inflection point at the concave-convex intersection of the SBH profile. As is typically assumed, the ZSV corresponds to the slope of the tangent in the inflection point located at the transition from the concave to the convex episode. In experiment 1 to 8, the obtained tangent lines appear sensible based on visual inspection (see Fig. 5 and the *Supplementary Information*). For experiment 9 the time needed for the transition from the zone settling phase to the compressed settling phase is extremely short which likely affects the accuracy of the estimated slope of the tangent line, as also discussed in van Loosdrecht et al. (2016).

All slopes of the identified tangent lines correspond to settling velocities and are shown in the top panel of Fig. 6. As expected, the obtained settling velocities follow a decreasing concave trend. In addition, the results obtained with visual estimates are generally consistent with the SCS-based results (maximum 27% relative difference). A conclusive validation would however requires additional samples. A fit of the Vesilind equation ($v(c) = v_0 e^{-r_v c}$, with c the sludge concentration, $v(c)$ the ZSV, and v_0 and r_v parameters) is shown as well and delivers an R2 value of 0.95. As an alternative, a fit of a rational equation (Eq. 28, Diehl, 2015) is also shown. This rational equation has three parameters and delivers an R2 value of 0.86. Both equations thus fit the data well and cannot not be discriminated easily. The bottom panel shows the corresponding settling flux ($q(c) = v(c) \cdot c$) which is typically used for the determination of the settling capacity of secondary settlers. *Note that the Vesilind and rational equations are designed to describe the settling*

behavior at relatively high concentrations where zone settling occurs. It is therefore not surprising that the curves have a different shape in the low concentration region.

4. Discussion

4.1. Main Results and Major Benefits of the Proposed Method

Image analysis was executed for the first time by means of a method for qualitative trend analysis, particularly on the basis of an SCS model. Furthermore, automated SBH registration is benchmarked for the first time against a pre-existing image analysis method for SBH registration and conventional SBH registration by human experimenters. Based on our results, several benefits of the proposed method when applied for image analysis taken during batch settling experiments have been demonstrated:

- The images were obtained with an off-the-shelf digital camera, all code is released publicly, and both experiments and image analysis can be executed in a standard laboratory environment. Consequentially, the method is accessible to many in the field, in contrast to alternatives which rely on equipment and software that is expensive to obtain and maintain.
- The SCS data model allows automatic SBH estimation based on light intensity profiles extracted from digital images recorded during batch settling experiments. As demonstrated, this is also possible despite the collection of rather noisy images. In contrast, the pre-existing maximum slope (MS) method is very sensitive to noise. Because of this, we

expect our method to fare well even in cases the supernatant remains turbid, e.g. when pin-point flocs are present.

- The SCS data model fits curves to the complete light intensity profiles and all color channels at once. Put otherwise, *(i)* all available information is incorporated in the image analysis, *(ii)* noise amplification due to differentiation is avoided, and *(iii)* information removal and biasing effects of data filtering are absent.
- The combined experimental and data-analytic method prevents human error and subjective analysis by automating the SBH registration via deterministic optimization. In contrast, conventional approaches may suffer from uncertainty stemming from subjectivity of human experimenters as well as variability of the exact method. Such variability may stem from the application of different practices in different regions, in individual wastewater treatment plants, by individual operators, and over time.

In its current form, our image-based analysis is considered attractive to academic experimenters primarily as a way to increase the efficiency of experimental data collection, possibly enabling the execution of measurement campaigns over long periods or with a high measurement frequency. It may also be useful in full-scale activated sludge WWTPs where an early-stage warning of deteriorating sludge settling properties is warranted. Importantly, routine application requires as much sample preparation as is necessary to obtain the diluted sludge settling index (DSVI) given that the time spent on sludge blanket registration with the human eye can be omitted.

At the same time, our results lead to acceptable but still considerable deviations between the results obtained with the SCS-method and those obtained with human-eye based SBH profiles. The number of experiments executed to develop and demonstrate the SCS-method are however too low to establish whether the observed deviations are of a systematic or random nature. In addition, it is unclear whether the larger errors should be expected in the human-eye SBH profiles or the SCS-based profiles. Our initial experiences suggest that human-eye SBH profiles can exhibit some lag during the time zone-settling dominates, especially when the sludge blanket is not defined well yet. Indeed, the fast-forming and fast-moving sludge blanket can be hard to track in time by the human eye. Regardless of such differences, the SCS-based method also offers the ability to obtain an objective SBH reading rather than a reading prone to human error and subjectivity.

4.2. Expanded Range of Qualitative Trend Analysis Applications

Historically speaking, qualitative trend analysis methods, including the original SCS method, were proposed to tackle extrapolation issues in fault diagnosis (see e.g. Maurya et al., 2007; Villez et al., 2013). Recent work expanded the range of applicability of such methods to fault detection in sequencing batch reactors (Villez and Habermacher, 2016), ammonia control (Thürlimann et al., 2015) and dynamic model identification (Masic et al., 2016b,a). SCS-based data modeling is especially valuable when models which are entirely mechanistic in nature are prohibitively expensive to obtain. The current study expands the application range of the SCS data model further into the field of image analysis and characterization of separation processes. Thus, our current results further demonstrate the general applicability of

550 qualitative trend analysis methods and SCS-based methods in particular.

