Phosphorus model of lake eutrophication

Dieter M. Imboden

Swiss Federal Institute for Water Resources and Water Pollution Control (EAWAG),
Swiss Federal Institutes of Technology, Dübendorf

Abstract

A two-box lake model is presented with the subsystems epilimnion (E) and hypolimnion (H) and phosphorus as the limiting nutrient factor. The mean O_2 consumption in H as a function of P loading is calculated and critical P-loading figures above which the lake turns toward eutrophy are given for varying lake mean depth z and hydraulic loading factor q . The results agree with an empirical relation between z and P loading given by Vollenweider. In addition, P retention factors are calculated using the model and compared to measured values.

Eutrophication of lakes represents one of the main problems of water pollution control. In most lakes nitrogen or phosphorus is the limiting nutrient factor. Increasing population density, the use of fertilizer in intensive agriculture, and the phosphorus content of detergents have raised the N and P concentrations in lakes and thereby their biological productivity. A quantitative study of this effect has to take into account not only biological and chemical but also physical processes (transport phenomena) within the water system.

Box models are useful and still mathematically simple tools for this kind of investigation. The choice of the subsystems (boxes) should correspond to regions of homogeneous chemical and biological conditions. The model then describes the temporal variation of the relevant concentrations (model functions) within each box depending on a set of model parameters which themselves can be different in each box. The external parameters are the flushing coefficient and the loading of the relevant substances; the internal parameters are the biochemical reaction rates (especially rate of photosynthesis), the mixing ratio between the different boxes, the sedimentation rate, and the exchange of matter at the water-sediment interface. Of course, these factors undergo seasonal variation. However, as an approximation it is possible to choose constant mean values for them for certain periods such as summer stagnation and winter circulation (method of temporally constant parameters).

In principle, all these chemical concentrations are interdependent through various biochemical reactions, making the mathematical treatment of the model highly complicated. However, in the case of a dominant controlling nutrient factor it is possible to decompose the system of equations. Furthermore, with a stoichiometric formula of the Redfield type (Redfield et al. 1963; Stumm 1964), the variation of other substances (e.g. O_2) can be calculated as a consequence of the P values. As Thienemann (1928) has pointed out, the analysis of O_2 concentration gives important information about the trophic condition of lakes.

Various workers have treated the nutrient balance using a one-box model for the lake. Vollenweider (1969, in press) has summarized the results, which in their most advanced form lead to an equation connecting mean nutrient concentrations to a few relevant lake parameters. However, disregarding the vertical variation of nutrient concentrations, as is done in the one-box model, may produce oversimplified and misleading results. For instance, within the frame of the one-box model the mean residence time of a substance has to be smaller than that of the water if the sediments serve as a sink for this substance (Vollenweider 1969). In fact, as pointed out by O'Melia (1972) the real situation is more complicated, depending on the grade of vertical inhomogeneity and the exchange rate between different depths.

The first step behind the one-box model

Fig. 1. Nutrient model for a lake with phosphorus as limiting nutrient factor. λ and π represent mean concentrations of phosphate-P and particulate-P. The indices E, H, and "in" represent epilimnion, hypolimnion, and inflow. Q_{in} and Q_{out} are inflow and outflow of water per unit time; g is velocity of sedimentation of particulate-P; Q is exchange of water between epilimnion (E) and hypolimnion (H) per unit time; S is exchange of phosphate at sediment-water interface per unit area and unit time; Ph is photosynthesis; and M is mineralization or respiration.

is made by the distinction between zones of photosynthesis (trophogenic zone) and zones of mineralization (tropholytic zone). Since during stratification these zones coincide fairly well with the epilimnion (E) and hypolimnion (H), the expressions E and H are used for the two subsystems.

In Fig. 1 a two-box model is presented in which phosphorus is the only limiting nutrient factor. This assumption itself cannot be proved within the model, but has to be elaborated by additional considerations. Within certain boundaries the P concentrations then do not depend on other concentrations.

The total P concentration $[P_t]$ is separated into two components

$$[P_t] = \lambda + \pi. \quad (1)$$

λ represents the dissolved P available for bioproduction (thus $\lambda \sim [PO_4-P]$), π the particulate-P. The question of the attribution of dissolved organic phosphorus is left open, as its role in organic production seems to be still unrevealed and rather of the order of a correction term (Gächter 1968; Lean 1973). Furthermore, it is assumed that the π component mainly consists of organic matter, although an exact inter-

pretation would include all P which is incorporated in or adsorbed to particulate matter.

