Geophysical Research Letters

RESEARCH LETTER

10.1029/2020GL087832

Key Points:

- Sedimentation of 3-D snow particles, obtained from snow microtomography and a phase field model, is computed in still air in Stokes regime
- Particles fall in preferred orientations either drifting or spiraling, drag and rotation torque coefficient are modeled with particle sphericity
- Terminal velocity in still air can be modeled with sphericity and provides effective dynamics and mean settling velocity in turbulence

Supporting Information:

- Supporting Information S1
- Table S1
- Movie S1
- Movie S2

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Citation:

Zeugin, T., Krol, Q., Fouxon, I., & Holzner, M. (2020). Sedimentation of snow particles in still air in stokes regime. *Geophysical Research Letters*, 47, e2020GL087832. <https://doi.org/10.1029/2020GL087832>

Received 5 MAR 2020

Accepted 24 JUN 2020

Accepted article online 8 JUL 2020

Sedimentation of Snow Particles in Still Air in Stokes Regime

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Abstract We compute the resistance matrices of realistic 3-D snow particle shapes obtained from microcomputed tomography data of snowpack samples and a phase field model. Using these resistance matrices, we calculate the sedimentation of the particles in still air in Stokes regime. We find that particles attain preferred orientations and the mode of motion is either drifting or spiraling. Simple laws, relating average drag and rotation torque coefficients to the particles' sphericity, are established which lead to a new formulation of a snow particle's average terminal velocity. The presented models are valid in Stokes regime in still air, corresponding to particle sizes up to $\sim 100 \mu\text{m}$ but can reasonably be extended to moderately higher Reynolds numbers ($\sim 1 \text{ mm}$) and used for estimating mean settling velocity in turbulent conditions. Though not all relevant atmospheric conditions are covered, our study provides the basis for addressing more complex conditions in the future.

Plain Language Summary How snow particles move through air has an important impact on a number of physical problems that affect the distribution and redistribution of snow on the earth surface. Snow particle motion also influences the lifetime of clouds and with that is an important factor for Earth's climate. Even though real snow particles feature complex shapes, their falling behavior is predominantly studied using simple geometries such as spheres or spheroids. In this study we use realistic, three-dimensional geometries of snow particles and study their sedimentation in still air. We find that the particles eventually fall with a preferred orientation and either spiral or drift. We test simple laws that allow us to describe snow particles sedimentation based on their shape.

1. Introduction

The motion of snow particles in atmosphere is a fundamental process for many physical phenomena, for example, collisions (Shaw, 2003), aggregation (Westbrook et al., 2006), or snow particle growth and sublimation (Pruppacher & Klett, 1978). The presence of snow crystals within clouds significantly influences cloud properties, among others, the albedo (e.g., Yang et al., 2015). Snow particle terminal velocity, u_∞ , also determines the lifetime of cirrus clouds (Sanderson et al., 2008). Given that snow clouds are a main contributor to Earth's energy balance (e.g., Harrison et al., 1990; Stephens et al., 2012), current climate models are highly sensitive to the fallout rate of snow particles in cirrus clouds (Mitchell et al., 2008). Therefore, an accurate description of snow particles' behavior in clouds, especially the reliable prediction of terminal velocities, is paramount to estimate climate scenarios.

Distribution and redistribution of snow on the earth surface is influenced by snow particles' motion in air. For example, Aksamit and Pomeroy (2016) demonstrate complex motion of blowing snow during saltation. Sommer et al. (2018) show that snow redistribution due to saltation and subsequent sublimation processes of snow particles in turbulent conditions impact the mass balance of the Antarctic snow cover. Current snow transport models used to predict snow distribution may be improved by more accurate description of the interaction between air and snow particles (Aksamit et al., 2017).

