Hypolimnetic density currents traced by sulphur hexafluoride (SF₆)

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ABSTRACT

The artificial tracer sulphur hexafluoride (SF₆) has been used to study the density-driven deep water exchange between two sill-separated basins of Lake Lucerne, Gersauersee and Urnersee. The sources of the density gradients between the two basins are (1) salinity differences between the major inlets due to the different geology of their drainage areas, and (2) temperature differences due to spatial variation of wind forcing. Wind speeds are generally larger in Urnersee, especially in spring during the so-called Föhn events, when winds blow from the south. In contrast, Gersauersee is protected from these winds. In spring 1989, a total of 630 g of SF₆ was released at 80 to 120 m depth in the small Treib Basin located between Urnersee and Gersauersee. During about 100 days the distribution of SF₆ in the lake was determined by gaschromatography. Two models are used to quantify the exchange flow, (1) a one-box mass balance model for SF₆ in the deep part of Treib Basin, and (2) a one-dimensional diffusion/advection model describing the temporal and vertical temperature variation in Urnersee. According to the first model, the flow into the deep hypolimnion of Urnersee, decreases from $21 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ at the end of March to about $8 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ in late April. The second model yields similar flow rates. The decrease of the flow rate during spring, confirmed by both approaches, is consistent (1) with the decreasing strength of the density gradient above the sill during spring and early summer, and (2) with hydrographic information collected in Lake Lucerne during other years.

1. Introduction

In fall 1985, physical investigations were carried out in Lake Lucerne (Switzerland) to study the simultaneous influence of river and wind induced mixing (Wüest et al., 1988). Since the wind exposure of two of the several basins, both about 200 m deep, is significantly different, Lake Lucerne appeared to be an ideal object for such a study. The wind-exposed upstream basin, Urnersee (Fig. 1), is known for its year-long rather large oxygen concentration in the whole water column, while the adja-

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Figure 1. Map of the southern basins of Lake Lucerne (Switzerland) showing the locations of the three underwater sills (V-G, G-T and T-U), the four thermistor string moorings (G, C, T and S) and the two weather stations (WS-Sisikon mounted on a buoy at the center of Urnersee, WS-Beckenried mounted on a dredge barge moored close to the shore of Gersauersee). Contour lines correspond to 100 and 150 m depth. The sill V-G connects Gersauersee to the rest of Lake Lucerne and its outlet. Inset: SF₆ was released into the deep water of the western part of Treib Basin (see Fig. 2)

cent basin, Gersauersee, showed strong oxygen depletion in the deep hypolimnion during the Seventies and Eighties, when Lake Lucerne had become mesotrophic. Furthermore, Urnersee is strongly influenced by the River Reuss, the largest inlet into Lake Lucerne (Fig. 1).

As a great surprise detailed measurements of temperature and electric conductivity (a measure for salinity) led to the conclusion that during spring and early summer of 1986, a density-driven flow must have transported considerable quantities of water from the upper hypolimnion of Gersauersee over two sills (Fig. 2) into the deep layers of Urnersee (Wüest, 1987; Wüest et al., 1988). It was estimated that during May and June 1986 in Urnersee between 50 and 70 percent of the water volume below 100 m was exchanged by lateral currents. A salinity-induced horizontal density gradient between the two basins was identified as the main driving force of the flow.

The electric conductivity in the River Reuss, mainly draining the crystalline parts of the Alps, lies between 80 and 100 $\mu\text{S} \cdot \text{cm}^{-1}$ (salinity $S \approx 70$ to 90 $\text{g} \cdot \text{m}^{-3}$). In contrast, the smaller inlets Muota and Engelberger Aa (Fig. 1), both originating from calcareous parts of the Alps, have electric conductivities between 210 and 250 $\mu\text{S} \cdot \text{cm}^{-1}$ ($S \approx 180$ to 220 $\text{g} \cdot \text{m}^{-3}$). In spite of the pronounced salinity differences

among the major inlets, electric conductivity is remarkably homogeneous throughout the lake (mean of about 205 $\mu\text{S} \cdot \text{cm}^{-1}$, Aeschbach-Hertig et al., 1996). Only a slight horizontal gradient remains at the basin boundaries.

Based on time series of temperature, molecular oxygen concentration, and electric conductivity obtained during the years 1964 to 1974, Van Senden et al. (1990) were able to show that horizontal density gradients between Urnersee and Gersauersee occur regularly in spring. Furthermore, a three-dimensional investigation on the spatial and temporal distribution of water temperature, electric conductivity, tritium, and helium isotopes has shown that lateral density-driven flow also occurs between other basins of Lake Lucerne (Aeschbach-Hertig et al., 1996).