551 4.3. Perspectives

552 Given the promising results in this study, several new questions can be
553 raised. The following topics are of primary interest:

- 554 1. Is the image analysis method robust enough to handle several sludge
555 types without further modification?
- 556 2. Can the image analysis method also be used for experimentation with
557 highly diluted sludges whose settling is of Stokesian nature?
- 558 3. Are the obtained SBH profiles useful for more complex modeling tasks
559 such as the joint modeling of hindered and compressed settling? This
560 may be the case but it is unclear yet whether the SCS-based SBH
561 profiles are of sufficient quality.
- 562 4. Can the time savings obtained by avoiding human-eye sludge blan-
563 ket reading be increased further by enabling the execution of multiple
564 simultaneous settling experiments and/or by providing a sludge sam-
565 ple preparation device that does not modify the flocculation state? If
566 possible, the SBH registration method combined with automated sam-
567 ple preparation will finally enable the evaluation of predictive control
568 strategies based on solid flux theory, including automated control of
569 the recycle flow rate, step feed flow rate, step sludge flow rates, and
570 temporary sedimentation in aeration tanks.

571 Based on current experience the authors are convinced that the answer to
572 each of the above questions is yes. However, further experimental evidence

is warranted. In any case, our experiments suggest that such experimental evidence can now be collected in an objective and time-efficient manner.

5. Conclusions

Automatic registration of the sludge blanket height in settling experiments is demonstrated to be feasible via image analysis. The image analysis procedure is based on a multivariate extension of the shape constrained spline method. Promising results were obtained with inexpensive equipment accessible to any laboratory. It is especially noteworthy that the shape constrained splines method appears fairly robust against large levels of noise and the obtained results compare fairly to conventional sludge blanket height registration methods. Most importantly, we consider our study the first step towards a fully automated, reliable, and economical alternative to existing methods for sludge blanket height registration.

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7. References

- Andersen, E. D., Andersen, K. D., 2000. The MOSEK interior point optimizer for linear programming: an implementation of the homogeneous algorithm. *High Performance Optimization* 33, 197–232.
- Balslev, P., Nickelsen, C., Lynggaard-Jensen, A., 1994. On-line flux-theory based control of secondary clarifiers. *Water Science and Technology* 30 (2), 209–218.
- Bredeweg, B., Linnebank, F., Bouwer, A., Liem, J., 2009. Garp3 – Workbench for qualitative modelling and simulation. *Ecological Informatics* 4(5), 263–281.
- Bürger, R., Diehl, S., 2013. Convexity-preserving flux identification for scalar conservation laws modelling sedimentation. *Inverse Problems* 29 (4), 045008.
- Bye, C. M., Dold, P. L., 1998. Sludge volume index settleability measures: effect of solids characteristics and test parameters. *Water Environment Research* 70 (1), 87–93.
- Cacossa, K. F., Vaccari, D. A., 1994. Calibration of a compressive gravity thickening model from a single batch settling curve. *Water Science and Technology* 30 (8), 107–116.
- CHDK, 2016.
URL <http://chdk.wikia.com>

- 617 Chen, J., Beck, M. B., 2001. Operational control of storm sewage at an
618 activated sludge process. *Water Science and Technology* 43 (7), 131–137.
- 619 Daigger, G. T., Roper Jr, R. E., 1985. The relationship between svi and ac-
620 tivated sludge settling characteristics. *Water Pollution Control Federation*
621 *Journal*, 859–866.
- 622 De Clercq, J., 2006. Batch and continuous settling of activated sludge: in-
623 depth monitoring and 1D compression modelling. Ph.D. thesis, Ghent Uni-
624 versity.
- 625 De Clercq, J., Jacobs, F., Kinnear, D. J., Nopens, I., Dierckx, R. A., De-
626 francq, J., Vanrolleghem, P. A., 2005. Detailed spatio-temporal solids con-
627 centration profiling during batch settling of activated sludge using a radio-
628 tracer. *Water Research* 39 (10), 2125–2135.
- 629 Diehl, S., 2007. Estimation of the batch-settling flux function for an ideal sus-
630 pension from only two experiments. *Chemical Engineering Science* 62 (17),
631 4589–4601.
- 632 Diehl, S., 2015. Numerical identification of constitutive functions in scalar
633 nonlinear convection-diffusion equations with application to batch sedi-
634 mentation. *Applied Numerical Mathematics* 95, 154–172.
- 635 Ekama, G. A., Barnard, J. L., Gunthert, F. W., Krebs, P., McCorquodale,
636 J. A., Parker, D. S., Wahlberg, E. J., 1997. Secondary settling tanks:
637 theory, modelling, design and operation. Scientific and technical report
638 no. 6. International Association on Water Quality, London, UK.