The motion of snow particles suspended in air is governed by gravity and external hydrodynamic forces dictated by the dynamics of the atmosphere. In general, determination of the hydrodynamic forces (drag and lift) and torques (pitching and rotation torques) acting on a particle (for an overview see Zastawny et al., 2012) is a nonlinear problem that requires solving the full Navier-Stokes equations of the fluid flow around the particle. For small relative velocities (Reynolds number $Re \ll 1$) the flow perturbation by the particle can

be described by Stokes flow (Happel & Brenner, 1983). In Stokes flow the nonlinear term of the Navier-Stokes equations is neglected. As a result the hydrodynamic forces and torques acting on a particle are determined from a linear combination of relative particle velocity, particle angular velocity, and the so-called particle resistance matrix consisting of the translation, coupling, and rotation resistance tensors (Brenner, 1967). The translation and rotation resistance tensors describe the resistance of the particle to the respective motion, whereas the coupling tensor characterizes the interplay of translational and rotational motion. The resistance matrix is known analytically only for a few simple particle shapes (e.g., sphere, spheroids, and discs) (Happel & Brenner, 1983). Symmetric particles exhibit a vanishing coupling tensor with respect to the particle centre. Either numerical simulations or experiments have to be performed to investigate the forces and torques acting on arbitrary particles.

Particles suspended in fluids and turbulence are well studied, though much of the literature focuses on simple geometries (Voth & Soldati, 2017). Commonly, the interaction between fluid and particle is expressed with dimensionless coefficients for the drag and lift forces and pitching and rotation torques. This enables easy comparison between different shapes and sizes. Most studies investigating nonspherical particles in fluids focus on the drag coefficient. (Haider & Levenspiel, 1989) correlate the drag coefficient of nonspherical particles to sphericity Φ , the ratio of the surface of a volume equivalent sphere to the actual particle surface area. Sphericity Φ is not to be confused with the same named parameter characterizing the shape of snow grains in snow packs (see Lehning et al., 2002; Bartlett et al., 2008). Leith (1987) proposes a model for the drag coefficient as a function of sphericity and the so-called crosswise sphericity, the ratio between the projected area of the volume equivalent sphere to the projected area of the particle perpendicular to the flow. Hölzer and Sommerfeld (2008) additionally introduce the lengthwise sphericity to improve on the model by Leith (1987). Others use anisotropy parameters to predict the drag coefficient (e.g., Bagheri & Bonadonna, 2016). Regarding the lift coefficient, there are studies which present models for specific particle shapes (e.g., Zastawny et al., 2012). Furthermore, the cross flow principle assumes that lift is proportional to drag and the incidence angle of a particle (Hoerner & Borst, 1985), which holds for simple geometries. To the best of our knowledge, no relation to reasonably estimate the lift coefficient exists for complex, irregular particle shapes. The situation is similar for the rotational and the pitching torque of arbitrary particles (Zastawny et al., 2012).

Snow particles exhibit a broad range of complex shapes initially mostly determined by the air temperature and supersaturation present during crystal growth (Libbrecht, 2005; Nakaya, 1951). During their fall, snow crystals are subject to collisions, mechanical damage, and early metamorphism, enhancing the range of particle shapes and especially increasing the irregularity of the shapes. The range of snow particle sizes (maximum extension of the grain, d_b) typically is 0.2–5.0 mm in proximity to the earth surface (Fierz et al., 2009). However, snow particles in the atmosphere as well as in the saltation layer can be considerably smaller (Gordon & Taylor, 2009; Kinne & Liou, 1989). The wide variety of complex snow particle shapes further complicates investigating snowflake motion. The physical interaction between fluid and particle becomes more complex for irregular particles, as coupling of translational and rotational motion is introduced in Stokes regime. Furthermore, experimentally obtaining realistic, three-dimensional snow particle geometries is difficult, since the ice crystals are inherently fragile. In the past, simple approximations such as spheres and ellipsoids were used to study snow particle motion (e.g., Aksamit et al., 2017; Gauer, 1999; Jucha et al., 2018). In a recent experimental study, Westbrook and Sephton (2017) use more sophisticated 3-D-printed snowflake geometries, though these still are conceptual approximations.

