Probability density of displacement and overturning length scales under diverse stratification

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[1] Vertical overturns, produced by turbulence in density-stratified lakes and oceans, are often quantified by the Thorpe scale, L_T . The correlation between L_T and the Ozmidov (energy-containing) scale can be used to estimate rates of turbulent dissipation and vertical diffusivities. Based on temperature microstructure measurements from several stratified lakes, the probability density functions (pdf's) of Thorpe displacements and overturning length scales are distinguished and discussed. The stratification was varying over five orders of magnitude between the different data sets, resulting in Thorpe scales between 1 cm and 100 m. It is shown that the analyzed pdf's follow a universal form that can be empirically described by an exponential function and parameterized by a single length scale. The functional form of the pdf of overturning length scales can be related to the inertial subrange, which is found in temperature, as well as in length-scale fluctuation spectra. As a result, the ratio between the maximum displacement length scale, L_{max} , and the Thorpe scale depends on the rate of turbulent kinetic energy dissipation. Based on the universal pdf of overturning length scales, we show with numerical simulations that L_{max} is a more appropriate macroscopic length scale than L_T for the estimation of turbulence based on displacements in temperature profiles.

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1. Introduction

[2] Small-scale turbulent mixing in lakes, estuaries and oceans is of great relevance for many processes ranging from global (climate) to regional (water quality) scales. Unfortunately, modeling and measuring at those scales contain many pitfalls and are still very challenging. Microstructure measurements, successfully used for two decades in lakes, estuaries and oceans, brought valuable insight into the nature and dynamics of these processes [e.g., Gregg, 1987; Imberger and Ivey, 1991]. One major outcome was the realization of the strong intermittency and patchiness of small-scale turbulence, especially under stratification. As a consequence, high temporal and spatial resolutions of measurements are required, which often conflicts with the large effort needed for acquiring those highly specialized data. Hence, there is a great interest to overcome this disadvantage by using theories and parameterizations for small-scale dynamics, which are based on larger scales. Those scales are more easily accessible by models and simpler to resolve by conventional profilers. The focus of this paper is to provide strong evidence on the uniformity of the distributions of

displacement and overturning length scales, which can be used as a tool to infer the small-scale dynamics of turbulent overturns from the largest overturns present in profiles.

[3] The vertical extent of turbulent overturns can be obtained from high-resolution temperature casts, such as temperature microstructure profiles. In the absence of any turbulence, temperature profiles become increasingly monotonic due to the smoothing effect of molecular diffusion. Turbulence, in contrast, produces vertical overturns that appear as inversions in measured temperature profiles. By reordering a measured temperature profile to the corresponding monotonic profile, the displacement length scale l (the vertical distance that each measured temperature value has to be moved up- or downward to its position in the monotonic profile) can be calculated.

[4] The root-mean square (rms) of the displacements l within an appropriate segment of the profile is the so-called Thorpe scale L_T [Thorpe, 1977]. It is a statistical measure of the vertical size of overturning eddies in that segment [Dillon, 1982]. Various measurements have shown that the Thorpe scale is nearly equal to the energy containing length scale, the Ozmidov scale L_O [e.g., Dillon, 1982; Peters and Gregg, 1988; Moum, 1996; Lorke, 1998]. This relation can be used to calculate the dissipation rate ϵ from L_T and the stability N^2 [Dillon and Park,

1987; Ferron *et al.*, 1998] using the definition of the Ozmidov length scale L_O [Ozmidov, 1965]:

$$L_O = \sqrt{\frac{\varepsilon}{N^3}} \quad (1)$$

1.1. Relation Between L_T and L_O

[5] Baumert and Peters [2000] showed that the ratio L_O/L_T is not constant but depends on the gradient Richardson number. The range of variability, however, is small compared to the overall error of the estimated value of ε , especially in low energetic systems such as lakes. Direct numerical simulations (DNS) of the temporal evolution of turbulent length scales within breaking Kelvin–Helmholtz billows revealed that the length scale ratio L_O/L_T can be interpreted as a “clock”, which increases monotonically as the turbulent event evolves [Smyth and Moum, 2000]. The ratio changes from $L_O/L_T < 0.5$ for young, preturbulent overturns to about 1 after the transition to turbulence and increases beyond 1 as turbulence finally decays. Wijesekera and Dillon [1997] sorted numerous turbulent patches observed in microstructure profiles from the oceanic thermocline into classes of different L_O/L_T ratios and showed that the Shannon entropy is higher for patches with larger L_O/L_T ratios. These two results are directly related as a temporal increase of L_O/L_T [Smyth and Moum, 2000] corresponds to an increase of entropy [Wijesekera and Dillon, 1997] as turbulence evolves. By analyzing the results of the DNS and microstructure measurements Smyth *et al.* [2001] found a parameterization for the mixing efficiency γ_{mix} in terms of the length scale ratio L_O/L_T . The ratio of the turbulent buoyancy flux to the turbulent kinetic energy dissipation, γ_{mix} , is crucial for determining the buoyancy flux and the turbulent diffusivity from dissipation rates ε [Osborn, 1980].