- 639 Hastie, T., Tibshirani, R., Friedman, J., 2001. The elements of statistical
640 learning. Data Mining, Inference, and Prediction. Springer, NY, USA.
- 641 Kim, Y. J., Choi, S. J., Bae, H., Kim, C. W., 2011. Sludge settleability
642 detection using automated sv30 measurement and its application to a field
643 wwtp. Water Science and Technology 64 (8), 1743–1749.
- 644 Kuipers, B. J., 1994. Qualitative Reasoning: Modeling and simulation with
645 incomplete knowledge. MIT Press, Cambridge, MA, USA.
- 646 Li, B., Stenstrom, M. K., 2014. Research advances and challenges in one-
647 dimensional modeling of secondary settling tanks—A critical review. Water
648 Research 65, 40–63.
- 649 Li, B., Stenstrom, M. K., 2016. Practical identifiability and uncertainty
650 analysis of the one-dimensional hindered-compression continuous settling
651 model. Water Research 90, 235–246.
- 652 Locatelli, F., François, P., Laurent, J., Lawniczak, F., Dufresne, M., Vazquez,
653 J., Bekkour, K., 2015. Detailed velocity and concentration profiles mea-
654 surement during activated sludge batch settling using an ultrasonic trans-
655 ducer. Separation Science and Technology 50 (7), 1059–1065.
- 656 Mancell-Egala, W. A., Kinnear, D. J., Jones, K. L., De Clippeleir, H., Takács,
657 I., Murthy, S. N., 2016. Limit of stokesian settling concentration charac-
658 terizes sludge settling velocity. Water Research 90, 100–110.
- 659 Masic, A., Srinivasan, S., Billeter, J., Bonvin, D., Villez, K., 2016a. Biokinetic
660 process model diagnosis with shape-constrained spline functions. In: 3rd

- 661 IWA Specialized International Conference Ecotechnologies for Wastewater
662 Treatment (ecoSTP16), Cambridge, UK, June 27-30, 2016, appeared on
663 USB stick.
- 664 Masic, A., Srinivasan, S., Billeter, J., Bonvin, D., Villez, K., 2016b. On the
665 use of shape-constrained splines for biokinetic process modeling. In: 11th
666 IFAC Symposium on Dynamics and Control of Process Systems, including
667 Biosystems (DYCOPS-CAB 2016), Trondheim, Norway, June 6-8, 2016,
668 appeared on USB-stick.
- 669 Maurya, M. R., Rengaswamy, R., Venkatasubramanian, V., 2007. Fault di-
670 agnosis using dynamic trend analysis: A review and recent developments.
671 Engineering Applications of Artificial Intelligence 20, 133–146.
- 672 Mines Jr, R. O., Vilagos, J. L., Echelberger Jr, W. F., Murphy, R. J., 2001.
673 Conventional and awt mixed-liquor settling characteristics. Journal of En-
674 vironmental Engineering 127 (3), 249–258.
- 675 MOSEK ApS, 2012. MOSEK Optimization Software for MATLAB, Version
676 6.0. Available from <http://www.mosek.com/>.
- 677 Ozinsky, A. E., Ekama, G. A., 1995a. Secondary settling tank modelling and
678 design. Part 1: Review of theoretical and practical developments. Water
679 S. A. 21 (4), 325–322.
- 680 Ozinsky, A. E., Ekama, G. A., 1995b. Secondary settling tank modelling and
681 design. Part 2: linking sludge settleability measures. Water S. A. 21 (4),
682 333–349.

- 683 Papp, D., 2011. Optimization models for shape-constrained function esti-
684 mation problems involving nonnegative polynomials and their restrictions.
685 Ph.D. thesis, Rutgers University, New Brunswick, NJ, USA.
- 686 Papp, D., Alizadeh, F., 2014. Shape-constrained estimation using nonnega-
687 tive splines. *Journal of Computational and Graphical Statistics* 23, 211–
688 231.
- 689 Plósz, B. G., Weiss, M., Printemps, C., Essemiani, K., Meinhold, J., 2007.
690 One-dimensional modelling of the secondary clarifier-factors affecting sim-
691 ulation in the clarification zone and the assessment of the thickening flow
692 dependence. *Water Research* 41 (15), 3359–3371.
- 693 Ramin, E., Wágner, D. S., Yde, L., Binning, P. J., Rasmussen, M. R.,
694 Mikkelsen, P. S., Plósz, B. G., 2014. A new settling velocity model to
695 describe secondary sedimentation. *Water Research* 66, 447–458.
- 696 Ramsay, J. O., Silverman, B. W., 2002. *Applied Functional Data Analysis:*
697 *Methods and Case Studies*. Springer, New York, NY, USA.
- 698 Ramsay, J. O., Silverman, B. W., 2005. *Functional Data Analysis*. Springer,
699 New York, NY, USA.
- 700 Shaich, D., Becker, R., King, R., 2001. Qualitative modelling for automatic
701 identification of mathematic models of chemical reaction systems. *Control*
702 *Engineering Practice* 9, 1373–1381.
- 703 The MathWorks Inc., 2014. *MATLAB Release 2014b*. Natick, MA, USA.