In this study we start from first principles, recap the full equations of motion of a particle in the Stokes regime, and compute the particle resistance matrices of 72 realistic snow particle shapes. The majority of the 3-D snow particle geometries are extracted from microcomputed tomography (μ CT) data of real, fresh snow using a segmentation algorithm. Additionally, geometries representing younger precipitation particles are obtained from a numerical phase field model conducted in another study (Demange et al., 2017), which simulates the growth of snowflakes in 3-D based on the governing physical mechanisms. Each particle's resistance matrix is computed by numerically solving the steady Navier-Stokes equations for small Reynolds numbers. Subsequently, particle tracking simulations using the equations of motion and computed resistance matrices are performed to investigate the transient motion of the snow particles settling in still air. The sedimentation would usually occur in the presence of turbulence. Still, sedimentation in air at rest, studied here, is very useful since it gives a reasonable estimate for the mean settling velocity in

turbulence. We deduce models of snow particles' drag and rotation torque coefficients as well as of the terminal velocity from our particle tracking simulations and discuss their validity. In this study, we focus on the Stokes regime for a number of reasons. First, a rough assessment shows that most of the snow particles investigated will sediment at Re below or of order 10. The fluid mechanical flow in this range is qualitatively similar to the low- Re flow (Happel & Brenner, 1983) where there is no wake, vortex shedding, randomness, or other phenomena associated with high- Re flows. Second, a fully resolved study of the sedimentation is computationally extremely expensive. This prohibits the investigation of large numbers of particles for practical reasons. Restricting our study to Stokes regime, enables us to study a broad range of complex snow particle geometries with a variety of initial conditions.

2. Theoretical Background

The motion of an arbitrarily shaped particle suspended in a viscous fluid is governed by the conservation of linear and angular momentum. Expressed in the body-fixed particle frame of reference, the equations of motion of a particle are given by

$$\begin{bmatrix} m_p & 0 \\ 0 & I \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{u}_p \\ \boldsymbol{\omega} \end{bmatrix} + \begin{bmatrix} m_p \boldsymbol{\omega} \times \mathbf{u}_p \\ \boldsymbol{\omega} \times (I \boldsymbol{\omega}) \end{bmatrix} = \begin{bmatrix} \mathbf{F}_D + \mathbf{F}_L \\ \mathbf{T} \end{bmatrix} + \begin{bmatrix} (m_p - m_f) \mathbf{g} \\ 0 \end{bmatrix}, \quad (1)$$

with m_p the particle mass, m_f the displaced fluid mass, $\mathbf{u}_p$ the velocity of the particle centre of mass relative to the fluid, $\mathbf{g}$ the gravitational acceleration (rotated into the body-fixed particle frame), $\mathbf{F}_{D,L}$ the hydrodynamic drag and lift forces, I the particle's moment of inertia, $\boldsymbol{\omega}$ the particle's angular velocity, and $\mathbf{T}$ the hydrodynamic torques acting on the particle. For a complete derivation of the equations of motion, we refer to the supporting information (SI). Stokes flow can be assumed in the limit that Reynolds number, $Re = 2ur_{eq}/\nu$, and rotational Reynolds number, $Re_\omega = 4\omega r_{eq}^2/\nu$, are small ($Re, Re_\omega \ll 1$). Here, u and ω are the magnitudes of the relative streaming and angular velocities and ν is the kinematic viscosity of the fluid. Following (Brenner, 1967), hydrodynamic forces and torques in Stokes regime are expressed with the particle resistance matrix $\mathcal{K}_p$ as

$$\begin{bmatrix} \mathbf{F}_D + \mathbf{F}_L \\ \mathbf{T}_P + \mathbf{T}_R \end{bmatrix} = -\mu \mathcal{K}_p \begin{bmatrix} \mathbf{u}_p \\ \boldsymbol{\omega} \end{bmatrix} = -\mu \begin{bmatrix} \mathcal{K} & \mathcal{C}_p^T \\ \mathcal{C}_p & \Omega_p \end{bmatrix} \begin{bmatrix} \mathbf{u}_p \\ \boldsymbol{\omega} \end{bmatrix}, \quad (2)$$

again expressed in the body-fixed particle frame of reference. The μ is the dynamic viscosity of the fluid, $\mathcal{K}$ is the translation tensor, $\mathcal{C}_p$ is the coupling tensor, Ω_p the rotation tensor, and $\mathbf{T}_{P,R}$ are the hydrodynamic pitching and rotation torques, all with respect to the particle center of mass.