For the rate of mineralization (or respiration) a first order equation is chosen

$$(d\pi/dt)_{Min} = -(d\lambda/dt)_{Min} = -R\pi, \quad (2)$$

where the rate coefficient R has the dimension time⁻¹. R is a function of temperature as a consequence of Van't Hoff's law. It changes considerably if the lake becomes anaerobic. The range of aerobic conditions can be calculated within the model.

The rate of photosynthesis depends on both the biomass concentration and the nutrient concentration. However, if P really is the limiting nutrient factor, the linear equation

$$(d\lambda/dt)_{Ph} = -(d\pi/dt)_{Ph} = -\alpha\lambda \quad (3)$$

is a useful approximation (Imboden 1973). It signifies that within the trophogenic layer practically all the available phosphate is steadily consumed independently of the actual biomass concentration. This is at least true for most lakes during summer and corresponds to a recent investigation by Lean (1973). The rate coefficient α can be determined by various field measurements (e.g. Gächter 1968).

From a biological point of view the above assumptions may look rough or even incorrect. Of course, one could replace equations 1 to 3 by more complicated ones; e.g. splitting $[P_t]$ into three components or choosing a more complex model of photosynthesis. However, such alterations would result in an increase of variables and system parameters. The aim of the model presented here is to describe a large class of lakes with a few common parameters in order to compare them in particular ways, and its assumptions therefore represent a compromise between simplicity and reality. Of course, the modeling of a specific lake would justify the introduction of more refined equations.

The model results in a system of four coupled linear differential equations for the two epilimnetic and hypolimnetic concentrations:

Table 1. Model parameters and their range of variation.

	Summer/ Stagnation	Winter/ Circulation	Unit
<u>External parameters</u>			
$q_s = Q_{in}/A$: Hydraulic loading (Q_{in} : Water inflow per unit time; A: Lake Surface)			
Extreme cases: Lake Tahoe	0.001		$m\ day^{-1}$
Zürichsee (Lake Zurich)	0.1		
$L_t = L_\lambda + L_\pi$: P-loading per lake surface and unit time (L_λ , L_π : Loading of dissolved and par- ticulate-P, respectively)			
Extreme cases: Lake Tahoe	0.1		$mg\ P\ m^{-2}\ day^{-1}$
Bodensee (Lake Constance)	11		
<u>Internal parameters</u>			
$\xi = Q/V_E$: Exchange coefficient (Q : Flux of water between E and H per unit time; V_E : Volume of E)	0.001-0.01	0.05-0.5	day^{-1}
g : Velocity of sedimentation	0.1-0.4	0.1-0.4	$m\ day^{-1}$
α : Rate coefficient of photosynthesis	1-2	0.06-0.3	day^{-1}
R : Rate coefficient of mineralization Hypolimnion R_H	0.003-0.02	0.003-0.02	day^{-1}
Epilimnion R_E^*	$3\ R_H$	R_H	
S : Exchange of phosphate at sediment- water interface †	0.27 ‡ up to 500 §		$mg\ P\ m^{-2}\ day^{-1}$

* If the temperature difference between E and H is 12°C in summer and 0 in winter.

† S is positive if the flux goes out of the sediments into the water.

‡ Theoretical estimate by Stumm and Leckie (1971).

§ Measurements of Mauensee, a very small and highly eutrophic Swiss lake (Internal Report of Swiss Federal Institute for Water Resources and Water Pollution Control, Dübendorf, Zürich, 1971).

$$\begin{aligned}
 d\lambda_E/dt &= -\lambda_E(q_s/z_E + \xi + \alpha) \\
 &\quad + \lambda_{II}\xi + \pi_E R_E + L_\lambda/z_E; \\
 d\lambda_{II}/dt &= \lambda_E(\xi/\epsilon) - \lambda_{II}(\xi/\epsilon) \\
 &\quad + \pi_{II} R_{II} + S/z_H; \\
 d\pi_E/dt &= \lambda_E\alpha - \pi_E(q_s/z_E + \xi + R_E + g/z_E) \\
 &\quad + \pi_{II}\xi + L_\pi/z_E; \\
 d\pi_{II}/dt &= \pi_E(\xi/\epsilon + g/z_{II}) \\
 &\quad - \pi_{II}(\xi/\epsilon + g/z_{II} + R_{II}). \quad (4)
 \end{aligned}$$

The system 4 can be solved numerically or analytically, although the latter leads to very complicated expressions.