- Thürlimann, C. M., Dürrenmatt, D. J., Villez, K., 2015. Qualitative trend analysis for pH based soft-sensing of ammonia in full scale continuous WWTPs. Proceedings of the 2nd IWA New Developments in IT & Water conference Rotterdam, The Netherlands (February 8-10, 2015), appeared on USB-stick.
- Torfs, E., Locatelli, F., Balemans, S., Laurent, J., Vanrolleghem, P. A., Bürger, R., Diehl, S., François, P., Mosse, R., Nopens, I., 2016. Concentration-driven models revisited: Towards a unified framework to model settling tanks in wwtps. In: Proceedings 5th IWA/WEF Wastewater Treatment Modelling Seminar (WWTmod2016). Annecy, France, April 2-6, 2016. 109-118.
- Traoré, A., Grieu, S., Thiery, F., Polit, M., Colprim, J., 2006. Control of sludge height in a secondary settler using fuzzy algorithms. Computers and Chemical Engineering 30 (8), 1235–1242.
- van Loosdrecht, M. C., Nielsen, P. H., Lopez-Vazquez, C. M., Brdjanovic, D., 2016. Experimental methods in wastewater treatment. Water Intelligence Online 15 (9781780404752).
- Vanderhasselt, A., Vanrolleghem, P. A., 2000. Estimation of sludge sedimentation parameters from single batch settling curves. Water Research 34 (2), 395–406.
- Vanrolleghem, P., Van der Schueren, D., Krikilion, G., Grijspeerdt, K., Willems, P., Verstraete, W., 1996. On-line quantification of settling prop-

- 726 erties with in-sensor-experiments in an automated settlometer. Water Sci-
727 ence and Technology 33 (1), 37–51.
- 728 Vanrolleghem, P. A., Clercq, B. D., Clercq, J. D., Devisscher, M., Kinn-
729 ear, D. J., Nopens, I., 2006. New measurement techniques for secondary
730 settlers: a review. Water Science and Technology 53 (4-5), 419–429.
- 731 Vesilind, P. A., 1968. Design of prototype thickeners from batch settling tests.
732 Water Sewage Works 115, 302–307.
- 733 Villez, K., Habermacher, J., 2016. Shape anomaly detection for process moni-
734 toring of a sequencing batch reactor. Computers and Chemical Engineering
735 91, 365–379.
- 736 Villez, K., Venkatasubramanian, V., Rengaswamy, R., 2013. Generalized
737 shape constrained spline fitting for qualitative analysis of trends. Com-
738 puters and Chemical Engineering 58, 116–134.

Table 1: List of batch settling experiments. Experiments marked with (*) are those experiments for which visual registration of the sludge blanket height was performed by two human experimenters (during the experiment) and by visual inspection of close-up images (after the experiment).

Experiment index	Original sludge sample	Concentration g/L	Pixel height J
1	1	2.40 (no dilution)	1275
2	1	1.92	1282
3	1	1.54	1285
4	1	1.22	1285
5 (*)	2	1.20	1286
6 (*)	2	1.00	1287
7	2	0.86	1289
8	2	0.74	1279
9	2	0.58	1287

Table 2: Standard error (in mL) obtained by considered the deviations between the SBH measurements and the curve fitted to the SBH measurement obtained by close-up image inspection (VO).

Experiment index	Method				
	SCS	MS	VO	VS1	VS2
5	18.3	37.9	2.51	7.37	32.6
6	23.9	36.3	5.19	41.1	21.5

Table 3: Coefficients of determination (R^2) for the curves fitted to each of the sludge blanket height profiles.

Experiment index	Method				
	SCS	MS	VO	VS1	VS2
1	0.9999	0.5607			
2	0.9999	0.9972			
3	0.9999	0.8036			
4	0.9997	0.8795			
5	0.9998	0.9972	1.0000	1.0000	0.9975
6	0.9990	0.9989	0.9994	0.9998	1.0000
7	0.9993	0.8027			
8	0.9990	0.8580			
9	0.9954	0.2353

Figure 1: Scheme of the experimental setup. Camera 1 is positioned level to the top surface of the sludge sample. Camera 2 is adjusted manually during each experiment to take close-up images of the sludge blanket. Human inspection is executed in such a way that the lines of sight of the cameras are not interrupted.

Figure 2: Experiment 5 - Image 42. Dots: Light intensity data; Dashed white lines: Fitted shape-constrained spline functions; Full black vertical lines: Identified inflection point location. The shape constrained splines method located the inflection point in the light intensity data successfully and is consistent with a visual assessment of the sludge blanket height.

Figure 3: Experiment 5 - Image 42. (Top) Dots: Averaged red channel light intensity data; Dotted vertical line: Pixel location obtained with maximal slope method considering every pixel; Dashed vertical line: Pixel location obtained with maximal slope method considering only the first 1225 pixels; Full vertical line: Pixel location obtained via shape-constrained spline function fitting; Circles and connecting full line: vizualization of the maximal slope when considering only the first 1225 pixels. (Bottom) Dots: Computed slopes; Dotted vertical line: Pixel location obtained with maximal slope method considering every pixel; Dashed vertical line: Pixel location obtained with maximal slope method considering only the first 1225 pixels; Full vertical line: Pixel location obtained via shape-constrained spline function fitting; Circles and horizontal full line: vizualization of the maximal slope when considering only the first 1225 pixels. The maximum slope method delivers results that are fairly different from the result obtained with the shape constrained splines method.