By studying internal seiches in Lake Lucerne, Van Senden and Imboden (1989) found additional evidence for the coupling between the two basins. The first internal horizontal mode of Gersauersee, which has a period of about 60 hours during early summer, propagates over the sills into the deep hypolimnion of Urnersee where it can be clearly distinguished from the eigenmodes of Urnersee. This observation was attributed to the so-called “seiche pumping”, i.e. a periodic spilling of water over the sills from Gersauersee into Urnersee.

Yet, in 1989 direct evidence for the exchange flow was still missing. Therefore, an experiment was designed to directly measure and quantify the flow of water over the sill. A rough estimate based on the total mass flow determined by Wüest et al. (1988) showed that the corresponding mean current velocities would lie well below 1 $\text{cm} \cdot \text{s}^{-1}$. Therefore, it was decided to conduct an experiment with the artificial tracer sulphur hexafluoride (SF₆). Such an experiment, if properly designed, should also allow to determine the integrated exchange volume of water for a given period of time. This article reports on the experiment conducted in spring 1989.

2. Tracer technique

Sulphur hexafluoride (SF₆) is a non-toxic gas of sole anthropogenic origin. It is mainly used as an insulator in electric equipment, especially in high-voltage underground transmission cables. SF₆ is chemically stable in water and only decomposed by UV-radiation in the stratosphere (Singh et al., 1977). In 1992, the average SF₆-concentration in the atmosphere was 2.8 pptv with an annual increase rate of 8.3% (Maiss and Levin, 1994). Due to its large Henry coefficient ($K_{\text{H}} = 3.1 \cdot 10^3 \text{ atm} \cdot \text{L} \cdot \text{mol}^{-1}$ at 15°C) the concentration of SF₆ in water at 15°C in equilibrium with a partial pressure of 2.8 pptv is rather small, i.e. 0.9 $\text{fmol} \cdot \text{L}^{-1}$ (1 $\text{fmol} = 10^{-15} \text{ mol}$). The non-dimensional volumetric mixing ratio between air and water (the so-called non-dimensional Henry coefficient) is 88 and 132 for water at 5 and 15°C, respectively.

In the concentration range of 0.4 to 500 $\text{pmol} \cdot \text{L}^{-1}$ (1 $\text{pmol} = 10^{-12} \text{ mol}$) SF₆ can be detected by gas chromatography combined with an electron capture detector (ECD) without previous extraction or dilution step (Schlatter, 1991; Watson et al., 1991). For the gas chromatographic analysis, 25 mL of water is drawn into a 50 mL glass syringe filled with 25 mL of N₂ (details in Schlatter et al., 1990). By shaking the syringe most of the SF₆ is transferred to the gaseous phase. 1 mL of the N₂/SF₆ gas

mixture is then injected into the gas chromatograph which consists of two columns to be used alternatively (3 m long, 2 mm internal diameter, packed with a 5 Å molecular sieve). The columns are kept at 60 °C. Retention time for SF₆ is about 40 s. The concentration is calculated by integration of the ECD output signal. The analytical error (1 σ) is about 20% for concentrations at 0.7 pmol · L⁻¹ and 9% at 4 pmol · L⁻¹.

If SF₆ is concentrated on a trap (e.g., a polystyrene adsorbent) the detection limit can be lowered to 0.03 fmol · L⁻¹ (Wanninkhof et al., 1991) which is 30 times smaller than the equilibrium concentration with the present atmospheric partial pressure.

SF₆ has been used in limnological and oceanographic investigations since more than ten years (Watson et al., 1987). Early applications were devoted to the quantification of gas exchange rates in lakes (Wanninkhof et al., 1985; Wanninkhof et al., 1987; Wanninkhof and Bliven, 1991) and in the ocean (Wanninkhof, 1992). Mixing processes in the ocean have been studied using deliberately added SF₆ such as in a recent experiment in the Santa Monica Basin off the coast of California (Ledwell et al., 1986; Ledwell and Watson, 1991; Watson et al., 1991; Ledwell and Hickey, 1995), Santa Cruz Basin (Ledwell and Bratkovich, 1995), in the open atlantic water (Ledwell, et al., 1993) and in Lake Constance (Maiss et al., 1994a and 1994b). Wüest et al. (1996) have used SF₆ in Alpachersee, a basin of Lake Lucerne, to compare diapycnal diffusivity measured by tracer and microstructure technique, respectively.