The definition of the model parameters q_s , ξ , α , R_E , R_{II} , L_λ , L_π , and g is given in Table 1. z , z_E , z_{II} are the total, epilimnetic, and hypolimnetic mean depths, $V = V_E + V_{II}$ the corresponding volumes, and A is the lake surface: $V = Az$. Since the aim of this work is mainly to compare different lakes, the number of morphometric parameters is reduced by introducing an idealized form of the lake which results in the equations $z = z_E + z_{II}$ and $z_{II}/z_E = V_{II}/V_E (= \epsilon)$, i.e. the surface of the epilimnion is assumed to be equal to the "surface" of the hypolimnion. Consequently, there is no di-

rect epilimnion-sediment interface, and direct phosphate exchange between E and the sediments is neglected in the first equation of 4. The hydraulic characterization of the lake is uniquely given by the loading q_s , and the total and the epilimnetic mean depth, z and z_E . In modeling a single lake, equation 4 could easily be rewritten for a general lake morphology.

A detailed mathematical analysis of the model (steady state concentrations, relaxation time, determination and sensitivity of parameters) is presented elsewhere (Imboden 1973). In connection with the eutrophication problem we are interested in the following questions.

(a) How long does the system take to reach the steady state during the summer?

(b) What is the hypolimnetic oxygen consumption corresponding to this steady state?

(c) To what degree does the circulation period destroy the lake's "memory" of the summer period?

As will be shown, the model allows us to answer the first two questions, but the third one requires a more refined knowledge of mixing processes within the lake.

Concerning (a), I have shown (Imboden 1973) that in summer the concentrations π_E and π_H approximately move to their steady state, π_E^0 and π_H^0 , in the same way as the function

$$x(t) = e^{-(t-t_0)/\tau} \{x(t_0) - x^0\} + x^0 \quad (5)$$

moves from its initial state $x(t_0)$ at time t_0 to its steady state x^0 for $t \rightarrow \infty$. The exponential relaxation is characterized by the relaxation time τ after which the function reaches its steady state x^0 within 37% of the original deviation $\{x(t_0) - x^0\}$.

The relaxation times, $\tau(\pi_E)$ and $\tau(\pi_H)$, for π_E and π_H are given approximately by

$$\begin{aligned} \tau(\pi_E) &= (q_s/z_E + g/z_E)^{-1} = z_E/(q_s + g) \\ \tau(\pi_H) &= (R_H + g/z_H)^{-1}. \end{aligned} \quad (6)$$

For most lakes $q_s \ll g$, so that the upper limit of $\tau(\pi_E) \sim z_E/g \sim 50$ days is valid. For deep lakes $\tau(\pi_H)$ is approximated by $1/R_H = 50$ to 300 days; for shallower lakes $\tau(\pi_H)$ is smaller. Since in summer α is

much larger than the other parameters such as ξ , q_s/z_E , and R_E , the steady state value of λ_E is approached together with π_E and π_H . Note that the value of α does not have a significant influence on the concentrations as long as α is great. Finally, λ_H only approaches its steady state very slowly, depending on the exchange rate ξ between E and H. In the extreme case $\xi = 0$, λ_H does not have a finite upper limit.

(b) As first pointed out by Hutchinson (1938) and Deevey (1940), a relation should exist between hypolimnetic oxygen consumption and epilimnetic processes (bioproduction). In the steady state, the rate of mineralization in H per unit volume is given by the input of particulate-P from E into H, namely (π^0 means steady state)

$$(d\pi/dt)_{Min} = \pi_E^0 \cdot g/z_H + (\pi_E^0 - \pi_H^0)\xi/\epsilon, \quad (7)$$

if we assume that the organic matter which is lost to the sediments is completely mineralized there as well. The last equation of 4 yields the following relation between π_E^0 and π_H^0 :

$$\pi_H^0 = \frac{\xi/\epsilon + g/z_H}{\xi/\epsilon + g/z_H + R_H} \times \pi_E^0. \quad (8)$$

Together with the averaged stoichiometric relation (π and O_2 in mg m^{-3})

$$(d[O_2]/dt)_{Min} = 140(d\pi/dt)_{Min} \quad (9)$$

we finally get the oxygen consumption in H per unit volume as a function of π_E^0 :

$$\left(\frac{d[O_2]}{dt} \right)_{Min} = -140 \times \pi_E^0 \left[\frac{g}{z_H} + \frac{\xi R_H}{\xi + g/z_E + \epsilon R_H} \right]. \quad (10)$$