Figure 4: Experiment 5. Composite image showing data from 127 consecutive images. Green arrows are used to indicate the sludge blanket at the front and back of the column recognized by close visual inspection. Yellow cross-hairs indicate the sludge blanket height estimates obtained by means of the shape constrained splines method and correspond to the front of the column. Red cross-hairs indicate the sludge blanket height estimates obtained by means of the maximum slope method. The red cross-hairs do not match any obvious feature in the image and follow an irregular pattern.

Figure 5: Registered batch settling curves: *Top*: Experiment 5; *Bottom*: Experiment 6. SCS-based sludge blanket height profiles (green dots, $\hat{h}_{SCS}$), MS-based sludge blanket height profiles (blue dots, $\hat{h}_{MS}$) and profiles based on visual registration (red, yellow, and purple dots; $\hat{h}_{VO}$, $\hat{h}_{VS1}$, and $\hat{h}_{VS2}$). A spline function with a concave-convex shape (full grey line) is fitted to the $\hat{h}_{SCS}$ data (SBH data - SCS). The tangent line in the inflection point of the shape constrained splines function is shown with a dashed black line. The modeling errors (grey circles) show that the curve fits the data well in both experiments.

Figure 6: Use of the ZSV to characterize dynamic sludge settling properties. *Top:* Zone settling velocity (ZSV) as a function of the sludge concentration. All ZSVs are shown together with a least-squares fit of the Vesilind and Diehl equations. The experiment number is added at the top of the image right above the corresponding solids concentration. *Bottom:* Settling flux curve obtained based on the fitted Vesilind and Diehl equations.

739 S. Supplementary Information

740 The Supplementary Information includes additional details regarding the
 741 experimental method (Section S.1), the SCS modeling method (Section S.2),
 742 additional figures (Section S.3), and all data and software to reproduce our
 743 results (separate .zip file).

744 S.1. Experimental protocol

745 The following description of a single experiment is added to ensure broad
 746 applicability of the SCS-based SBH registration method.

747 S.1.1. Hardware and consumables

748 Prior to the experiment, collect and prepare the following materials:

- 749 1. A 2 L clear glass cylinder with coloured tape added to mark the 200
 750 mL and 1000 mL levels. Take note of the distance between these two
 751 levels.
- 752 2. A white panel to place behind the 2 L cylinder.
- 753 3. An off-the-shelf digital camera, equipped with a continuous power sup-
 754 ply and programmed to continuously collect an image at a fixed time
 755 interval.
- 756 4. A diluted sludge sample of at least 2 L with known solids concentration.
- 757 5. A stirring rod

758 S.1.2. Protocol

759 Execute the following steps in the laboratory:

- 760 1. Place the camera, cylinder, and panel on a single line on a horizontal
 761 platform.

- 762 2. Position the camera so that the line-of-sight corresponds to a horizontal
763 line aligned with the 2000 mL mark on the cylinder.
- 764 3. ensure that the positions of the camera, cylinder, and panel do not
765 change during the experiment.
- 766 4. Start the image collection program on the camera.
- 767 5. Fill the cylinder with the 2 L of the diluted sludge sample.
- 768 6. Stir the sample with the stirring rod.
- 769 7. Stop stirring right before the recording of an image and record the time
770 stirring was halted.
- 771 8. Let the sludge settle for as long as considered necessary (e.g., 30 min-
772 utes).
- 773 9. Stop the image collection program on the camera.
- 774 10. Empty and clean the cylinder.

775 The following steps are executed after the experiment is executed.

- 776 1. Collect all images from the camera.
- 777 2. Select all images of interest starting with the image where stirring was
778 stopped first.
- 779 3. Select a section in the images corresponding to the center of the cylinder
780 and covering the 200 mL to 2000 mL range of the column.
- 781 4. Apply the SCS method to find the pixel corresponding to the sludge
782 blanket in an image section. Apply this to every image section.
- 783 5. Convert the sludge blanket pixels to a sludge blanket height (SBHs) by
784 linear interpolation.

785 *S.2. Description and proofs of applied bounding procedures*

786 In the next paragraphs, the necessary elements contributing to the glob-
 787 ally optimal solution of the SCS optimization problem discussed in the text
 788 are explained in detail. Additional symbols not used in the main text are
 789 given in Table S.1.

