Title: Individual variation and abrupt collapses in mutualistic networks

Gaurav Baruah[1,2*]

[1]Department of Evolutionary Biology and Environmental Studies, University of Zurich, Winterthurerstrasse 190, 8057 Zurich, CH

[2]Department of Fish Ecology and Evolution, Center for Ecology, Evolution and Biogeochemistry, Eawag, Seestrasse 79, 6047 Kastanienbaum, CH

*Correspondence to: gbaruahecoevo@gmail.com

Supporting information 1: The eco-evolutionary model:

We model the dynamics of pollinators P and plants N in an ecologically relevant quantitative trait z. Each individual belonging to a guild of pollinators or plants can be described with its trait z, but each of the species belonging to the guild of pollinators or plants are comprised of individuals with different trait values. Now the number of individuals with within species i at time t for pollinators will be N_i^A and for the plants it will be N_i^P, and the distribution of their traits within each species i can be given by a function $p_i(z,t)$ and by definition this function satisfies as

$$\int p_i(z,t)dz = 1$$

at every time t. The limits of integration encompass the whole trait axis. $N_i^P p_i^P(z,t)dz$ is then the population density of species i's individuals with phenotype value between z and $z+dz$ for the plants and for the animals we can write $N_i^A p_i^A(z,t)dz$ which is the population density with phenotype value betweeen z' and $z'+dz'$.

The model is in the quantitative genetic limit, i.e., the trait in question is determined by many independent loci. In this case the trait distribution is normal and variance of the trait does not change in response to selection (Falconer & Mackay 1996):

$$p_i^P(z,t) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{\left(z-u_i^A(t)\right)^2}{2\sigma_i^2}}$$

where $u_i^P(t)$ is the mean trait value for the species i and σ_i^2 is the trait variance. In this scenario, only the mean of the trait responds to selection and the trait variance remains constant. This also means that the distribution of the trait remains normal.

The governing dynamical equations of population dynamics can be written with Lotka-Volterra equations. The per-capita growth rate of can be written, for instance for the pollinator species as (Barabas & D'Andrea 2016):

$$r_i^A(\vec{N},\vec{p},z,t) = b_i - \sum_j^{S_A} a_{ij}^A(t)N_j^A(t) + \sum_k^{S_P} N_k^P(t) \int \frac{A_{ik}\gamma(z,z')}{1 + \tau\, A_{ik}\gamma(z,z')N_k^P}\, p_k^P(z',t)dz'$$

Where a_{ij}^A captures the the inter- and intraspecific competition among species belonging to the same guild; S_A is the total number of species belonging to the pollinator community; A_{ik} is the adjacency matrix that captures whether a pollinator with trait z interacts with a plant with trait z', i.e., $A_{ik} = 1$, if individual with trait z belonging to the pollinator community interacts with another individual belonging to the plant community and 0 if they don't interact; h is the handling time; $\gamma(z,z')$ is the mutualistic interaction kernel as $\gamma(z,z') = \frac{\gamma_0}{q_i} e^{-\frac{(z-z')^2}{w^2}}$, where γ_0 is the average strength of co-evolutionary mutualistic interaction when individuals belonging to two different groups of species have same traits; w^2 is the width of the interaction kernel that controls the strength of interaction; q_i is the degree of species i. When trade-off between the number of interactions and average mutualistic strength is relaxed, $q_i = 1$ for all species, or else q_i is the degree of species i.
If traits of two individuals belonging to two different guilds of species are similar, the stronger is the mutualistic benefit. b_i is the species growth rate which is fixed at -0.05 to signify obligate mutualism.

Now the population dynamics of species i over all trait space z can be written as:

$$\frac{dN_i^A(t)}{dt} = N_i^A(t) \int r_i^A(\vec{N},\vec{p},z,t)p_i^A(z,t)dz$$

$$\frac{dN_i^A(t)}{dt} = N_i^A(t) \int \left(b_i - \sum_j^{S_A} a_{ij}^A(t)N_j^A(t) + \sum_k^{S_P} N_k^P(t) \int \frac{A_{ik}\gamma(z,z')}{1+\tau A_{ik}\gamma(z,z')N_k^P} p_k^P(z',t)dz' \right) p_i^A(z,t)dz$$

(S2)

Following (Barabas & D'Andrea 2016) formulation of evolutionary trait dynamics of competitive communities, we can similarly derive the evolutionary trait dynamics for mutualistic communities given the per capita growth rate $r_i^A(\vec{N},\vec{p},z,t)$ for an individual with trait z,

$$\frac{du_i^A}{dt} = h_i^2 \int (z - u_i^A) r_i^A(\vec{N}, \vec{p}, z, t) p_i(z,t) dz$$

$$= h_i^2 \int (z - u_i^A) \left(b_i - \sum_j^{S_A} a_{ij}^A(t) N_j^A(t) + \sum_k^{S_P} N_k^P(t) \int \frac{A_{ik}\gamma(z,z')}{1+\tau A_{ik}\gamma(z,z')N_k^P} p_k(z',t)dz' \right) p_i(z,t)dz$$

(S3)

Equation S3 and equation S2 are not analytically solvable, however, numerically we can solve the two

equations by using Gaussian quadrature which can be implemented by using the R package *statmod*

(Smyth *et al*. 2020). Gaussian quadrature is a quadrature rule used to approximate a definite integral

within the domains of an integral. Here, we could approximate numerically the integral within the

bounds of the trait axis. For that we ensure that the bounds of the trait axis go from -z to z where we

assume z goes from -1 to 1. Numerically we can then solve the integrals and simulate the eco-

evolutionary dynamics in response to small changes in the strength of co-evolutionary interactions.

Supporting Information 2: Co-evolutionary dynamics of empirical networks

I specifically assume obligate mutualism which meant that plants and pollinators are completely

dependent on each other. However, this particular assumption does not hinder in the simulation

experiments where networks were forced to collapse. If obligate mutualism is not assumed, the only

difference would be that the alternative state which in the case of facultative mutualism would be

positive total abundance devoid of any mutualistic benefits. Facultative mutualism would only shift

the alternative stable state without altering the general results.

In the following example (see below figure S1-S2) for two different network types of varying

nestedness (network 1 has nestedness 0.9 and network 2 has nestedness 0.2), I show the co-

evolutionary dynamics of plants and pollinators in response to three different co-evolutionary

mutualistic strength γ_0. In figure S1a, nestedness (NODF) estimated was 0.9 and network size was 26.