Assuming that $S = 0$ (which means that phosphate is neither adsorbed nor redissolved at the sediment-water interface), the only inhomogeneous terms in equation 4 are proportional to the specific phosphorus input L_A and L_P . Therefore, the steady state concentrations also have to be proportional to L_A and L_P . In fact, as can be shown, these values mainly depend on L_t and are insensitive to the partition of the P input between phosphate and particulate form. One can write

Fig. 2. Tolerable annual phosphorus loading per lake area $L_{t,max}$ as a function of mean depth z for different hydraulic loading factors q_s . Comparison of theoretic $L_{t,max}$ for oligotrophy with the empirical curve by Vollenweider (1968). Choice of model parameters: $\alpha = 1 \text{ day}^{-1}$; $g = 0.2 \text{ m day}^{-1}$; $z_E = 10 \text{ m}$; $R_H = 0.02 \text{ day}^{-1}$; $R_E = 3 R_H$; $\xi = 0.001 \text{ day}^{-1}$. In addition, two extreme cases with $q_s = 0.02 \text{ m day}^{-1}$ are plotted to demonstrate the sensitivity due to the variation of R and ξ . Curve A: $R_H = 0.02 \text{ day}^{-1}$, $\xi = 0.01 \text{ day}^{-1}$; Curve B: $R_H = 0.003 \text{ day}^{-1}$, $\xi = 0.001 \text{ day}^{-1}$. The points show the present condition of the lake, the end points of the arrow the theoretic $L_{t,max}$. Abbreviation for the lakes as in Table 2.

$$\pi^0_E = fL_t, \quad (11)$$

where π^0_E can be calculated from an analytical expression (Imboden 1973) containing all the model parameters. Thus, we get f by normalizing π^0_E by the corresponding loading L_t . As can be shown, the mineralization rates R_H and R_E are the only internal parameters to which f is really sensitive: by shifting R_H from 0.02 day^{-1} to 0.003 (and doing the same for R_E using $R_E = 3R_H$) we decrease f by 30 to 50% depending on q_s and z .

Replacing the time dependent π_E by its steady state value π^0_E and assuming a mean

duration of the stagnation period T_{st} (~ 180 days), we get an average oxygen depletion in H at the end of the stagnation period:

$$\Delta[O_2] = 140T_{st}f \left[\frac{g}{z_H} + \frac{\xi R_H}{\xi + g/z_E + \epsilon R_H} \right] \times L_t, \quad (12)$$

which is proportional to the total specific P-loading L_t . The various model parameters enter into 12 both directly and through the factor f . Given an upper limit for the tolerable O_2 consumption $\Delta[O_2]_{max}$ for an oligotrophic lake, we get the corresponding up-

per limit $L_{t,\max}$ above which a lake starts to show eutrophic characteristics:

$$L_{t,\max} = \Delta[\text{O}_2]_{\max} \times \left\{ 140T_{st} f \left[\frac{g}{z_{\text{H}}} + \frac{\xi R_{\text{H}}}{\xi + g/z_{\text{E}} + \epsilon R_{\text{H}}} \right] \right\}^{-1}. \quad (13)$$

In Fig. 2 $L_{t,\max}$ is plotted as a function of z and q_s for fixed internal parameters and $\Delta[\text{O}_2]_{\max} = 1,000 \text{ mg m}^{-3}$. The resulting curves show surprisingly good agreement with Vollenweider's (1968) empirical relation between mean depth and tolerable P loading. Although the choice of $\Delta[\text{O}_2]_{\max}$ and with it the absolute values of $L_{t,\max}$ may seem arbitrary, the theoretical and empirical variations of $L_{t,\max}$ with z show a similarity which cannot be reached with a simple one-box model. Regarding the relatively low value of $\Delta[\text{O}_2]_{\max}$ compared to the O_2 saturation, we have to take into account the effectively inhomogeneous O_2 depletion within the hypolimnion, which normally has its maximum at the bottom of the lake and may produce anaerobic zones even with a low mean O_2 deficit.

In addition to the mean depth z , I have introduced the hydraulic loading q_s as a second factor determining the P tolerance of a lake. The result seems to agree with experimental knowledge. For instance, Lake Tahoe with a mean depth of 300 m, which would have a high $L_{t,\max}$ in the pure L_t/z diagram, in this model has a very low P tolerance (in accordance with observations) due to its extremely low $q_s = 0.001 \text{ m day}^{-1}$. Vollenweider (in press) has come to a similar conclusion by introducing a modified diagram in which he plots $L_{t,\max}$ as a function of q_s instead of z .