Table S.1: List of symbols used only within the Supplementary Information

Symbol	Definition
β^L	Lower bound values for of all spline function coefficients
β^U	Upper bound values for of all spline function coefficients
ϵ	Bounding gap tolerance
$\underline{\theta}_t$	Lower bound for θ_t
$\overline{\theta}_t$	Upper bound for θ_t
θ_{QP}	Upper bound solution for θ
Θ_l	l th considered set for θ during optimization
$\underline{g}$	Lower bound to the objective function
$\overline{g}$	Lower bound to the objective function

790 The complete optimization problem is written as follows:

$$\hat{\beta}, \hat{\theta} = \arg \min_{\beta, \theta} g(\beta, \theta) = \sum_{j=1}^J \sum_{k=1}^K \left(\tilde{Y}_{j,k} - Y_{j,k}(\beta) \right)^2 \quad (\text{S.1})$$

$$\text{s.t. } \forall k = 1, \dots, K :$$

$$Y(\beta) = f(\beta, x_k) \quad (\text{S.2})$$

$$\beta \in \Omega(\theta, S) \quad (\text{S.3})$$

$$\theta \in \Theta \quad (\text{S.4})$$

791 *S.2.1. Solving for β*

792 The SCS function fitting problem discussed in the main text is a pseudo-
 793 convex program as long as values for the transitions (θ) are fixed and known.
 794 Depending on the applied sign constraints and the exact objective function,
 795 the problem can be reduced to a semi-definite program, a second order cone
 796 program, or even a quadratic program (QP). This is discussed at length in
 797 Papp (2011); Villez et al. (2013); Papp and Alizadeh (2014).

798 *S.2.2. Solving for β and θ*

799 The original problem described in the main text requires simultaneous
 800 optimization of the transitions θ . This is a nonlinear problem. However, and
 801 similar to prior work, this kind of problem can be solved to global optimal-
 802 ity in a deterministic manner by means of the branch-and-bound algorithm
 803 (Villez et al., 2013). To this end, the algorithm repeatedly divides the set
 804 of feasible values for θ (Θ) into smaller subsets until convergence. We refer
 805 to the l th generated subset during algorithm execution as Θ_l ($\Theta_l \subset \Theta$). For
 806 each subset, a lower and upper bound value to the objective function is com-
 807 puted. These bounds allow ignoring branches in the resulting solution tree

during the remainder of the optimization algorithm execution as soon as it is guaranteed that those branches cannot include the global optimum. The exclusion of such branches from the algorithmic search is known as *fathoming*. In what follows, the bounding procedures enabling such fathoming are explained and proven.

S.2.2.1. Step 1: Finding a feasible solution for θ . Consider a candidate solution set, Θ_l . Any such set corresponds to a hyper-rectangular set within the feasible solution space and can be described completely as follows:

$$\theta \in \Theta_l \Leftrightarrow \forall t \in \{1, 2, \dots, T\} : \underline{\theta}_t \leq \theta_t \leq \overline{\theta}_t \quad (\text{S.5})$$

with $\underline{\theta}_t$ and $\overline{\theta}_t$ describing the interval containing the considered values for θ_t .

In addition, each feasible solution within this set satisfies the following order relationship:

$$\forall t \in \{1, 2, \dots, T-1\} : \theta_t \leq \theta_{t+1} \quad (\text{S.6})$$

A practical way to propose a feasible solution is obtained by solving the following QP subject to the above conditions (Eq. S.5–S.6):

$$\min_{\theta} \sum_{t=1}^T (\theta_t - \underline{\theta}_t)^2 + (\theta_t - \overline{\theta}_t)^2 \quad (\text{S.7})$$

The solution, if the problem is feasible, is further referred to as θ^{QP} . If the set defined by Eq. S.5–S.6 is empty, one cannot find a feasible solution. This case is dealt with separately.

825 *S.2.2.2. Step 2a: No feasible solution available.* When no feasible solution
826 for θ can be found, the bounding procedures are trivial.

827 **Procedure.** In this case, the bounding procedures are very straight-
828 forward. As in prior work, both the upper bound ($\overline{g(\Theta_l)}$) and lower bound
829 ($\underline{g(\Theta_l)}$) are set to $+\infty$:

$$\underline{g} = \underline{g(\Theta_l)} = \bar{g} = \overline{g(\Theta_l)} = +\infty \quad (\text{S.8})$$

830 **Proof.** The proof of these bounds is straightforward. Indeed, if no
831 feasible solution can be found θ , then there no solution can be found with
832 any objective function value lower than $+\infty$. This automatically also defines
833 the upper bound at the same value. This concludes the proof.

834 *S.2.2.3. Step 2b: A feasible solution is found.* Computing the upper and
835 lower bounds is more involved when a feasible solution for θ , namely θ^{QP} ,
836 has been found.

837 **Upper bound – Procedure.** An upper bound value for the objective
838 function is computed by replacing θ with the proposed solution (θ^{QP}) in the
839 original problem (Eq. S.1–S.4). This means the following problem is now
840 solved:

$$\hat{\beta}^U = \arg \min_{\beta} \overline{g(\beta)} = \sum_{j=1}^J \sum_{k=1}^K \left(\tilde{Y}_{j,k} - Y_{j,k}(\beta) \right)^2 \quad (\text{S.9})$$

$$\text{s.t. } \forall k = 1, \dots, K :$$

$$Y(\beta) = f(\beta, x_k) \quad (\text{S.10})$$

$$\beta \in \Omega(\theta^{\text{QP}}, S) \quad (\text{S.11})$$

841 This problem is again at least pseudo-convex and can thus be solved to
 842 deterministic global optimality by means of interior-point algorithms. The
 843 corresponding vector containing all spline coefficients is referred to as $\hat{\beta}^U$.
 844 The resulting objective function is an upper bound to the objective function:

$$\exists \theta \in \Theta_l, \exists \beta \in \Omega(\Theta_l, S) : g(\beta, \theta) \leq \overline{g} = \overline{g(\hat{\beta}^U)} \quad (\text{S.12})$$

845 **Upper bound – Proof.** Any feasible solution, including the computed
 846 pair $(\beta^U, \theta^{\text{QP}})$, corresponds to an upper bound as its existence automatically
 847 implies that at least one solution exists which delivers an objective function
 848 value which is the same or lower value than the computed one, $g(\beta^U, \theta^{\text{QP}})$.
 849 As such, this proves the validity of the computed upper bound.