As seen from the figure S1a, there was a sharp transition from high positive abundance of plants and

pollinators from $\gamma_0 = 3.25$ to almost zero abundances at co-evolutionary strength, $\gamma_0 = 2.9$. However, in figure S1b, nestedness estimated was around 0.15 and network size 22. Here, in the figure S1b, we do not see an abrupt shift in community abundance when co-evolutionary strength goes from $\gamma_0 = 3.25$ to $\gamma_0 = 2.9$.
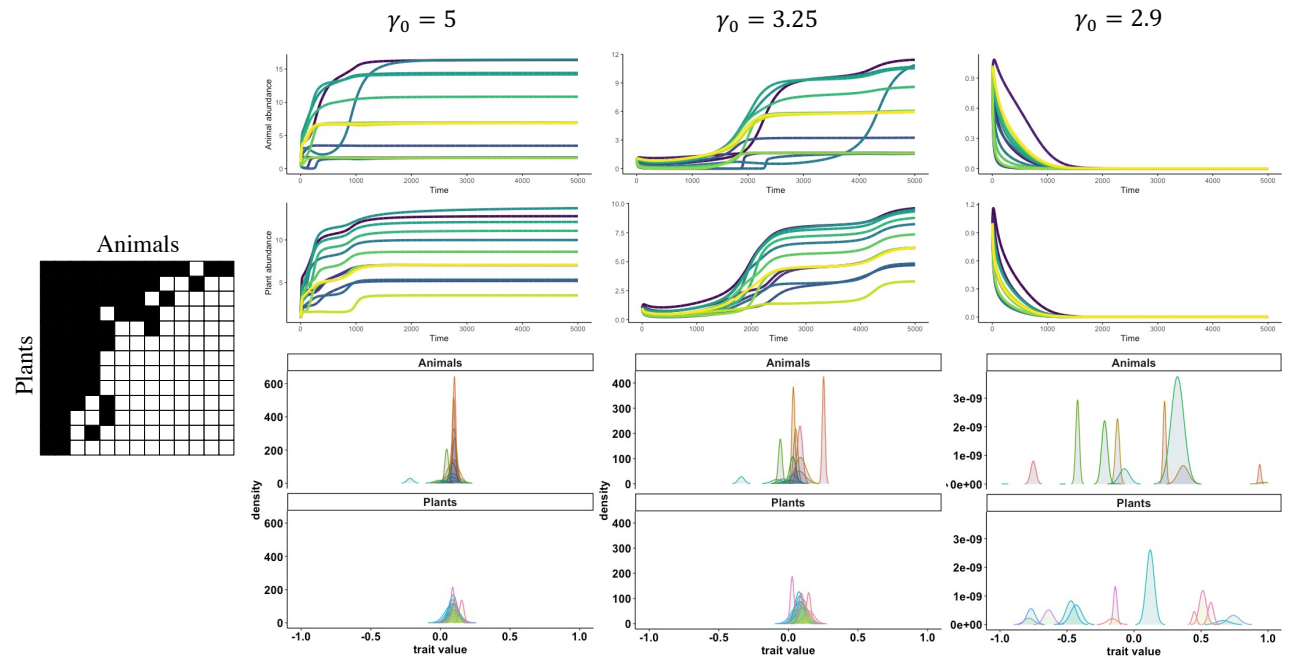
Figure S1a: Eco-evolutionary dynamics of a plant-pollinator network with high nestedness (NODF) of 0.85. Number of plants in this network were 13 and pollinators were 13 too. Shown here are the timeseries of animal, plant abundance; phenotypic distribution of animals and plants at the end of the simulation for three different values of co-evolutionary mutualistic strength γ_0. Intraspecific trait variance was randomly sampled from U[0.01,0.05].
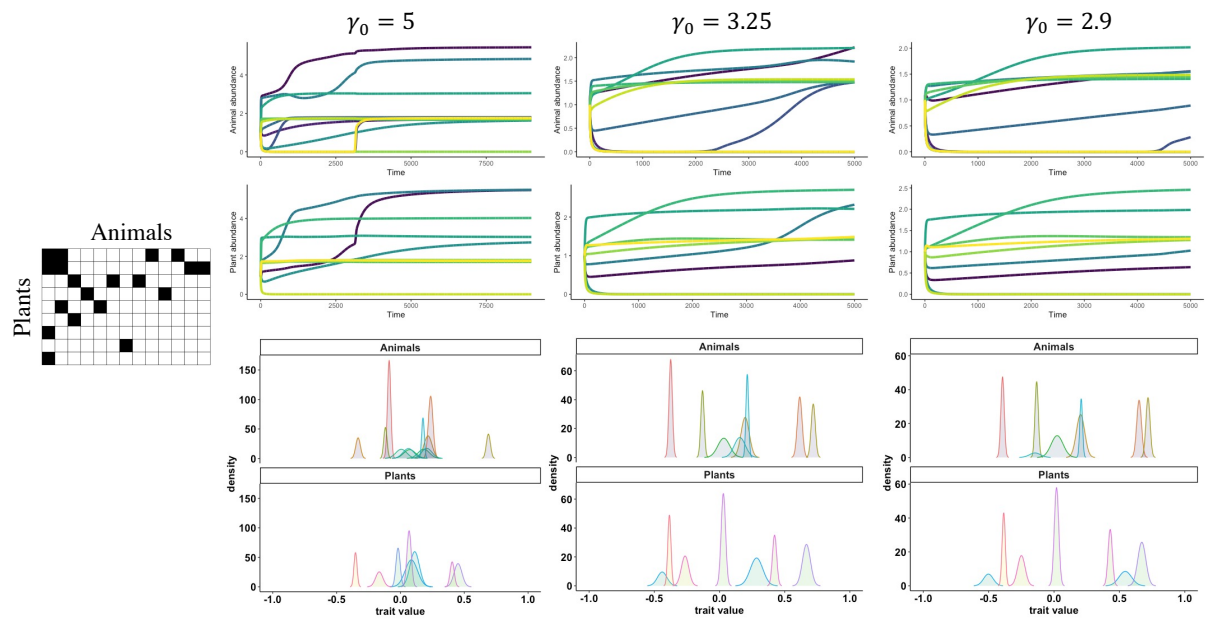
Figure S1b: Eco-evolutionary dynamics of a plant-pollinator network with low nestedness (NODF) of 0.15. Number of plants in this network were 9 and pollinators were 13. Shown here are the timeseries of animal, plant abundance; phenotypic distribution of animals and plants at the end of the simulation for three different values of co-evolutionary mutualistic strength γ_0. Intraspecific trait variance was randomly sampled from $U[0.01, 0.05]$.

References

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Falconer, D.S. & Mackay, T.F.C. (1996). *Introduction to Quantitative Genetics (4th Edition)*.

Smyth, G., Hu, Y., Dunn, P., Phipson, B. & Chen, Y. (2020). *statmod: Statistical Modeling*.

Table S1: References of datasets used in the study:

ID	Species	Reference
M_PL_003	61	Arroyo, M.T.K., R. Primack & J.J. Armesto. 1982. Community studies in pollination ecology in the high temperate Andes of central Chile. I. Pollination mechanisms and altitudinal variation. Amer. J. Bot. 69:82-97.
M_PL_007	52	Dicks, LV, Corbet, SA and Pywell, RF 2002. Compartmentalization in plant,Äiinsect flower visitor webs. J. Anim. Ecol. 71: 32,Äi43.
M_PL_008	49	Dupont YL, Hansen DM and Olesen JM (2003) Structure of a plant-flower-visitor network in the high-altitude sub-alpine desert of Tenerife, Canary Islands. Ecography 26:301-310.
M_PL_011	27	Olesen, J.M., Eskildsen, L.I. & Venkatasamy, S. (2002). Div. Distr., 8:181-192.
M_PL_013	65	Ollerton, J., S. D. Johnson, L. Cranmer, and S. Kellie. 2003. The pollination ecology of an assemblage of grassland asclepiads in South Africa. Annals of Botany 92:807-834.
M_PL_022	66	Medan, D., N. H. Montaldo, M. Devoto, A. Mantese, V. Vasellati, and N. H. Bartoloni. 2002. Plant-pollinator relationships at two altitudes in the Andes of Mendoza, Argentina. Arctic Antarctic and Alpine Research 34:233-241.
M_PL_024	29	Mosquin, T., and J. E. H. Martin. 1967. Observations on the pollination biology of plants on Melville Island, N.W.T., Canada. Canadian Field Naturalist 81:201-205.
M_PL_025	57	Motten, A. F. 1982. Pollination Ecology of the Spring Wildflower Community in the Deciduous Forests of Piedmont North Carolina. Doctoral Dissertation thesis, Duke University, Duhram, North Carolina, USA; Motten, A. F. 1986. Pollination ecology of the spring wildflower community of a temperate deciduous forest. Ecological Monographs 56:21-42.
M_PL_032	40	Schemske, D. W., M. F. Willson, M. N. Melampy, L. J. Miller, L. Verner, K. M. Schemske, and L. B. Best. 1978. Flowering Ecology of Some Spring Woodland Herbs. Ecology 59:351-366.
M_PL_033	47	Small, E. 1976. Insect pollinators of the Mer Bleue peat bog of Ottawa. Canadian Field Naturalist 90:22-28.
M_PL_036	22	Olesen unpubl.
M_PL_037	50	Montero AC (2005).The Ecology of Three Pollination Networks. MSc thesis (Univ of Aarhus, Aarhus, Denmark).
M_PL_038	50	Montero AC (2005).The Ecology of Three Pollination Networks. MSc thesis (Univ of Aarhus, Aarhus, Denmark).
M_PL_039	68	Stald L (2003). Struktur og dynamik i rum og tid af et best√∏vningsnetv√¶rk p√• Tenerife, De Kanariske √∏er. Msc thesis (Univ of Aarhus, Aarhus, Denmark).
M_PL_042	18	Philipp M, B√∂cher J, Siegismund HR, Nielsen LR (2006) Ecography 29:531-540. (Philipp, M., B√∂cher, J., Siegismund, H. R. and Nielsen, L. R. 2006. Structure of a plant-pollinator network on a pahoehoe lava desert of the Galapagos Islands. ,Äi Ecography 29: 531,Äi 40.)
M_PL_045	43	Lundgren R, Olesen JM (2005). The Dense and Highly Connected World of Greenland's Plants and Their Pollinators. Arc Antarc Alp Res 37:514-520.
M_PL_046	60	Bundgaard, M. (2003). Tidslig og rumlig variation i et plante-best√∏vernetv√¶rk. Msc thesis. University of Aarhus. Aarhus, Denmark.
M_PL_050	49	Stald, L., Valido, A. & Olesen, J. M. 2003. Struktur og dynamik i rum og tid at et best√∏vningsnetv√¶rk p√• Tenerife, De Kanariske √∏er. MSc-thesis, Univ. of Aarhus, Denmark.
M_PL_052	54	Witt P (1998) BSc thesis. Univ of Aarhus, Aarhus, Denmark.
M_PL_059	26	Bezerra ELS, Machado ICS, Mello MAR. 2009. Pollination networks of oil-flowers: a tiny world within the smallest of all worlds. Journal of Animal Ecology 78:1096,Äi1101.
M_PL_060_01	50	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_02	50	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_03	58	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_04	67	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.

M_PL_060_07	68	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_08	47	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_09	58	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_10	39	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_11	34	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_12	37	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_13	38	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_14	48	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_15	51	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_16	56	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_17	52	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_18	48	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_19	31	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_20	30	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_21	28	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_22	44	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_23	39	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_060_24	38	Kaiser-Bunbury, C. N., S. Muff, J. Memmott, C. B. Müller, and A. Caflisch. 2010. The robustness of pollination networks to the loss of species and interactions: A quantitative approach incorporating pollinator behaviour. Ecology Letters 13:442-452.
M_PL_061_01	17	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_02	22	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_03	20	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_04	22	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_05	34	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_06	35	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_07	32	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_08	25	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_09	16	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_10	19	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.

M_PL_061_11	18	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_12	19	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_13	26	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_14	17	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_15	24	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_16	27	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
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M_PL_061_18	29	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_19	32	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
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M_PL_061_21	29	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_22	28	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_23	36	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_24	29	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_25	16	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
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M_PL_061_27	19	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_28	27	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_29	26	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_30	20	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_31	24	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_32	24	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_33	8	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_34	10	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_35	22	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_36	24	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_37	30	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_38	33	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.