The P-retention factor R_P is defined by the relative amount of P which is lost to the sediments. Under steady state conditions the P outflow of the lake per unit time is given by $Q_{\text{out}} (\lambda^0_{\text{E}} + \pi^0_{\text{E}})$ with $Q_{\text{out}} = Q_{\text{in}}$; the P input is $A \times L_t$ (A : lake surface). Thus,

$$R_P = 1 - (q_s/L_t)(\lambda^0_{\text{E}} + \pi^0_{\text{E}}). \quad (14)$$

R_P decreases with increasing hydraulic loading q_s and (through the variation of

λ^0_{E} and π^0_{E}) with increasing mean depth z , but is independent of the total P-loading L_t , as λ^0_{E} and π^0_{E} themselves are proportional to L_t . From a mathematical point of view this result is an obvious consequence of the linear system 4, just as the steady state concentrations are proportional to L_t , as mentioned in equation 11.

Of course, equation 14 is only valid for steady state conditions and for $S = 0$. With increasing L_t phosphate redissolution at the sediment-water interface may produce a breakdown of the retention capacity of the lake; R_P may even become negative. As an important conclusion it follows that the constancy of R_P with varying L_t reflects a normal adaptation of the lake to changing external conditions, while a change in R_P signalizes a basic alteration in the nutrient balance of the lake.

In Table 2, calculated and measured retention factors of a few lakes are compared. Bodensee (Lake Constance) is the only lake with a theoretical value significantly lower than the measured one. Since the corresponding theoretical π^0_{E} is very high (128 mg m^{-3}) and above measured concentrations, the steady state represents a poor approximation in this case. Three lakes (Baldeggersee, Hallwilersee, Greifensee) show lower measured than theoretical values. For Baldeggersee, Vollenweider (1968, p. 79 and table 4.7) has computed from experimental data that redissolution of P from the sediments, which may even exceed the P input from the river, may occur during periods on the order of several months. The breakdown of the P retention capacity seems to be even more pronounced for Hallwilersee.

The theory of maximum tolerable depletion of O_2 is only reasonable if the hypolimnetic O_2 reserves are completely renewed during the circulation period, i.e. if the annual O_2 deficits do not accumulate from one year to the next. Thus, we are left with question (c) about the "memory" of the lake.

Obviously, a detailed answer is not possible within the frame of this two-box model for the simple reason that the hypolimnion

Table 2. Phosphorus retention factors and P loading of some lakes (abbreviations are for use with Fig. 2). Data from Vollenweider (1968).

Lake and abbreviation	q_s (m day^{-1})	z (m)	Retention factor (%)		P loading ($\text{mg m}^{-2} \text{ year}^{-1}$)		$L_{\text{act}} / L_{\text{tol}}$	State*
			theory	measured	actual	tolerable		
Aegerisee A	0.019	50	67	68	160	170	0.94	O
Baldeggersee B	0.015	34	79	61	1,800	130	13.8	E
Constance Co	0.058	90	27	65	4,000	480	8.3	E
Greifensee G	0.037	19	72	62	1,600	100	16.0	E
Hallwilersee H	0.019	21	83	36	550	92	6.0	E
Lake Geneva L	0.036	154	26	20	700	470	1.5	O/M
Pfaffikersee P	0.033	18	74	77	1,360	100	13.6	E
Sempachersee	0.0074	46	94	--	770	250	3.1	E
Türlersee T	0.018	14	84	80	300	70	4.3	M/E
Zürichsee Z	0.098	44	31	25	1,320	370	3.6	E
Lake Tahoe Ta	0.001	300	86	93	40	160	0.25	O
Vänern V	0.001	36	99	--	150	90	1.7	O/E
Zellersee Ze	0.038	37	65	--	1,200	180	6.7	E

* O-oligotrophic, M-mesotrophic, E-eutrophic.

is not in fact homogeneous. Therefore the circulation of a lake cannot be simply described by increasing the exchange parameter ξ , as the exchange may for example be great at the epilimnion-hypolimnion interface (~ 10 m) but practically zero in a depth of 100 m in a meromictic lake. However, as the aim of this model is only to synthesize a general rough behavior for lakes characterized by a few parameters such as L_t , z , and q_s , it cannot be expected to describe single events within a lake (e.g. temporal algal blooms) nor all kinds of lakes (e.g. amictic lakes where transport of dissolved substances only occurs by diffusion processes).