850 **Lower bound – Procedure.** A lower bound can be computing by
 851 means of the following relaxation of the problem. For the considered subset
 852 Θ_l , one applies only those sign constraints which are applied universally for
 853 all solutions θ within the set Θ_l . Practically, the original problem is relaxed
 854 by replacing the instances of θ_t with either $\underline{\theta}$ or $\overline{\theta}$ as follows:

$$\hat{\beta} = \arg \min_{\beta} \underline{g(\beta)} = \sum_{j=1}^J \sum_{k=1}^K \left(\tilde{Y}_{j,k} - Y_{j,k}(\beta) \right)^2 \quad (\text{S.13})$$

$$\text{s.t. } \forall k = 1, \dots, K :$$

$$Y(\beta) = f(\beta, x_k) \quad (\text{S.14})$$

$$\beta \in \Omega^L(\underline{\theta}, \bar{\theta}, S) \quad (\text{S.15})$$

855 where $\Omega^L(\underline{\theta}, \bar{\theta}, S)$ is the relaxed feasible set for β , which is defined as
856 follows:

$$\beta \in \Omega^L(\underline{\theta}, \bar{\theta}, S) \quad \Updownarrow \quad (\text{S.16})$$

$$\forall d = 0, \dots, D_k, \forall e = 1, \dots, E, \forall t = 1, \dots, T, \forall k = 1, \dots, K :$$

$$f_k^{(d)}(\beta_k, x) \begin{cases} \geq 0 & \text{if } \underline{b}_e \leq x \leq \bar{b}_e \wedge s_{e,d+1} = +1 \\ = 0 & \text{if } \underline{b}_e \leq x \leq \bar{b}_e \wedge s_{e,d+1} = 0 \\ \leq 0 & \text{if } \underline{b}_e \leq x \leq \bar{b}_e \wedge s_{e,d+1} = -1 \end{cases}$$

$$\begin{aligned} \mathbf{b}^L &= \begin{bmatrix} b_1^L & b_2^L & \dots & b_e^L & \dots & b_{E-1}^L & b_E^L \end{bmatrix} \\ &= \begin{bmatrix} x_1 & \bar{\theta}_1 & \dots & \bar{\theta}_{t-1} & \dots & \bar{\theta}_{T-1} & \bar{\theta}_T \end{bmatrix} \\ \mathbf{b}^U &= \begin{bmatrix} b_1^U & b_2^U & \dots & b_e^U & \dots & b_{E-1}^U & b_E^U \end{bmatrix} \\ &= \begin{bmatrix} \underline{\theta}_1 & \underline{\theta}_2 & \dots & \underline{\theta}_t & \dots & \underline{\theta}_T & x_H \end{bmatrix} \end{aligned}$$

857 This relaxed problem is again pseudo-convex and can thus be solved to
858 deterministic global optimality by means of interior-point algorithms. The

859 obtained spline function parameters are referred to as β^L . The resulting
860 objective function is a lower bound to the objective function:

$$\forall \theta \in \Theta_l, \forall \beta \in \Omega(\Theta_l, S) : \underline{g} = \underline{g}(\beta^L) \leq g(\beta, \theta) \quad (\text{S.17})$$

861 **Lower bound – Proof.** Because the applied constraints in the modi-
862 fied lower bounding problem are always applied for any particular choice of θ
863 for the original problem, one can write that the feasible set for β in the lower
864 bound case, $\Omega^L(\underline{\theta}, \bar{\theta}, S)$, includes the feasible set for any feasible proposal for
865 θ for the original problem:

$$\forall \theta \in \Theta_l : \Omega(\theta, S) \subseteq \Omega^L(\underline{\theta}, \bar{\theta}, S) \quad (\text{S.18})$$

866 Given that the objective function and remaining constraints remain un-
867 changed in the lower bound procedure, it holds that this procedure leads to
868 a proven lower bound. This proves the validity of the lower bound.