3. Field experiment

The goal of the artificial tracer experiment with SF₆ conducted in Lake Lucerne were:

- to identify the postulated lateral exchange between the two basins, Gersauersee and Urnersee;
- to quantify the water flow across the sill;
- to relate the flow to hydrographic information such as the spatial distribution of water temperature and electric conductivity;
- to compare these results with the volume flow which was estimated earlier.

Morphological and hydrographic data of the lake basins are summarized in Table 1. The small Treib Basin located between Urnersee and Gersauersee was chosen for the release of SF₆ (Figs. 1 and 2). It is separated from the adjacent basins by sills of 93 m and 85 m depth (Fig. 2). Two weather stations (WS) and 6 thermistor strings (at 4 locations) were installed during the experiment. Their locations and depth ranges are shown in Figures 1 and 2.

On March 14 and 15, 1989 (Julian days #73 and #74), a total amount of 630 g SF₆ was released from pressure tubes on board the research boat *Thalassa* by means of a linear diffuser (width 2 m) which was lowered to a depth of 120 m and towed back and forward through the Treib Basin (inset of Fig. 1). The gas mixture contained 99% of N₂ and 1% of SF₆. The bubbles in the plume above the diffuser were completely dissolved in the layer between 80 and 120 m depth. The injected tracer mass

Table 1. Morphology and hydrography of basins at southern end of Lake Lucerne (see Fig. 1)

Location (center of basin)	Urnersee	Treib Basin	Gersauersee
	N 46°57' E 8°36.5'	N 46°59.5' E 8°34'	N 46°59' E 8°29'
Surface area (km ²)	22.0	4.78	25.49
Volume (km ³)	3.16	0.42	3.98
Mean depth (m)	144	87	156
Maximum depth (m)	195	125	214
Length (km)	10	3	10
Mean width (km)	2.2	1.6	2.5
Drainage area ^a (km ²)	932 ^b	1254	1531
Major inlets (Fig. 1)	Reuss	Muota	Engelberger Aa
Major inlets discharge (m ³ · s ⁻¹) ^c	44.7	19.0	11.6
Sills (Figs. 1, 2)	sill T-U	sill G-T	sill V-G
Net outflow at sill ^d (m ³ · s ⁻¹)	50	75	90
Mean residence time (yr) ^e	2.0	0.18	1.4

^a Excluding lake area.^b Excluding the drainage area of the river Muota. Depending on the depth at which the river intrudes into the stratified lake, the water flows into Urnersee or Treib Basin, respectively.^c According to Hydrologisches Jahrbuch der Schweiz (1989).^d Net flow due to river inlets (without exchange across sills caused by wind shear and/or density gradients).^e Calculated from basin volumes and the net flow across the corresponding sill. **Figure 2.** Depth contour along the thalweg of the southern part of Lake Lucerne (see Fig. 1) showing the locations of the two weather stations and the four Aanderaa thermistor strings including length of strings and depth of their lower end (number in italics). The western end of Gersauersee (left) corresponds to the bay near the entrance of the river Engelberger Aa. The hatched area indicates the volume of water into which the tracer was released

Figure 3. Isolines of SF_6 concentration (dark lines; [$\text{pmol} \cdot \text{m}^{-3}$] or [$\text{fmol} \cdot \text{L}^{-1}$]) in Urnersee (right) and Gersauersee (left) between March 20 (panel A) and May 24, 1989 (panel E). Numbers in parantheses are Julian days. Black dots indicate locations of SF_6 sampling. The tracer was released into the Treib Basin on March 14 and 15. Isopycnals are shown as thin lines and labelled as $\sigma_t = \rho - 1000 \text{ kg} \cdot \text{m}^{-3}$ (ρ = water density)

was determined such as to allow detection in the optimum concentration range of the chosen analytical procedure during several weeks. Based on an estimated size of the initial tracer cloud (10 m wide, 30 m thick, total track length 9 km) the average initial SF₆ concentration is calculated to be about 1400 pmol · L⁻¹. The first water samples analyzed for SF₆ showed that dilution by mixing had quickly decreased the *in situ* concentration to values to which the analytical method was tuned.