M_PL_061_39	30	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_40	35	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_41	13	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
M_PL_061_42	17	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
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M_PL_061_44	21	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
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M_PL_061_46	34	Kaiser-Bunbury CN, Vazquez DP, Stang M, Ghazoul J. 2014. Determinants of the microstructure of plant-pollinator networks. Ecology, 95: 3314-3324.
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M_PL_069_03	11	Kohler, G. (2011) Redes de intera√ß√£o planta-beija-flor em um gradiente altitudinal de Floresta Atl√¢ntica no Sul do Brasil. MSc. Thesis. Universidade Federal do Parana, Brazil.
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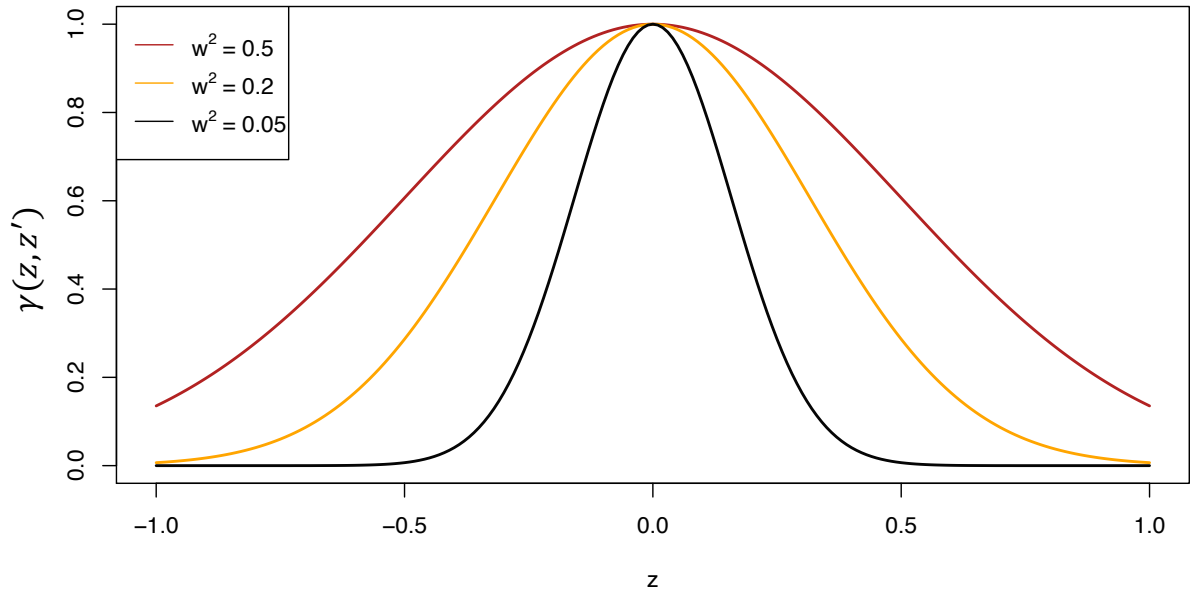
Figure S2: Strength of mutualism for different values of w^2. When w is small ($w^2 = 0.05$), the range of mutualistic benefits is narrow such that individuals belonging to different guilds of species with dissimilar trait values will have almost no fitness benefits. In contrast, when $w^2 = 0.5$, individuals having extreme trait values can still gain fitness benefits from each other if they interact.
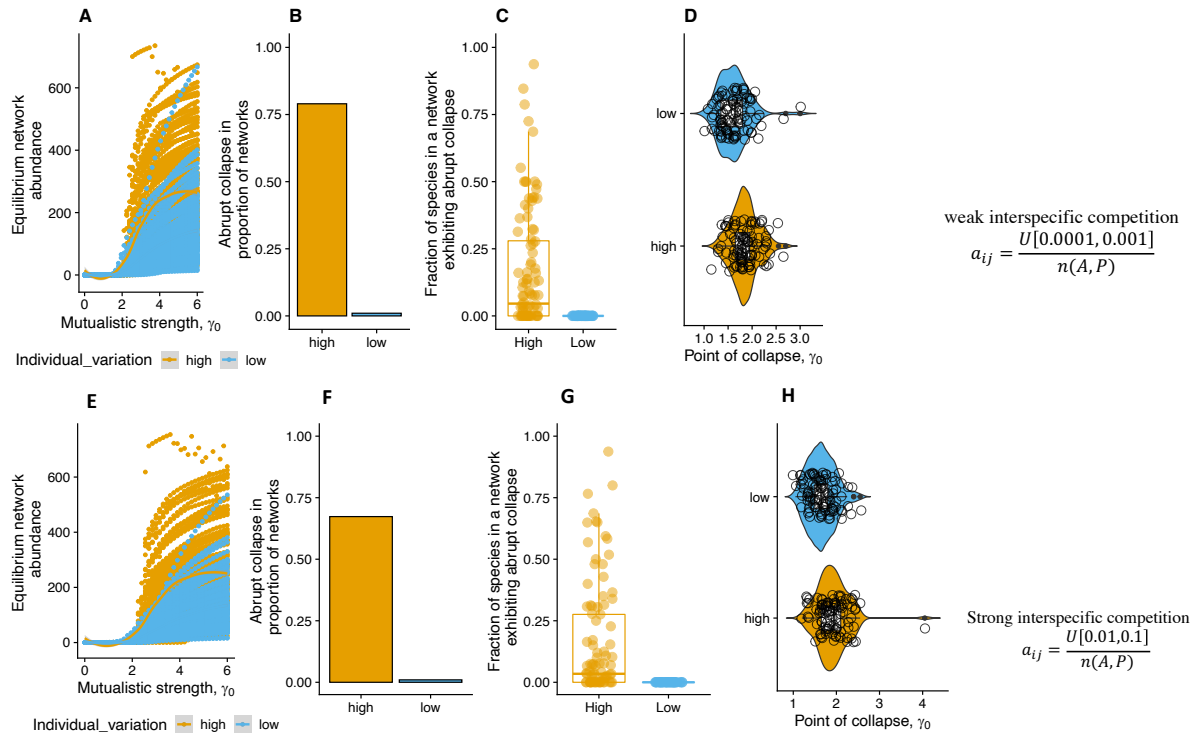
Figure. S3: Effects of strong interspecific competition on the occurrence of abrupt tipping points. Interspecific competition also decreases the average community abundance (see total abundances). Mean trait values of species were sampled from U[-1,1]. Strong interspecific competition decreases the chances of abrupt collapses drastically especially when species had high intraspecific trait variation. B & F: Proportion of networks that collapsed in the presence of high variation was lower when interspecific competition was stronger. C & D: In the presence of high intraspecific variation, on average the number of species that went through an abrupt collapse were lower for when species competed strongly (15.4%) versus when they competed weakly (17.1%). H: In addition, strong interspecific competition leads to an earlier abrupt collapse (point of collapse) when species had high individual trait variation (1.92) versus when species had low trait variation (1.65). Heritability was fixed 0.4.
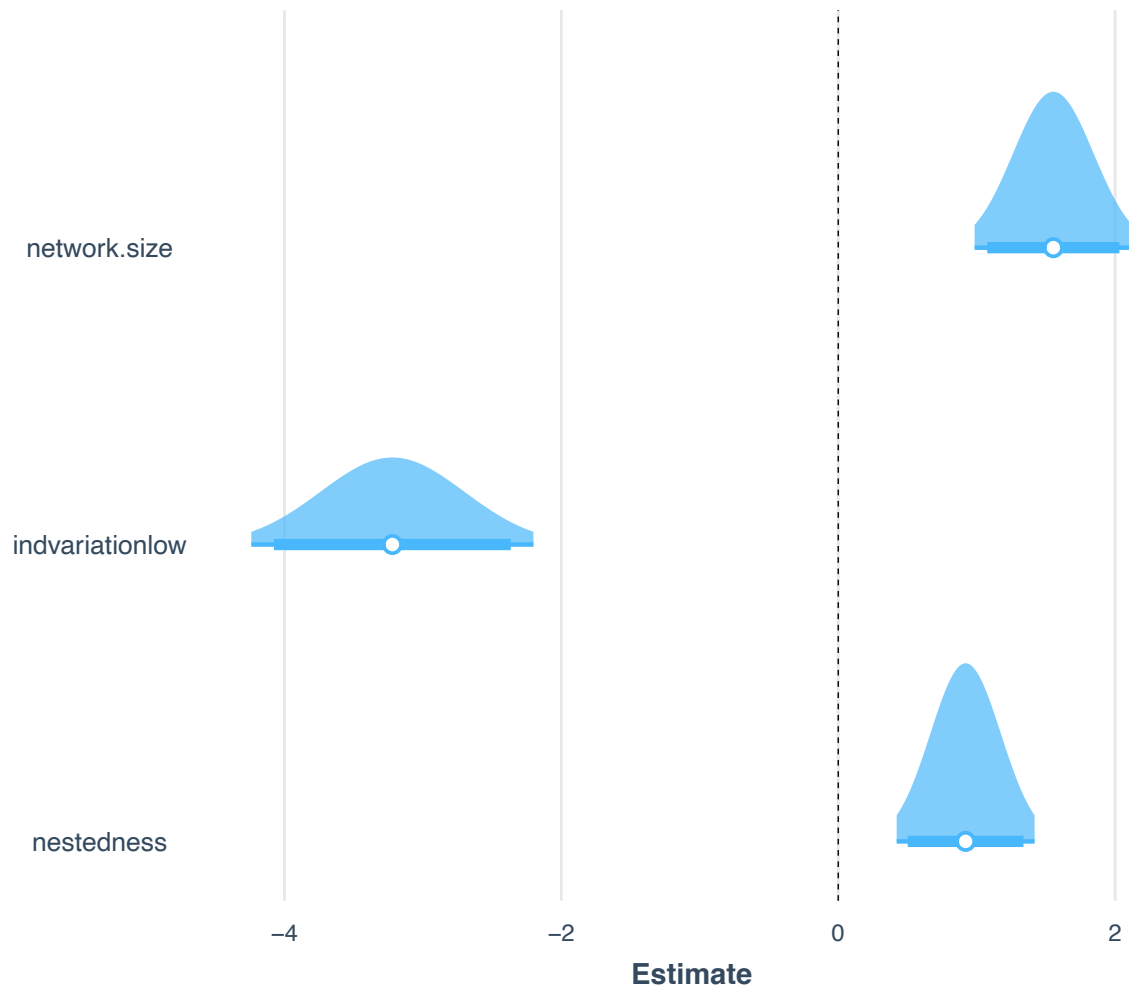
Figure S4: Estimates of predictor variables from generalized linear model with chances of abrupt collapse as the response variable. The distribution shows 95% confidence intervals. The generalized linear model consisted of abruptness of collapse as the response variable, individual variation, nestedness and network size as the predictor variables. Note that connectance as a predictor variable was not included as it was not a significant predictor. From the figure, low individual variation

negatively impacted the occurrence of abruptness of network collapses whereas nestedness and network size positively impacted the occurrence of abrupt collapses.