The drag and lift coefficients are commonly formulated as

$$C_D = \frac{\|\mathbf{F}_D\|}{0.5u_p^2 \rho_f \pi r_{eq}^2}, \quad C_L = \frac{\|\mathbf{F}_L\|}{0.5u_p^2 \rho_f \pi r_{eq}^2}, \quad (3)$$

where r_{eq} is the volume equivalent spherical radius (Landau & Lifshitz, 1987). Analogously, rotation torque and pitching torque coefficients are defined as

$$C_R = \frac{\|\mathbf{T}_R\|}{0.5\omega^2 \rho_f \pi r_{eq}^5}, \quad C_P = \frac{\|\mathbf{T}_P\|}{0.5u_p^2 \rho_f \pi r_{eq}^3}. \quad (4)$$

Leith (1987) proposed a model to estimate the average drag coefficient of arbitrary particles in Stokes regime as a function of Stokes drag of the volume equivalent sphere and shape factor K . K is given by $K = 1/3\Phi_\perp^{-0.5} + 2/3\Phi^{-0.5}$, where $\Phi_\perp$ is the crosswise sphericity. Analyzing the individual terms, we see that Φ is at minimum 2.7 times more important in determining K than $\Phi_\perp$. Furthermore, obtaining $\Phi_\perp$ a priori for complex shaped particles proves very difficult, since final orientation is not known. For these reasons, we adopt a simplified model for the average drag coefficient with a shape factor based solely on Φ . Analogously, we introduce a model for the rotation torque coefficient. Both models have the same basic structure and are given by

Figure 1. Examples of snow particles obtained from a modified phase field model (a, b) provided by Demange et al. (2017) and segmented from μ CT data of snow samples No. 2 (c, d) and No. 5 (e, f) from Schlee et al. (2014).

$$\overline{C}_D = \frac{24}{Re} \Phi^{b_D}, \quad \overline{C}_R = \frac{16\pi}{Re_\omega} \Phi^{b_R}, \quad (5)$$

where $b_D = -0.5$ and $b_R = -1.5$ are obtained from theoretical considerations (see Leith, 1987). The two models strictly hold only for Stokes regime. Outside Stokes regime, the nonlinear term in the Navier-Stokes equations is not negligible anymore. However, up to $Re \leq 10$ the force acting on the particle can be effectively described as the force resulting from Stokes equations times a modifying factor f of order 1, for example, for a sphere $f = 1 + 0.15Re^{0.687}$ (Ayala et al., 2008). For more complex particles it is reasonable to assume that a similar factor of order one holds too. However, the exact form and range of applicability is a topic for future work. Here, for particles outside of Stokes regime, our study is only qualitative and determines $\overline{C}_D$ and $\overline{C}_R$ up to factors of order 1.

Happel and Brenner (1983) introduced an average resistance to translation $\overline{\mathcal{K}}$ of an arbitrarily shaped particle. It can be obtained by averaging the resistances to translation for any orientation assuming that all particle orientations are equally probable. $\overline{\mathcal{K}}$ is defined as the harmonic mean of the translation tensor eigenvalues. The average settling velocity when neglecting coupling between translation and rotation can be obtained through balancing of the hydrodynamic and hydrostatic forces and is given by

$$\overline{u}_\infty = \mu^{-1} \overline{\mathcal{K}}^{-1} V_p (\rho_p - \rho_f) \mathbf{g}. \quad (6)$$

3. Methodology

3.1. Snowflake Geometries

To account for the complex shapes of snow particles, we consider geometries obtained from μ CT scans of fresh snow samples (Schlee et al., 2014) as well as geometries simulated with a phase field model (Demange et al., 2017).

The experimental snow particle geometries are extracted from μ CT data of two distinct samples of fresh, sieved snow (snow samples No. 2 and 5 from Schlee et al., 2014). Details on the sampling and the snow samples themselves can be found in Schlee et al. (2014). The 3-D binary μ CT images are segmented into individual particles by applying an algorithm based on watershedding (see SI). Thirty particles from each data set

are chosen randomly for the use in the hydrodynamic simulations. We chose samples No. 2 and 5 because they are not subjected to any external impacts and allow us to study a broad variability of particle geometries. Particles of snow sample No. 2 exhibit plate-like features and some dendritic structures, while sample No. 5 contains highly dendritic particles. The average maximal extension of the chosen particles is $827 \pm 329 \mu\text{m}$. Figures 1c–1f show two examples of snow particles from both snow samples.