Water samples for the gas chromatographic analysis aboard the research boat *Thalassa* were taken using 5-liter Niskin bottles. Figure 3 shows isolines of SF₆-concentration (bold lines) along the thalweg (the path along the deepest section of the basins) of the basins from Gersauersee to Urnersee measured 5 to 70 days after tracer release. The black dots indicate the locations where SF₆ samples were taken. Also shown are isopycnals (thin lines) labelled by their σ_t -value defined by $\sigma_t = \rho - 1000 \text{ kg} \cdot \text{m}^{-3}$. Water density ρ was calculated from CTD profiles which were taken at least weekly at several stations along the thalweg. The influence of electric conductivity κ_{20} on ρ was calculated by the equation $\rho(T, \kappa_{20}) = \rho_o(T) \cdot [1 + \beta_\kappa \cdot \kappa_{20}]$, where $\rho_o(T)$ is the density of pure water at temperature T and the coefficient of haline contraction β_κ is defined by (Imboden and Wüest, 1995)

$$\beta_\kappa = \frac{\partial \rho}{\partial \kappa_{20}} = 0.68 \cdot 10^{-3} (\text{kg} \cdot \text{m}^{-3}) (\mu\text{S} \cdot \text{cm}^{-1})^{-1} \quad (1)$$

As shown by the sequence of graphs in Figure 3, within less than a month the major portion of the SF₆ was washed out of the Treib Basin into the deep hypolimnion of Urnersee. A small fraction of the SF₆ was mixed upwards into the thermocline from where, depending on the wind conditions, it was transported into both neighbouring basins. The tracer in the upper layers (20 to 60 m depth) was flushed back from Urnersee through the Treib Basin into Gersauersee.

For a better understanding of the dynamics of SF₆ in the lake, the wind forcing and the structure of water temperature and electric conductivity have to be analyzed in more detail. There are remarkable differences between the two basins, although they are close to each other: In both basins the prevailing wind direction is along the lake axis (Fig. 4), i.e. north/south for Urnersee and east/west for Gersauersee, but wind speeds are generally larger in Urnersee, especially during the so-called Föhn events when strong winds blow from the south. In Urnersee three such events can be identified: The first one lasting from April 1 to 4 (Julian day #91 to #94), albeit shortly interrupted on day #93, was less strong than the second (#99 to #102, April 9 to 12) and third one (#114 to 117, April 24 to 27). These were the only episodes when wind speeds at Urnersee exceeded 8 m · s⁻¹. The Föhn events were less pronounced in Gersauersee, although the 2nd and 3rd event are picked up by the wind sensors. Wind speeds over Gersauersee did not exceed 7.5 m · s⁻¹.

The effect of the wind on the vertical temperature structure is different in the two basins (Fig. 5). In the hypolimnion of Urnersee both the greater wind speed and the weaker density stratification favour large vertical oscillations of the isotherms as well as intense vertical mixing. The latter reduces the stratification even further. The effect of the Föhn events can be clearly seen between days #100 and #103 (April 10 and 13) and between days #114 and #118 (April 24 and 28). Apart from the oscillations every storm causes an irreversible deepening of the isotherms.

Figure 4. Wind speed and wind direction (wind blowing *from*) recorded 8 m above the water surface by the two weather stations WS-Beckenried and WS-Sisikon (Figs. 1 and 2). Wind roses (insets) show that in both basins winds are predominantly blowing along the basin axes. Winds are generally stronger in Urnersee. Wind speeds exceeding $8 \text{ m} \cdot \text{s}^{-1}$ in Urnersee and $5 \text{ m} \cdot \text{s}^{-1}$ in Gersauersee occur only during Föhn events when winds are generally blowing from the south

Figure 5. Isotherms in the hypolimnia of Gersauersee and Urnersee calculated from temperature data recorded by the thermistor strings TS Gersau and TS Schiller, respectively (Figs. 1 and 2). Values are smoothed by a 3 day moving average. See text for further explanations

In contrast, the hypolimnion of Gersauersee remains clearly stratified during the whole observation period. Although the Föhn events trigger larger internal displacements (days #94 and #116: April 4 and 26), the overall development of the temperature structure can be best characterised by a slow but steady sinking of the isotherms caused by the downward transport of heat by turbulent mixing. This process is also reflected by the depth of the isohalines (lines of constant salinity, approximated by the lines of constant κ_{20} , see Fig. 6): In Gersauersee, isohalines slowly

Figure 6. Isohalines in Gersauersee and Urnersee represented by electric conductivity κ_{20} ($\mu\text{S} \cdot \text{cm}^{-1}$) calculated from CTD profiles (indicated by dashed vertical lines)

Figure 7. (a) Temperature T and electric conductivity κ_{20} measured in Gersauersee (G) and Urnersee (U) on 19 April 1989 (#109), 35 days after tracer release. The vertical arrows indicate the effect of water flowing from Gersauersee over the G-T/T-U sills into Urnersee. (b) Contribution of water temperature and electric conductivity (salinity), respectively, to the horizontal density difference between Gersauersee and Urnersee calculated from the profiles shown in (a)

move downwards if averaged over the whole observation period. This corresponds to an upward transport of salt.