Supplementary 3: Relaxation of trade-off between number of mutualistic interactions and strength of mutualistic interactions
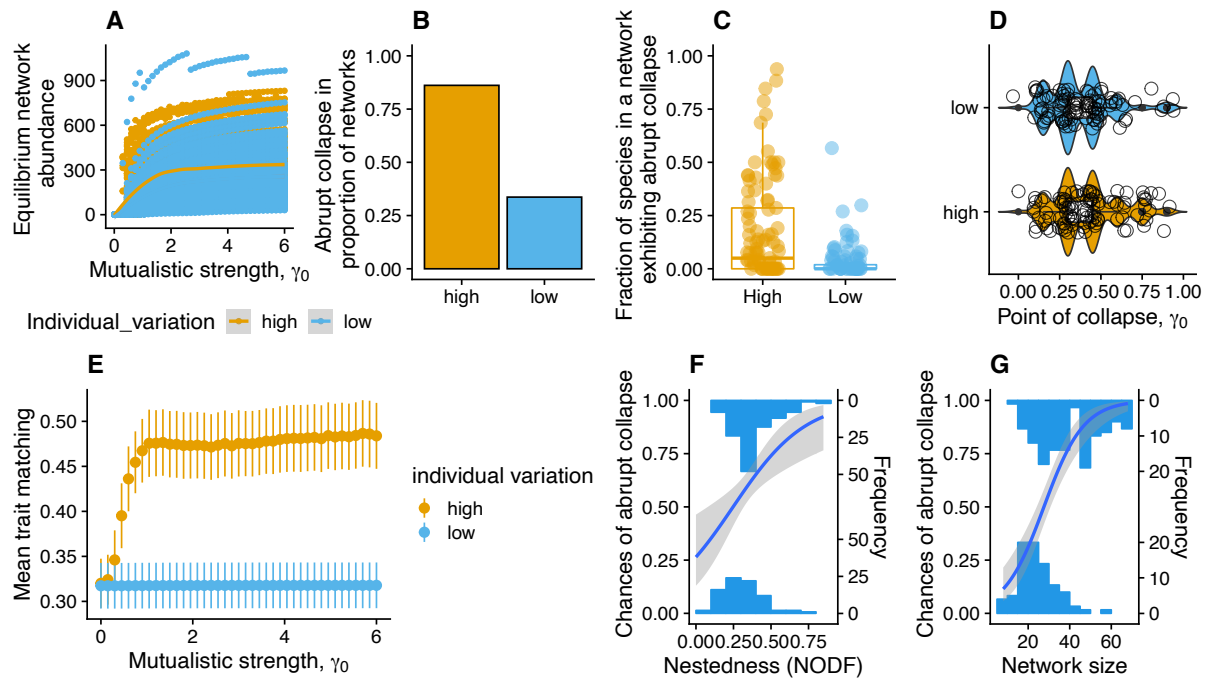
Figure S5: When trade-off between number of mutualistic interactions and average mutualistic interaction was relaxed i.e., $\delta=0$ for all species in a network. (B) 86% of networks with high individual variation went through an abrupt collapse whereas 33% of networks with low individual variation went through an abrupt collapse. C) On average, 17.1% (17.1%$\pm$ 2.3%) of species in a network goes through an abrupt transition when exhibiting high individual variation. In contrast 3.36% (3.36% $\pm$ 0.7%) of species goes through an abrupt transition in a network when they have low individual variation. D) Mutualistic strength at which the networks collapse when species had high variation (0.416 $\pm$ 0.038) versus low individual variation (0.374 $\pm$ 0.0360) did not differ significantly. E) Mean trait matching in networks on average was higher when species exhibited high individual variation in contrast to when species had low individual variation. F) & G) As nestedness and network size increased chances of abrupt collapses increased. Heritability was fixed 0.4. Mean trait values of species were sampled from U[-1,1].
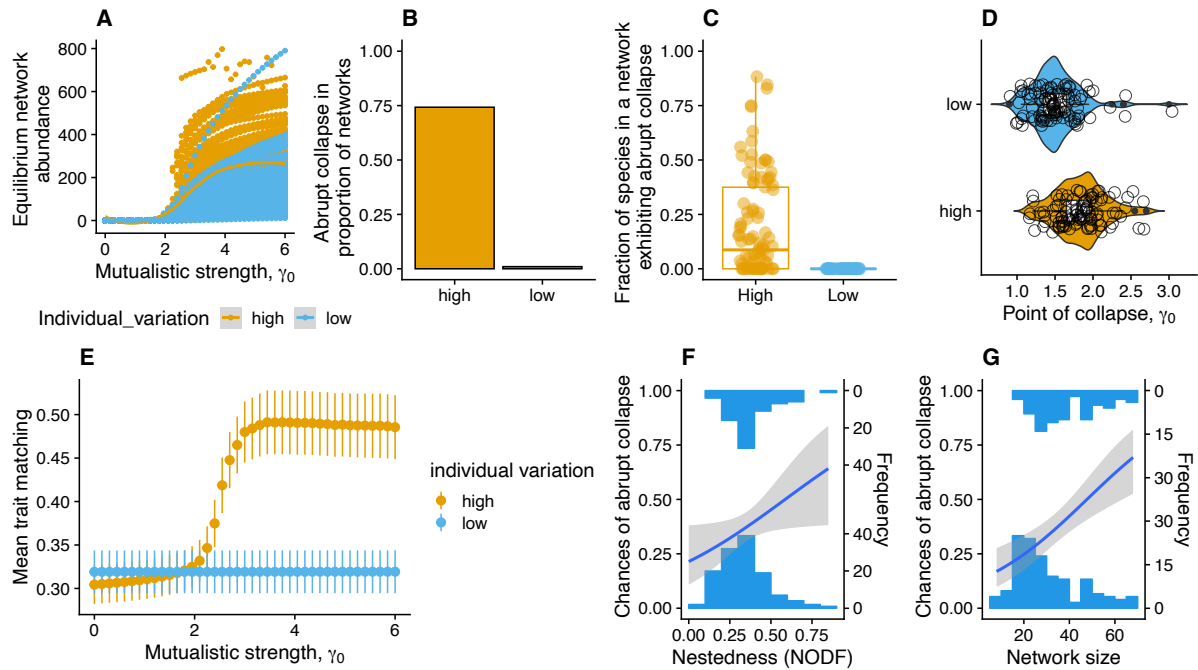
Supporting Information 4: **Addition of multiplicative and additive noise**

Multiplicative noise is added to growth rates of species b_i in equation 2. Rewriting equation 2 with multiplicative noise as:

$$r_i^A(\vec{N}, \vec{p}, z, t) = b_i - \sum_j^{S_A} a_{ij}^A(t)N_j^A(t) + \sum_k^{S_P} N_k^P(t) \int \frac{A_{ik}\gamma(z,z')}{1 + \tau A_{ik}\gamma(z,z')N_k^P} \, p_k(z',t)dz' \quad (2)$$

The intrinsic rate of growth of a species b_i is influenced by random environmental fluctuations that are normal distributed with mean of $b_i(t)$ and variance $\sigma_b^2(t)$ such that:

$$b_i(t) \sim Normal\{ b_i(t), \sigma_b^2(t)\}$$

At each time point intrinsic rate of growth of species was randomly sampled from a normal distribution with mean of $b_i(t) = -0.05$ and $\sigma_b^2(t) = 0.1$ for all species in a network. Results of including multiplicative noise are shown in figure S8.