Twelve elementary snowflake geometries generated with a modified phase field model are provided by Demange et al. (2017). The phase field model simulates the physical growth of snowflakes in 3-D and yields a broad range of precipitation particle geometries. The phase field particles feature regular, almost symmetric particles with varying dendricity. In Figures 1a and 1b two examples of phase field snow particles used for the hydrodynamic simulations are shown. All particles can be found in Figures S5–S7 (SI).

3.2. Simulation of the Resistance Matrix

Steady-state simulations of the laminar streaming and rotating flows around the particle are conducted to extract the resistance matrix of a snow particle. The simulations are performed using the open source software OpenFOAM. The steady Navier-Stokes equations are solved using the Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) for the velocity-pressure coupling. Two distinct simulation types are conducted; one for the translation of the particle and one for the rotation. Each simulation solves for the coupled velocity and pressure field. The hydrodynamic forces and torques are then computed (Equation S4 in the SI) and subsequently transformed into the body-fixed particle frame. The respective resistance matrix can be computed by solving Equation 2. The setup and validation studies for both simulation types are described in detail in the SI.

3.3. Particle Free Fall Simulations

To investigate the sedimentation behavior of the examined snow particles, their motion is calculated numerically for the case of free-falling particles in still air. The numerical procedures are adopted from (Zhang et al., 2001). A pseudo-code of the simulation can be found in the SI. The previously computed resistance matrices are used to calculate the hydrodynamic forces and torques acting on the particles, using Equation 2. Furthermore, only hydrostatic and hydrodynamic forces and torques are considered. Euler's four parameters are numerically integrated using the fourth-order Runge-Kutta scheme, while the explicit Euler forward scheme is used to obtain the new particle position, velocity, and angular velocity. The integration time step is set to 10^{-5} s, and each 1,000th time step is registered; that is, the temporal resolution of the stored time series is 10^{-2} s. Since we investigate the sedimentation of particles in still air, no background flow field is assumed and all particles are at rest initially. To ensure that all particles remain in Stokes regime throughout their complete sedimentation time, all particles are rescaled to $r_{\text{eq}} = 10 \mu\text{m}$.

We assign 125 different initial orientations for each particle and the simulation is terminated for a given particle-initial orientation combination once a steady state is reached. The criteria for steady state are defined by a maximal relative change of $10^{-6}\%$ for both u and ω . Note that this is a less strict definition of steady state compared to (Happel & Brenner, 1983) since it concerns magnitudes instead of vectors. The first initial orientation of each particle is chosen randomly, while the other 124 initial orientations are described by all possible combinations of the three Euler angles, where each Euler angle may be assigned a multiple of $\pi/5$. Thus, approximately all particle orientations are probed.

4. Results

During free fall in still air, all investigated snow particles eventually attain preferred orientations in which they reach steady state; that is, the magnitudes of both translational and angular velocities stay constant over time. We observe two different modes of motion in steady state. Ten particles exhibit a drifting motion while rotating only slightly (see also Figure S8 in the SI). The remaining 62 particles all spiral while sedimenting, where the center of mass describes a downward spiral and the particle itself generally also rotates. Example videos of the sedimentation are provided in the supporting information. The type of motion and the preferred orientation is only dependent on particle geometry and does not change with initial orientation.

We calculate drag, lift, pitching torque, and rotation torque coefficients for all particles in steady state. All four coefficients are trivially dependent on either Re or Re_ω . In the simulation all particles sediment with

Figure 2. Top: Average drag coefficient at $Re=0.1$ as a function of sphericity. The dashed line shows Equation 5 with $b_D = -0.5$. Bottom: Average rotation torque coefficient at $Re_\omega = 0.1$ as a function of sphericity. The dashed line represents Equation 5 with $b_R = -1.5$.

different terminal velocities; thus, the coefficients are extracted at different Reynolds numbers $Re' = 2u_\infty r_{eq}/\nu$ and $Re'_\omega = 4\omega_\infty r_{eq}^2/\nu$. To assess the impact of the shape, we control for Re and rescale the coefficients at $Re = Re_\omega = 0.1$ using $\bar{C}_{D,L,P}^s = \bar{C}_{D,L,P}' Re'/0.1$ and $\bar{C}_R^s = \bar{C}_R' Re'_\omega/0.1$. Figure 2 (top) shows $\bar{C}_D^s$ as a function of Φ . Also shown is Equation 5 with $b_D = -0.5$. Figure 2 (bottom) shows $\bar{C}_R^s$ as a function of Φ . Equation 5 with $b_R = -1.5$ is also shown. Residuals to both models are shown in Figures S10 and S11 (SI).