The opposite phenomenon is observed in Urnersee (Fig. 6): The isohalines tend to move upwards, i.e. salt is transported downwards. This cannot be explained by vertical diffusive mixing since the Fickian flux would be directed against the salinity gradient. As postulated earlier by Wüest et al. (1988) the density flow over the T-U sill into the deep hypolimnion of Urnersee would cause an upward flow (upwelling) in that basin.

Indeed, these authors observed an upward dislocation of the isotherms and took this as an indirect confirmation of their postulated density flow. A similar effect is thus to be expected for the isohalines.

The lateral density current so clearly seen in the SF_6 experiment must be driven by the horizontal density gradient between the basins. As shown in Figure 3, the density gradient above the sill loses its strength from spring to early summer. In Figure 7a temperature and electric conductivity profiles from both basins are compared. The corresponding horizontal density differences given in Figure 7b are splitted into the contributions originating from the temperature and salinity difference, respectively. According to Figure 7b, in mid April two thirds of the horizontal density difference between Gersauersee and Urnersee is caused by temperature, one third by electric conductivity (salinity).

The following section is focused on the quantification of the density-driven flow between the two basins. Other aspects of the dynamics of the tracer distribution will not be further discussed. Additional information can be found elsewhere (Schlatter, 1991; Van Senden and Imboden, 1989; Wüest, 1987; Wüest et al., 1988).

4. Quantifying the inter-basin lateral exchange by mathematical models

Two rather simple models will be used to quantify the lateral flow of water into the deep hypolimnion of Urnersee: (1) a one-box model to be applied to the mass balance of SF_6 in the deep part of Treib Basin, and (2) a one-dimensional diffusion/advection model to describe the temporal and vertical temperature variation in Urnersee.

One-box model for SF_6 in Treib Basin

The temporal rate of change of the average SF_6 -concentration C_T in the deep water of Treib Basin (volume V_T , defined by the water mass below the isopycnal $\sigma_t = 0.08 \text{ kg} \cdot \text{m}^{-3}$) is described by taking account of the following processes (Fig. 8):

- (a) Temporal rate of change of water volume V_T .
- (b) Inflow of water with $\sigma_t \geq 0.08 \text{ kg} \cdot \text{m}^{-3}$ from Gersauersee into V_T (flow rate Q_G). This water does not contain SF_6 ($C_G = 0$ in Fig. 8).
- (c) Outflow of water from V_T into the deep part of Urnersee (flow rate Q_U , SF_6 -concentration βC_T with $\beta = C_{90}/C_T$, C_{90} is the SF_6 -concentration in Treib Basin at 90 m depth, i.e. at depth of the T-U sill).

- (d) Vertical diffusive flux of SF₆ into the surface layer of the Treib Basin, F_T . The boundary between V_T and the surface layer is defined by $\sigma_t = 0.08 \text{ kg} \cdot \text{m}^{-3}$.

With the variables defined above the mass balance equation for SF₆ in the water volume V_T is given by

$$\frac{d}{dt}(V_T C_T) = -\beta Q_T C_T - F_T \quad (1)$$

which leads to

$$\frac{dC_T}{dt} = -\frac{C_T}{V_T} \left(\beta Q_T + \frac{dV_T}{dt} \right) - \frac{F_T}{V_T} \quad (2)$$

All quantities except Q_T can be determined from the measured SF₆-concentrations and the depth of the isopycnal $\sigma_t = 0.08 \text{ kg} \cdot \text{m}^{-3}$ in the Treib Basin:

$$Q_T = -\frac{1}{\beta} \left(\frac{V_T}{C_T} \cdot \frac{dC_T}{dt} + \frac{dV_T}{dt} + \frac{F_T}{C_T} \right) \quad (3)$$

Equation (3) is evaluated in Table 2. The flow into the deep hypolimnion of Urnersee, Q_T , decreases from $(21 \pm 15) \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ at the end of March to about $(8 \pm 6) \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ in late April. Obviously, the errors attributed to the different variables that are determined from the field data sum up to a rather large uncertainty in the final result of the flow rate Q_T . For instance, the depth of the isopycnal $\sigma_t = 0.08 \text{ kg} \cdot \text{m}^{-3}$ is oscillating due to internal seiches and has therefore to be calculated as an average from several density profiles. The vertical flux F_T is estimated from the

Figure 8. Box model for SF₆ in the deep water of Treib Basin (volume V_T , defined by $\sigma_t \geq 0.08 \text{ kg} \cdot \text{m}^{-3}$). Note that the depth of the $\sigma_t = 0.08 \text{ kg} \cdot \text{m}^{-3}$ isopycnal – and thus of the volume V_T – is not constant as a function of time (Table 2). See text for definition of symbols