Additive noise is added to growth rates of species b_i in equation 2. Rewriting equation 2 with additive noise as

$$r_i^A(\vec{N}, \vec{p}, z, t) = b_i - \sum_j^{S_A} a_{ij}^A(t)N_j^A(t) + \sum_k^{S_P} N_k^P(t) \int \frac{A_{ik}\gamma(z,z')}{1 + \tau A_{ik}\gamma(z,z')N_k^P} \, p_k(z',t)dz' \quad + \sigma dW$$

Where, dW is a gaussian white noise process with mean 0 and $\sigma^2 = 0.1$. Results of adding additive white noise to species dynamics are shown in figure S9.
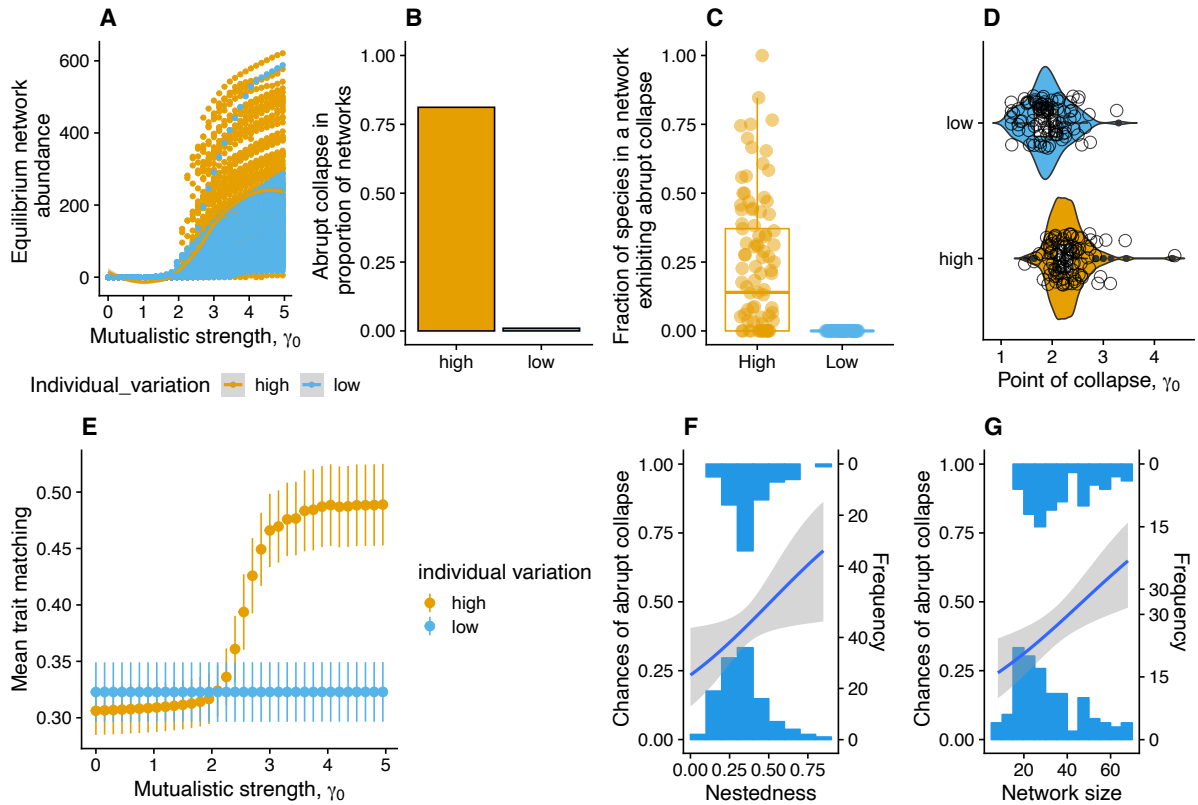
Figure S6: Effects of addition of "multiplicative noise" on the occurrence of abrupt tipping points. Multiplicative noise was added to the growth rates of species i.e., b_i in equation 2. Every time point in a simulation b_i was sampled from a random normal distribution with mean $b_i = -0.05$ and variance of

0.1, such that growth rates of species fluctuated over time. Multiplicative noise had a very slight impact on the occurrence of abrupt tipping points. Fraction of networks with high individual variation that went through an abrupt collapse was slightly higher than when noise was not added (see figure 1) (81% for multiplicative noise to 78% for no noise). C) On average, 16.8% of species in a network goes through an abrupt transition when exhibiting high individual variation. In contrast, 0% of species goes through an abrupt transition in a network when they have low individual variation. D) Mutualistic strength at which the networks collapse when species had high versus low individual variation did not differ significantly. E) Mean trait matching in networks on average was higher when species exhibited high individual variation in contrast to when species had low individual variation. F) & G) As nestedness and network size increased chances of abrupt collapses increased. Heritability was fixed 0.4. Mean trait values of species were sampled from U[-1,1].
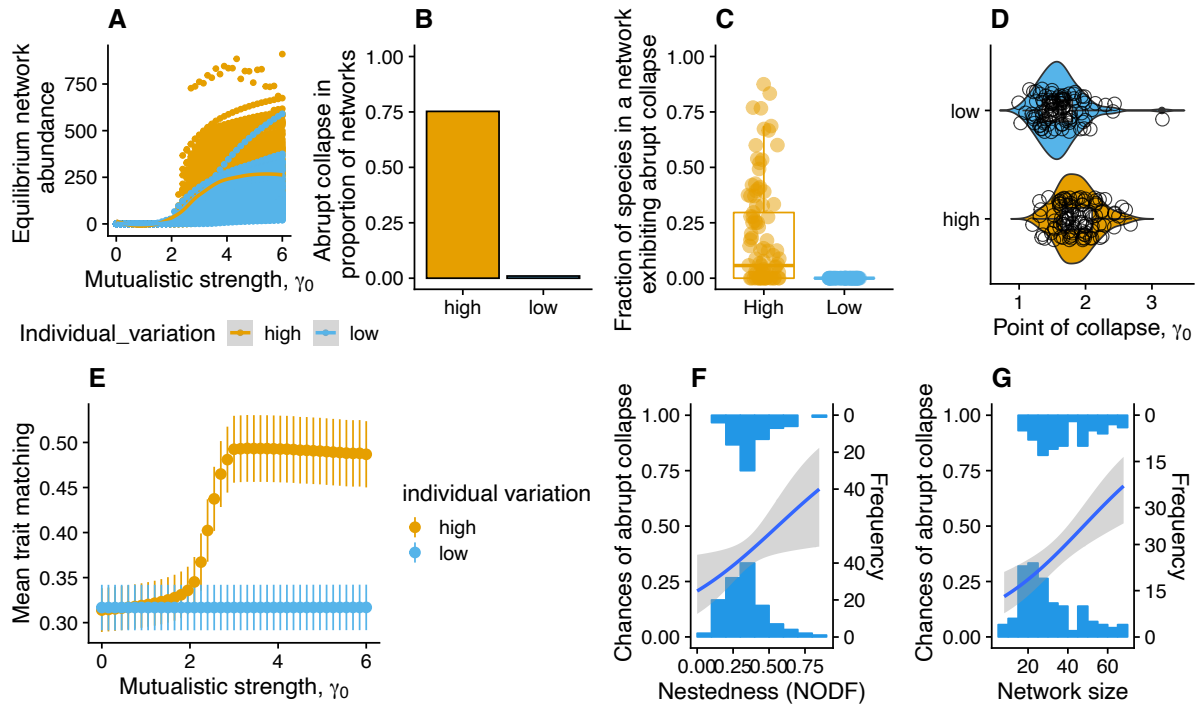