The model may be extended by replacing the hypolimnetic box by a one-dimensional diffusion model for O_2 consumption as a function of time and depth during stratification (Imboden 1973). In the same way the renewal of O_2 during circulation could be calculated, to give a more detailed answer about the maximum O_2 depletion capacity of the lake. However, we cannot expect this kind of extended model to be applicable in a general way, with a few fixed parameters, to a group of different lakes. Instead, it could describe indi-

vidual properties due to specific morphometric and meteorological conditions, such things as the eddy diffusion coefficient as a function of depth and season that has recently been calculated for the Zürichsee by Li (1973), using monthly temperatures measured over 10 years.

This kind of investigation lies beyond the aim of the general P model presented here. However, from our box-model calculation we are at least able to draw the following qualitative conclusions concerning the memory and the rate of response of a lake to changes in nutrient input:

1. A lake, oscillating between two (or more) annual states, each described by a different set of model parameters, reaches the corresponding steady states faster if its mean depth is small and its hydraulic loading factor is large. In this case its memory does not persist over the seasonal changes. The response to changed input factors (mainly P loading) is felt within 1 year in its P concentration values.

2. A deep lake may have a memory in its hypolimnion consisting of both slow approach to steady state and incomplete renewal of its oxygen reserves. Its persistence against increase of nutrient input

may keep the lake oligotrophic for a certain time. The inverse effect would slow down a desired response to reduction of nutrient input.

3. Both the shallow (quickly responding) and deep (inert) lake may in addition have an important and long lasting memory in the sediments. Even after lowering the P input, once the lake is anaerobic for part of the year, P could be returned to the water as a consequence of redissolution out of the sediments until the sediment surface is depleted allowing a new (aerobic) state.

References

- DEEVEY, E. S. 1940. Limnological studies in Connecticut. Part 5. *Am. J. Sci.* **238**: 717-741.
- GÄCHTER, R. 1968. Phosphorhaushalt und planktische Primärproduktion im Vierwaldstättersee (Horwer Bucht). *Schweiz. Z. Hydrol.* **30**: 1-66.
- HUTCHINSON, G. E. 1938. On the relation between oxygen deficit and the productivity and typology of lakes. *Int. Rev. Gesamten Hydrobiol.* **36**: 336-355.
- IMBODEN, D. M. 1973. Limnologische Transport- und Nährstoff-Modelle. *Schweiz. Z. Hydrol.* **35**(1): in press.
- LEAN, D. R. S. 1973. Phosphorus dynamics in lake water. *Science* **179**: 678-680.
- LI, Y. H. 1973. Diffusion in lakes. *Schweiz. Z. Hydrol.* **35**(1): in press.
- O'MELIA, C. R. 1972. An approach to the modeling of lakes. *Schweiz. Z. Hydrol.* **34**: 1-34.
- REDFIELD, A. C., B. H. KETCHUM, AND F. A. RICHARDS. 1963. The influence of organisms on the composition of seawater, p. 26-77. In M. N. Hill [ed.], *The sea*, v. 2. Interscience.
- STUMM, W. 1964. Discussion (Methods for the removal of phosphorus and nitrogen from sewage plant effluents by G. A. Rohlich), p. 216-229. In W. W. Eckenfelder [ed.], *Advances in water pollution research*. Proc. 1st Int. Conf., London, 1962, v. 2. Pergamon.
- , AND J. O. LECKIE. 1971. Phosphate exchange with sediments: its role in the productivity of surface waters, p. III-26/1-16. In S. H. Jenkins [ed.], *Advances in water pollution research*. Proc. 5th Int. Conf., San Francisco and Hawaii, 1970, v. 2. Pergamon.
- THIENEMANN, A. 1928. Der Sauerstoff im eutrophen und oligotrophen See. *Binnengewässer* **4**.
- VOLLENWEIDER, R. A. 1968. Water management research. OECD—Rep. 68.27. Paris. 159 p.
- . 1969. Möglichkeiten und Grenzen elementarer Modelle der Stoffbilanz von Seen. *Arch. Hydrobiol.* **66**: 1-36.
- . In press. Input-output models of lakes. In W. Stumm and K. Wuhrmann [eds.], *Proc. Kastanienbaum Conf.*, April 1972.

Submitted: 21 June 1973

Accepted: 30 November 1973