Table 2. Evaluation of SF₆ balance for the deep water of Treib Basin^a

	Date (# = Julian day)		
	March 30 (#89)	April 11 (#101)	May 2 (#122)
Treib Basin			
Depth of $\sigma_t = 0.08 \text{ kg m}^{-3}$ isopycnal (m)	35 ± 8	60 ± 8	80 ± 5
Volume of water with $\sigma_t \geq 0.08 \text{ kg m}^{-3}$ V_T (10^6 m^3)	262 ± 34	163 ± 29	93 ± 17
Mean SF ₆ -concentration in V_T C_T ($10^{-9} \text{ mol} \cdot \text{m}^{-3}$)	7.05 ± 1.16	2.26 ± 0.34	0.67 ± 0.11
SF ₆ -concentration at sill C_{90} ($10^{-9} \text{ mol} \cdot \text{m}^{-3}$)	10.3 ± 2.0	2.4 ± 0.5	1.03 ± 0.38
Concentration ratio $\beta = C_{90}/C_T$	1.5 ± 0.4	1.1 ± 0.3	1.5 ± 0.6
Calculation of Q_T: Contributions to Eq. (3)^b			
$V_T \frac{dC_T}{dt}$ ($10^6 \text{ m}^3 \cdot \text{d}^{-1}$)	-18 ± 8	-6.6 ± 2.8	
dV_T/dt ($10^6 \text{ m}^3 \cdot \text{d}^{-1}$)	-8.3 ± 3.7	-3.3 ± 1.6	
F_T/C_T ($10^6 \text{ m}^3 \cdot \text{d}^{-1}$)	<1	<1	
Flow into Urnersee, Q_T ($10^6 \text{ m}^3 \cdot \text{d}^{-1}$)	21 ± 15	7.7 ± 5.9	

^a Errors given as 2 σ -values (95% probability).^b β , V_T , C_T calculated as the arithmetic mean of values at the beginning and the end of the time interval, respectively.

1st Fickianlaw using the vertical gradient of SF₆ at the upper boundary of the Treib Basin deep water $\left(\frac{\partial C}{\partial z} < 70 \text{ pmol} \cdot \text{m}^{-4}\right)$. As shown in Table 2, the corresponding 3rd term of Eq. (3), F_T/C_T , is negligible even if a rather large vertical diffusivity of $5 \text{ m}^2 \cdot \text{d}^{-1}$ is used, a value determined by Wüest (1987) for the more wind exposed Urnersee.

In principle, the model for the Treib Basin could be checked with a corresponding mass balance model for the Urnersee deep water. Due to the much larger volume of Urnersee and the difficulty to calculate an average SF₆ concentration from the rather inhomogeneous tracer distribution in this basin (see panels A, B in Fig. 3), the uncertainty of such a model calculation would be very large. Furthermore, during the later part of the experiment (observations on days #101 and #122; April 11 and Mai 2), a significant portion of the SF₆ is found in the upper layers of Urnersee (depth less than 60 m). Based on the available SF₆ data it is not possible to decide whether this tracer mass originates mainly from upward mixing in Urnersee (flux F_U in Fig. 8) or from lateral mixing at the surface between Treib Basin and Urnersee. This uncertainty would introduce an additional error into the calculation of Q_T .

One-dimensional diffusion/advection model for temperature in Urnersee

There is an alternative way to check the mass balance of Treib Basin. Wüest et al. (1988) have used the upward shift of the vertical temperature profile in the hypolimnion of Urnersee to calculate the density-driven flow from Treib Basin into Urnersee. For May and June 1986, they obtained a total mass flow between 500 and $700 \times 10^6 \text{ m}^3$. This corresponds to a mean flow rate of 8 to $12 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$, in good agreement with the results of Table 2.

Since the vertical temperature gradient in Urnersee is very small and variable, especially during the initial phase of the experiment (Fig. 5), it is not possible to apply this method to the period of the SF₆-experiment. However, later in the year 1989 the vertical gradients were better defined. Thus, the temperature profiles recorded in Urnersee between May and August 1989 can be used as input for the following one-dimensional diffusion/advection temperature model (Fig. 9):

$$A \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(AK \frac{\partial T}{\partial z} \right) - \frac{\partial}{\partial z} (AwT) + qT^* \quad (4)$$

with the following depth-dependent variables:

- z: vertical coordinate, positive upwards [m]
- T: water temperature
- A: area of Urnersee at depth z
- K: vertical turbulent diffusivity [$\text{m}^2 \cdot \text{s}^{-1}$]
- w: vertical advection velocity, positive upwards [$\text{m} \cdot \text{s}^{-1}$]
- q: lateral in/outflow per unit depth [$\text{m}^2 \cdot \text{s}^{-1}$]