Figure S7: Effects of addition of "additive noise" on the occurrence of abrupt tipping points. Additive noise was added as a white noise with mean 0 and standard deviation of 0.1 to population dynamics via equation 2. Additive noise had a very slight impact on the occurrence of abrupt tipping points. B) Fraction of networks with high individual variation that went through an abrupt collapse was slightly lower than when noise was not added (see figure 1) (81% for multiplicative noise, 74% for additive noise, to 78% for no noise). C) On average, 18.9% of species in a network goes through an abrupt transition when exhibiting high individual variation. In contrast, 0% of species goes through an abrupt transition in a network when they have low individual variation. D) Mutualistic strength at which the networks collapse when species had high versus low individual variation did not differ significantly. E) Mean trait matching in networks on average was higher when species exhibited high individual variation in contrast to when species had low individual variation. F) & G) As nestedness and network size increased chances of abrupt collapses increased. Heritability was fixed 0.4. Mean trait values of species were sampled from U[-1,1].
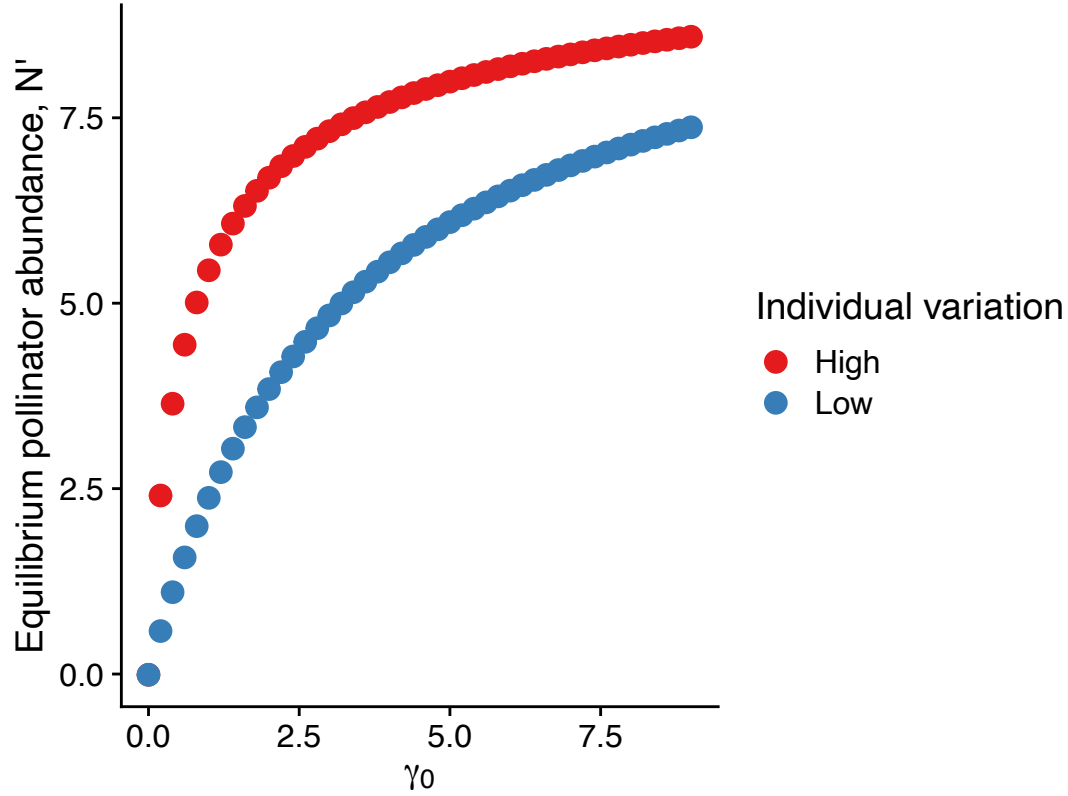
Figure S8: Effects of different sampling distribution for competition coefficients $a_{ij}, i \neq j$. Competitive coefficients were sampled from a random exponential distribution of the form $\lambda e^{-\lambda x}, x > 0$, where λ was fixed at 70. Mean of the exponential distribution is $\frac{1}{\lambda} = 0.014$. Intraspecific competition was fixed at 1 but interspecific competition coefficients were scaled and sampled from random exponential distribution as in the main-text. B) Fraction of networks with high individual variation that went through an abrupt collapse was 75% and for networks with low individual variation was 0.9%. C) On average, 17.6% of species in a network goes through an abrupt transition when exhibiting high individual variation. In contrast, 0% of species goes through an abrupt transition in a network when they have low individual variation. D) Mutualistic strength at which the networks collapse when species had high versus low individual variation did not differ significantly. E) Mean trait matching in networks on average was higher when species exhibited high individual variation in contrast to when species had low individual variation. F) & G) As nestedness and network size increased chances of abrupt collapses increased. Heritability was fixed 0.4. Mean trait values of species were sampled from U[-1,1].