869 *S.2.2.4. Bounding gap.* In a number of special cases, it can be shown that
870 the lower bound solution will deliver the globally optimal solution within
871 a considered set, Θ_l . This is only possible when the considered intervals
872 defining Θ_l do not contain any spline basis knot inside their boundaries.
873 Furthermore, this is only guaranteed when the transitions correspond only
874 to changes in the signs of derivatives which are piece-wise linear or piece-wise
875 quadratic in the function's argument. In the case of cubic spline functions,
876 as used in this work, this corresponds to inflection points (2nd derivative is
877 piece-wise linear) and extrema (1st derivative is piece-wise quadratic). This

878 was demonstrated in Villez et al. (2013) for the univariate case ($J = 1, K =$
 879 1). This property of the optimization problems means that the bounding gap
 880 during branch-and-bound optimization becomes zero in a finite number of
 881 steps, leading to absolute precision of the reported globally optimal solution.
 882 This property also holds for the extended SCS model studied in this work,
 883 however only when the number of considered spline functions is 1 ($J = 1,$
 884 without proof). No restrictions are required for the number of measured
 885 variables (K , without proof). In the general case ($J \geq 1$), an ϵ -optimal
 886 solution can be found in a finite number steps, with ϵ an arbitrary small
 887 strictly positive number.

888 *S.2.2.5. Discontinuous trends.* Locally discontinuous trends are not consid-
 889 ered explicitly in this study, unlike ?. To allow the fitting of SCS functions
 890 with discontinuities one only needs to apply the additional relaxations of the
 891 optimization problem discussed in ? to the multivariate case studied here.
 892 This leads again to a valid lower bound (without proof). The upper bound
 893 provided in this work remains valid in its current form (without proof). Even
 894 though such adjustments are not studied in detail in this work, they are im-
 895 plemented within the provided software toolbox for SCS function fitting.

896 *S.3. Additional figures*

897 *S.3.1. Setup*

Figure S.1: Experiment 6 - Image 42. This image is registered with camera 1. The yellow rectangle indicate the selected area for analysis. Camera 2 is visible at the bottom of the image left of the column.

898 *S.3.2. Experiment 1*

Figure S.2: Experiment 1 – Composite image.

Figure S.3: Experiment 1 – Composite image with indications of the sludge blanket height identified via shape constrained spline fitting.

Figure S.4: Experiment 1 – Batch settling curve, inflection point, and tangent line obtained with the SCS and MS method for SBH registration.

899 *S.3.3. Experiment 2*

Figure S.5: Experiment 2 – Composite image.

Figure S.6: Experiment 2 – Composite image with indications of the sludge blanket height identified via shape constrained spline fitting.

Figure S.7: Experiment 2 – Batch settling curve, inflection point, and tangent line obtained with the SCS and MS method for SBH registration.

900 *S.3.4. Experiment 3*

Figure S.8: Experiment 3 – Composite image.

Figure S.9: Experiment 3 – Composite image with indications of the sludge blanket height identified via the SCS (yellow) and MS (red) method.

Figure S.10: Experiment 3 – Batch settling curve, inflection point, and tangent line obtained with the SCS and MS method for SBH registration.

901 *S.3.5. Experiment 4*

Figure S.11: Experiment 4 – Composite image.

Figure S.12: Experiment 4 – Composite image with indications of the sludge blanket height identified via the SCS (yellow) and MS (red) method.

Figure S.13: Experiment 4 – Batch settling curve, inflection point, and tangent line obtained with the SCS and MS method for SBH registration.

902 *S.3.6. Experiment 5*

Figure S.14: Experiment 5 – Composite image.

Figure S.15: Experiment 5 – Composite image with indications of the sludge blanket height identified via the SCS (yellow) and MS (red) method.

Figure S.16: Experiment 5 – Batch settling curves, inflection points, and tangent lines obtained with all SBH profiles.

903 *S.3.7. Experiment 6*

Figure S.17: Experiment 6 – Composite image.

Figure S.18: Experiment 6 – Composite image with indications of the sludge blanket height identified via the SCS (yellow) and MS (red) method.

Figure S.19: Experiment 6 – Batch settling curves, inflection points, and tangent lines obtained with all SBH profiles.

904 S.3.8. Experiment 7

Figure S.20: Experiment 7 – Composite image.

Figure S.21: Experiment 7 – Composite image with indications of the sludge blanket height identified via the SCS (yellow) and MS (red) method.

Figure S.22: Experiment 7 – Batch settling curve, inflection point, and tangent line obtained with the SCS and MS method for SBH registration.

905 *S.3.9. Experiment 8*

Figure S.23: Experiment 8 – Composite image.

Figure S.24: Experiment 8 – Composite image with indications of the sludge blanket height identified via the SCS (yellow) and MS (red) method.

Figure S.25: Experiment 8 – Batch settling curve, inflection point, and tangent line obtained with the SCS and MS method for SBH registration.

906 *S.3.10. Experiment 9*

Figure S.26: Experiment 9 – Composite image.

Figure S.27: Experiment 9 – Composite image with indications of the sludge blanket height identified via the SCS (yellow) and MS (red) method.

Figure S.28: Experiment 9 – Batch settling curve, inflection point, and tangent line obtained with the SCS and MS method for SBH registration.

include 3 to 5 bullet points (maximum 85 characters, including spaces, per bullet point)

Highlights:

- A new experimental method to register sludge blanket profiles is demonstrated
- Qualitative trend analysis is used for image analysis for the first time
- The applied qualitative trend analysis model is based on shape constrained splines
- The use of only inexpensive hardware and open software makes the method accessible
- The method is compared favorably against pre-existing methods