The last term on the right-hand side of Eq. (4) describes the lateral inflow or outflow of water at depth z (see Fig. 9). It is quantified by $q(z)$, the flow per unit depth increment and takes account of the lateral flow from an adjacent lake basin or of a “density flow along the lake bottom”. Since q can have either sign, the temperature of the transported water T^* is given by:

$$T^* = \begin{cases} T_{\text{in}} & \text{for } q > 0 \quad (\text{inflow of water from Treib Basin}) \\ T(z) & \text{for } q < 0 \quad (\text{outflow of water from Urnersee into Treib Basin}) \end{cases} \quad (5)$$

The continuity equation for water requires

$$\frac{\partial}{\partial z} (Aw) = q \quad (6)$$

Inserting Eq. (6) into (4) and evaluating the derivatives yields

$$\frac{\partial T}{\partial t} - K \frac{\partial^2 T}{\partial z^2} - aK \frac{\partial T}{\partial z} - \frac{\partial K}{\partial z} \frac{\partial T}{\partial z} \equiv G = \frac{q}{A} (T^* - T) - w \frac{\partial T}{\partial z} \quad (7)$$

with $a = \frac{1}{A} \frac{dA}{dz} = \frac{d \ln A}{dz}$ (characteristic topographic function of the lake). The left-hand side of the equation, abbreviated by G , is the temporal temperature gradient corrected for the influence of vertical turbulent mixing (second to fourth term on the left-hand side of Eq. (7)). This correction is specially relevant above 50 m where the vertical temperature gradients are larger (Fig. 7 and further explanations below). For the following we assume $K(z) = \text{const.}$, thus the last term on the left-hand side of Eq. (7) becomes zero.

For the evaluation of Eq. (7) we are led by the following conceptual picture (Fig. 9): Deep water is flowing along the thalweg from the Treib Basin into Urnersee. Depending on its density and mixing with local water, the deep water from Treib Basin either penetrates to the deepest layer of Urnersee and/or spreads horizontally in Urnersee at some intermediate depth thus producing a positive $q(z)$ at those depths. Above the depth of the sill T-U water can, in principle, flow back into Treib Basin. This results in a negative $q(z)$ -value and a decrease of $w(z)$. At some intermediate depth z_{max} the vertical upwelling flow ($A \cdot w$) must be maximal. The corresponding flow is an estimate for the total water transport from Treib Basin into Urnersee, i.e. $Q_T \approx A(z_{\text{max}}) \cdot w(z_{\text{max}})$.

To solve Eq. (7) for $w(z)$ an iterative procedure is adopted. First it is assumed that the temperature difference between the inflowing and the local Urnersee water is zero, i.e. $T^* - T \approx 0$. This is a reasonable assumption, as the water temperature in Gersauersee at the T-U sill depth is about equal to the temperature in the deep layers of Urnersee, the potential endpoint of the flow (see arrows in Fig. 7a). With this assumption Eq. (7) can be solved for $(A \cdot w)$ which, according to Eq. (6), is

Figure 9. One-dimensional vertical diffusion/advection model for water temperature with lateral input or outflow of water. The exchange is quantified by the lateral water flow per unit depth $q(z)$ [$\text{m}^2 \cdot \text{s}^{-1}$]

equal to the integral of $q(z)$ from the lake bottom z_b to depth z :

$$A w = -A \frac{G}{\partial T / \partial z} = \int_{z_b}^z q(z') dz' \quad (8)$$

and

$$q(z) = \frac{\partial}{\partial z} (A w) = \frac{\partial}{\partial z} \left(-A \frac{G}{\partial T / \partial z} \right) \quad (9)$$

with the definition introduced in Eq. (7) and $\frac{\partial K}{\partial z} = 0$

$$G = \frac{\partial T}{\partial t} - K \frac{\partial^2 T}{\partial z^2} - aK \frac{\partial T}{\partial z} \quad (10)$$

To check the iteration the calculation is repeated by additionally taking into account the term $\frac{q}{A} (T^* - T)$ where the preliminary values obtained for $q(z)$ and the sign rule (Eq. (5)) are used. Then Eq. (8) is modified to

$$A w' = - (A G - q (T^* - T)) \left(\frac{\partial T}{\partial z} \right)^{-1} \quad (11)$$

Figure 10. Vertical water flow in Urnersee ($A \cdot w$) calculated from lateral water flow integrated from the bottom of Urnersee (Eq. 8). The lateral flows are obtained from the one-dimensional diffusion/advection model of Urnersee as described in the text. The total lateral flow over the sills into Urnersee, Q_T , can be estimated from the maximum of the $(A \cdot w)$ -curve. Model calculations are made for late spring (May 2 to 24, 1989) and summer (May 24 to August 4, 1989)

The procedure can then be repeated until a consistent solution for $q(z)$ is obtained. The result is shown in Figure 10 for two time periods by plotting the lateral volume flow ($A \cdot w$). A constant vertical diffusivity of $K_z = 2 \text{ m}^2 \cdot \text{d}^{-1}$ and an estimated lateral temperature difference of $(T^* - T) = 0.05 \text{ K}$ are adopted.