Figure S9: Transition from steady-state to collapse of animal abundance as co-evolutionary interaction strength is gradually decreased in the presence of high and low individual trait variation. Plotted on Y-axis is N' as a function of γ_0 (equation 4a). For the figure, $b_1 = b_2 = -0.05$ (obligate mutualism), $a_{11}^A = a_{11}^P = 1$, equilibrium plant abundance was fixed at $P' = 80$. For the high individual trait variation case, trait variance for the distribution of plant traits $p_1(z,t)$ and animal traits $p_2(z,t)$ i.e., σ_N^2 and σ_P^2 were fixed at 0.5. For the low individual variation case, σ_N^2 and σ_P^2 were fixed at 0.0005. Mean trait values for plant traits and animal traits were $u_N = -1, u_P = -0.1$. Note that the transition to collapse occurs at the same co-evolutionary strength γ_0, although in the presence of high trait variation, the transition was more abrupt for the two species plant-animal system.
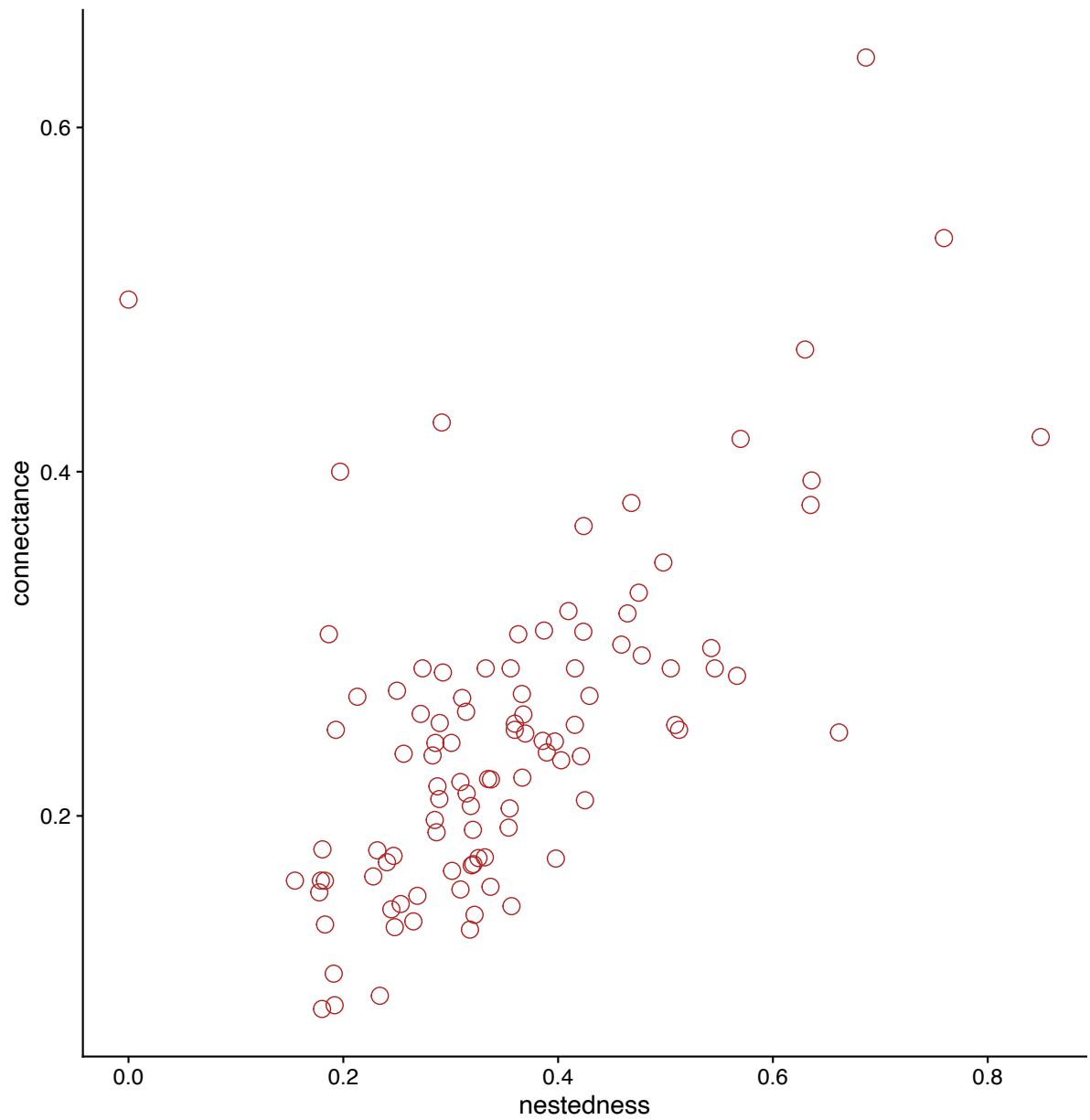
Figure S10: Interrelationship between Nestedness (NODF) and network connectance. Pearson correlation coefficient was estimated to be 0.54
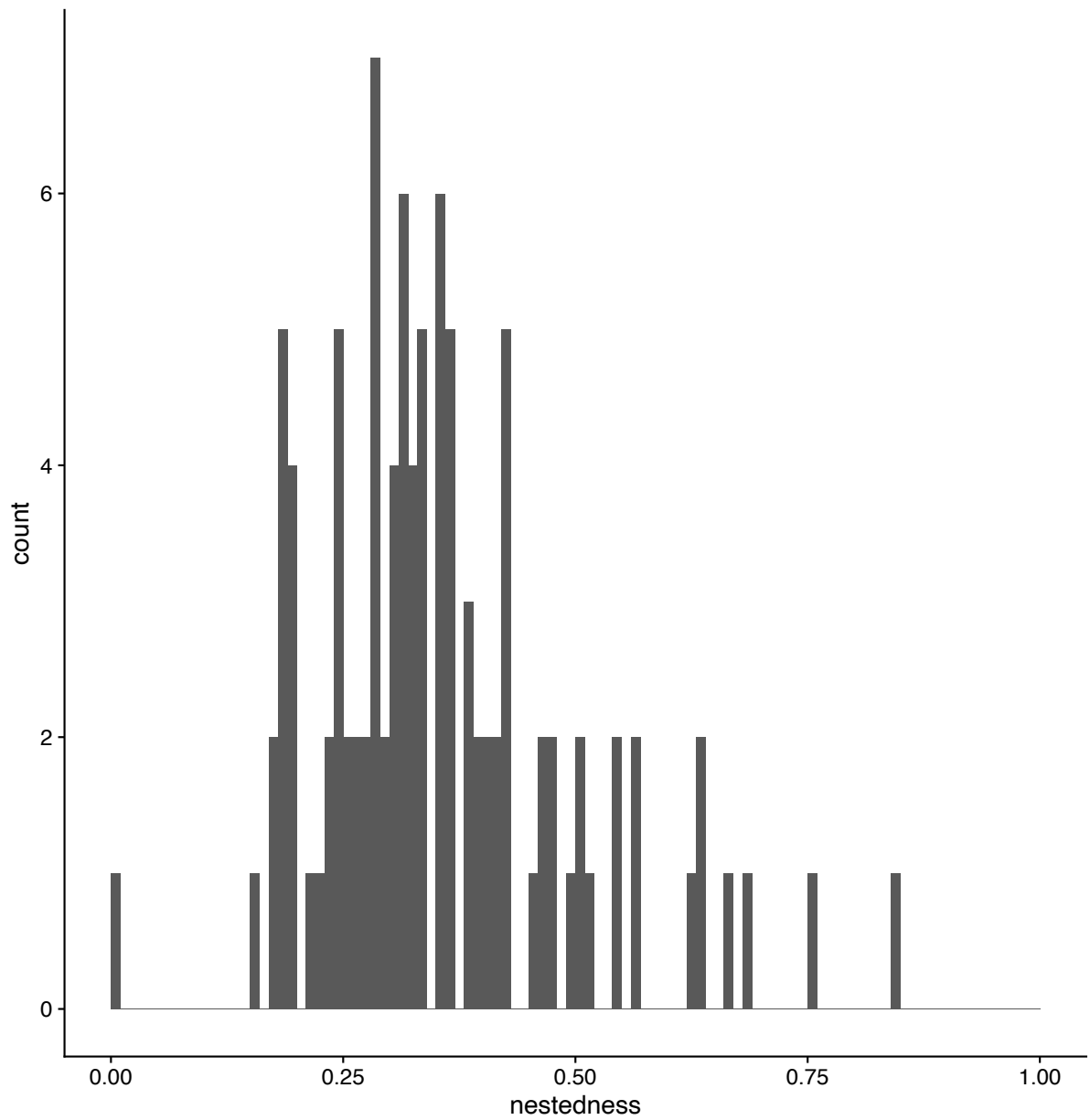
Figure S11: Frequency distribution of nestedness values (NODF) of the 101 mutualistic networks used in the study.