We could not find any correlation for $\bar{C}_P$, while $\bar{C}_L$ is weakly correlated to $\bar{C}_D$ and anisotropy. We refer to the SI for more details on the pitching torque and lift coefficients.

Correct prediction of the terminal velocity of a particle in a viscous fluid is of great importance for many practical applications. (Happel & Brenner, 1983) proposed Equation 6 as an estimate of $\bar{u}_\infty$ provided that one knows the particle's translation tensor. When $\bar{C}_D$ of a particle is known, $\bar{u}_\infty$ may also be predicted through balancing hydrostatic and hydrodynamic forces. Using the drag relation presented in this study, Equation 5 with $b_D = -0.5$, we can formulate the average settling velocity of an irregular particle as

$$\bar{u}_\infty = \frac{2gr_{eq}^2 \Phi^{0.5}}{9\nu} \frac{\rho_p - \rho_f}{\rho_f}. \quad (7)$$

Examining Equations 6 and 7, we see that $\bar{u}_\infty$ can be modeled as a function of Φ . Figure 3 shows the simulated average terminal velocity compared to predictions using Equations 6 and 7.

We also investigated the importance of coupling between translational and rotational motion on the terminal velocity. On average terminal velocity is 5.0% smaller when coupling is artificially neglected, while the maximum relative difference is approximately 20%. For more details we refer to the SI.

5. Discussion

Naturally, particles with a drift mode of motion in the steady state translate without significantly rotating. Seven of these particles are phase field particles with clear symmetry features. The three μ CT particles with drift motion exhibit no obvious geometric features grouping them together. Figure S8 (SI) shows that also spiralling particles may sediment while barely rotating. These particles include all remaining phase field and 14 μ CT particles. Most of those μ CT particles are either predominantly planar or have a highly irregular mass distribution. Since the phase field particles have a high degree of symmetry, those particles are expected to translate with little to no rotation.

Equation 5 with $b_D = -0.5$ exhibits an adjusted R^2 of 0.73, indicating a reasonable representation of $\bar{C}_D$. Thus, the simplified version of the model proposed by Leith (1987) holds also for complex, irregular snow particle shapes. A least squares fitted parameter $b_D = -0.45$ deviates from its theoretical value by 10% and has an adjusted R^2 of 0.73 as well, showing good agreement of the data with theory. In literature, it is suggested that $\bar{C}_D$ is also dependent on particle anisotropy (e.g., Bagheri & Bonadonna, 2016). However, introduction of various anisotropy measures (e.g., flatness and elongation) does not improve the correlation significantly (not shown). For the rotation torque coefficient, the adjusted R^2 of Equation 5 with $b_R = -1.5$ is a mere 0.04. However, excluding the three marked outliers results in an adjusted R^2 of 0.54. A simple least squares fit of the parameter b_R to the data gives $b_R = 1.46$, a deviation of 2.7% from the theoretical value. The adjusted R^2 remains unchanged. Thus, for the majority of the snow particles, $\bar{C}_R$ can reasonably be predicted by Re_ω and Φ . The largest outlier was a needle-like phase field particle. The two μ CT particles also excluded from consideration resemble a two-bladed propeller and a helix, respectively, which are expected to have geometries optimized to high rotational resistance.

Figure 3. Comparison of simulated average terminal streaming velocity with models based on $\mathcal{K}$ (Equation 6) and Φ (Equation 7).

occur in clouds and in the saltation layer, though particles close to the earth surface, such as the particles from the snow samples used in this study, are typically larger. However, as mentioned above, Equation 7 can reasonably be extended up to $Re \approx 10$ using modifying factors f . Applying this extended model to the original sized μ CT particles we find that roughly two thirds of the studied particles are expected to sediment at $Re \leq 10$ (see Figure S13 in the SI). Future work can be dedicated to investigate a possible extension to higher Re (10–1,000) also present in the atmosphere. For this, we propose to find expressions for f , g , and c in the generalized model of Clift and Gauvin (1971), $C_D = 24/Re f \Phi^{-0.5} + g \Phi^c$, where g is a modifying factor depending on Re .