As mentioned before, the best estimate of Q_T is given by the maximum value of $(A \cdot w)$. A closer analysis of the numerical iteration shows that for the first period (May 2 to May 24) the influence of turbulent mixing on the maximum value of $(A \cdot w)$ is negligible, but the lateral temperature difference shifts the maximum by about +10% to $23 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$. For the second period (May 24 to August 4) the turbulent diffusivity is more important while the lateral temperature difference is negligible. The maximum of $(A \cdot w)$ is less distinct and smaller than in spring, i.e. about $5 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$.

5. Summary and conclusion

The SF_6 experiment in the Gersauersee/Urnersee basin of Lake Lucerne performed in spring 1989 has clearly demonstrated the existence of a horizontal current from Gersauersee and Treib Basin into the deep water of Urnersee. The flow was postulated by Wüest et al. (1988) based on the temporal evolution of the vertical temperature profile in Urnersee during summer and on the horizontal density gradient across the sills derived from CTD measurements. The tracer experiment provided direct evidence for the occurrence of the density-driven exchange between the two basins in spring and summer.

It is more difficult to quantify the flow. The SF_6 mass balance applied to the deep part of Treib Basin at the end of March yields a total flow of about $20 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ which decreases to about $10 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ in late April. Although this numbers are similar to the flow calculated by Wüest et al. (1988) (8 to $12 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$) as a mean value for May and June 1986), the result from the tracer experiment is rather uncertain (relative error about 75%). Furthermore, application of a one-dimensional diffusion/advection scheme, the model applied by Wüest et al. (1988) to the temperature profiles of spring and summer 1989, yields a flow of $23 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ in May and of $5 \cdot 10^6 \text{ m}^3 \cdot \text{d}^{-1}$ for June to early August.

It is plausible that both the agreement and disagreement of these results is not accidental. Although the estimated flow rates are all of the same order, they seem to differ from month to month and year to year since they are driven by a subtle interplay of several processes which, among other factors, depend on the weather and, as shown by Van Senden and Imboden (1989), may be intermittent in time. This raises the question of the driving forces that keep the lateral density current running.

As discussed earlier, the horizontal density gradient between Gersauersee and Urnersee above the two sills results from both horizontal temperature as well as salinity gradients. For instance, for the situation shown in Figure 7 (April 19) two thirds of the density difference comes from temperature, on third from salinity. Both components make the water in Gersauersee denser than the water in Urnersee at the same depth. This applies to all available data sets. Since horizontal density currents tend to erode the gradients, processes must exist which continuously renew these gradients. As discussed in the introduction, the different chemical composition of the major tributaries, represents one process which keeps the salinity

gradient alive. In addition, the upper hypolimnion of Gersauersee is generally colder by a few tenth of a degree than in Urnersee, although the annual variation of the aerial heat content in both basins is nearly identical. The reason for this difference is the larger wind forcing in Urnersee, i.e. the more efficient vertical heat transport. Consequently, compared to Gersauersee water temperature in Urnersee is generally smaller at the surface and greater in the upper hypolimnion.

Based on this conclusion it is hypothesised that the phenomenon of density-induced lateral exchange between different parts of a lake must be rather common where ever the shape of a lake and the surrounding topography causes differential mixing, i.e. spatial variation of wind mixing from one basin to the next. Indeed, such lateral density gradients are frequently observed at a sill or a sea strait in the ocean. Examples are the Strait of Gibraltar where the more saline Mediterranean water meets the water of the North Atlantic, or the connection between less saline estuaries and the open sea. Although mathematical models have been developed for the quantification of the exchange through such straits (Farmer and Armi, 1986) the assumptions made for such models (two-layer critical flow, large density differences) are not applicable to a fresh water situation as "delicate" as the one found in Lake Lucerne with a relatively large opening between two water bodies and a rather small density gradient. In fact, the difficulty to develop physical mixing models for such situations is one important reason why tracer experiments are valuable to quantify the density-driven exchange flow.

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