In the turbulent airflow $\mathbf{u}(\mathbf{x}, t)$ the simplified equation of motion of snow particles is $d\mathbf{u}_p/dt = -(\mathbf{u}_p - \mathbf{u}(\mathbf{x}, t))/\tau_{\text{eff}} + \mathbf{g}$ if the smallest scale of flow variations is much larger than the snow particle size. Here, $\tau_{\text{eff}} = \bar{u}_\infty/g$ is the effective particle response time and $\bar{u}_\infty$ can be obtained from Equation 7, if necessary in conjunction with f . Averaging this simplified equation of motion, we find that the mean settling velocity in turbulence $\bar{u}_\infty t$ is equal to the average settling velocity in still air $\bar{u}_\infty$ plus the average flow velocity in the particle frame $\bar{\mathbf{u}}_z(\mathbf{x}, t)$. Wang and Maxey (1993) studied the case for homogeneous isotropic turbulence and found that $\bar{u}_\infty t$ equals $\bar{u}_\infty$ times a factor of order 1 whose magnitude depends on the ratio of τ_{eff} and the Kolmogorov time scale. As such, $\bar{u}_\infty$ serves as a good estimation for $\bar{u}_\infty t$.

6. Conclusions

The behavior of snowflakes in the atmosphere is a complex phenomenon and important for many physical processes. Even though there are several previous studies investigating the interaction of particles with the surrounding fluid, it is unknown for snow particles, due to the high degree of complexity and irregularity exhibited in their shapes. In this study, the particle resistance matrix of 72 distinct, realistic snow particles (12 from a phase field model, 30 particles each from μ CT data of two snow samples) were calculated using numerical simulations of the steady Navier-Stokes equations. The resistance matrices were then used in particle tracking simulations to investigate the motion of the snow particles sedimenting in still air in Stokes regime. We found that all snow particles attain preferred orientations in which they reach a steady state. In these steady states the particles exhibit either a drifting or a spiralling mode of motion.

We tested a drag relation based on the particle Reynolds number and the particle sphericity (Equation 5 with $b_D = -0.5$) and found that it describes the behavior of complex snow particles rather well. A new relation of the same basic structure (Equation 5 with $b_R = -1.5$) was presented as an estimate of the average rotation torque coefficient based on the rotational Reynolds number and sphericity. Finally, we presented a new formulation to estimate the average terminal velocity of irregular particles based solely on sphericity and we formulated a simplified equation of motion for realistic snow particles. From considerations on the Reynolds number, we established an upper limit ($r_{\text{eq,max}} \approx 20\mu\text{m}$, $d_{\text{b,max}} \approx 100\mu\text{m}$) to the direct applicability of our results. The results can be reasonably extended to moderately higher Re (~ 10) by using modifying factor f , and possibly even to high Re (~ 100 –1,000) by introducing modifying factors along the lines of the generalized model of Clift and Gauvin (1971). In this study, we imposed the surrounding air be at rest. Thus, a logical next step would be to adopt this framework to real-life atmospheric or saltation layer flow conditions.

This study's findings are not restricted to snow particles, but are expected to hold generally for complex shaped particles sedimenting in viscous fluids.

Data Availability Statement

The snow particle geometries as well as a table summarizing the main parameters and results can be downloaded in the ETH Zurich Research Collection (<https://doi.org/10.3929/ethz-b-000401995>) as stl-files and as a csv-file, respectively. The table is also available in the supporting information accompanying this article. Provided in the supporting information are as well details on the particle segmentation algorithm and on the numerical procedures, additional findings, and example videos of the simulated snow particle sedimentation.

Acknowledgments

The authors would like to acknowledge G. Demange for providing the geometries of the phase field particles and H. Lowe for providing the experimental μ CT data sets. We acknowledge P. Corso for inputs on the numerical methods. Furthermore, we would like to thank Prof. M. Lehning for discussions. Markus Holzner acknowledges financial support from Swiss National Science Foundation SNF under Grant 172916.

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