1.2. Maximum Displacement Length Scales

[6] In some cases, the maximum displacement length scale L_{max} (the maximum value of l within the respective segments of temperature profiles) was considered as an appropriate measure of the overturning scale. Moum [1996] and Peters *et al.* [1995] found a linear relationship between L_T and L_{max} for profiles from the equatorial undercurrent. However, Lorke [1998] found a $L_{max} \sim L_T^{0.85}$ power law from microstructure profiles in a strongly stratified lake. This relation also holds for microstructure profiles from other lakes, as demonstrated in Figure 1, which shows L_{max} versus L_T as obtained from three different lake data sets, collected under very different conditions of mixing and stratification. The strong correlation holds over four orders of magnitude and indicates that (i) the Thorpe scale is determined by the large overturns near to the maximum displacement length and (ii) the size distribution of displacement length scales, which determines the ratio between L_{max} and L_T , has a universal form. A universal length-scale distribution was assumed in a recent publication by Stansfield *et al.* [2001] based on the analysis of the probability distribution of displacements in CTD profiles from the Juan de Fuca Strait. The DNS of Smyth *et al.* [2001] showed that the ratio L_{max}/L_T remained almost constant during the temporal evolution of turbulence and hence

Figure 1. Maximum versus *root-mean square* displacement length scales within segments of temperature microstructure profiles from Lake Baikal (Russia), Lake Neuchâtel (Switzerland) and Müggelsee (Germany). The relationship $L_{max} = L_T$ is indicated. Data from Lake Neuchâtel and Müggelsee were ensemble averaged over several subsequent profiles. The inset shows a comparison of the individual (each single profile) and ensemble averaged estimates of the Müggelsee data.

independently confirm the expectation of a universal distribution of displacement length scales.

1.3. Spectrum of Displacement Length Scales

[7] The displacement length scales l obtained by Thorpe reordering of temperature profiles were estimated from temperature fluctuations T' . The two are related by the mean local temperature gradient $\partial T/\partial z$; i.e., $T' = l \cdot \partial T/\partial z$. The spectrum of temperature fluctuations follows a universal form with an inertial subrange [Grant and Moillet, 1968] and a sharp high-wave number cut-off in the viscous-convective subrange [Batchelor, 1959]. The high wave number spectrum, called the Batchelor spectrum, can be used to estimate the rate of viscous dissipation of kinetic energy ε by fitting the theoretical Batchelor spectrum to the measured T' spectrum [Batchelor, 1959; Dillon and Caldwell, 1980]. A sample from Lake Baikal (Figure 2) shows a comparison of a spectrum of temperature fluctuations T' and a spectrum of displacement length scales (transformed to the corresponding temperature fluctuations $l \cdot \partial T/\partial z$ using the local mean temperature gradient $\partial T/\partial z$). Both spectra agree well for length scales larger than the diffusive cut-off (at about 100 rad/m in Figure 2). The roll-off in the length-scale fluctuation spectrum, however, is less steep. The difference at these small scales is expected, because a local mean temperature gradient $\partial T/\partial z$ is not appropriate for calculating temperature fluctuations beyond the diffusive cut-off scale. The effective temperature gradients $\partial T'/\partial z$ within such small structures become highly variable and much larger than the local background gradient $\partial T/\partial z$. However, the inertial subrange and the transition to the diffusive subrange are perfectly represented in the displacement spectrum and hence small-scale turbulence features are contained in the displacement spectrum.

Figure 2. Spectra of temperature fluctuations T' and displacement length scales multiplied by a mean temperature gradient, i.e., $l \cdot \partial T / \partial z$. The corresponding theoretical spectra for the inertial subrange and the Batchelor diffusive cut-off are indicated. The data are from Lake Baikal, near 250 m depth. The dissipation rate ε estimated from Batchelor fitting was $9 \cdot 10^{-12}$ W/kg. For wave numbers larger than 200 rad/m, noise is dominating the spectra.

[8] In this paper, the size distribution of displacement length scales will be examined in terms of its probability density function. Using temperature-microstructure data from three different lakes with stratification varying over five orders of magnitude, we will show that the occurrence probability of displacement and overturning length scales has a universal form. This can partially be explained by the inertial-convective subrange of the energy spectrum of temperature fluctuations. The data sets, covering four orders of magnitude of overturning length scales, from a few centimeters up to 100 m in Lake Baikal, demonstrate the scale-independence of the results. In all data sets, the universal size distribution of displacement length scales yields a strong relationship between the Thorpe and the maximum displacement length scales, which allows inferring the Thorpe scale from maximum displacement length scales. From this it will be possible to estimate Thorpe scales more properly from well-resolved CTD data even without resolving the high-end temperature variance. By bearing in mind the limitations as depicted above, the Thorpe scales can be used to estimate rates of dissipation of turbulent kinetic energy or mixing efficiencies.

2. Data Sets and Instrumentation

[9] The following three sets of microstructure data were selected for this analysis because they span almost the entire range of turbulent length scales occurring in natural stratified water bodies and cover different stratified mixing conditions. The turbulence was shear-driven in all cases and convective regions were excluded from the analysis.

2.1. Lake Baikal

[10] A set of 52 temperature microstructure profiles was collected in 1996 in the three basins of Lake Baikal down to

a maximum depth of about 625 m [Ravens *et al.*, 2000]. Although below 250 m, the water column of Lake Baikal is permanently stratified, the stability is extremely low. Due to the lack of significant salinity gradients, the water column stability is determined by temperature only. Because the vertical temperature gradients are small and the thermal expansivity near the temperature of maximum density is low too, the stability N^2 (typically $N^2 = 10^{-8} \text{ s}^{-2}$ [Ravens *et al.*, 2000]) is very weak compared to oceanic conditions. The weak stratification gave rise to extremely large vertical displacements in the temperature microstructure.

2.2. Lake Neuchâtel

[11] A total of 966 temperature microstructure profiles were measured in Lake Neuchâtel (Switzerland) in spring 1996 [Kocsis *et al.*, 1999] (A. Simon *et al.*, Momentum and turbulent kinetic energy balance in the surface boundary layer for under-developed waves, submitted to *Journal of Geophysical Research*, 2001, hereinafter referred to as Simon *et al.*, submitted manuscript, 2001). The profiles were measured in the upward-rising mode from a maximum depth of 30 m to the surface. The sampling interval of sequential profiles was 15 min. Global radiation lead to near-surface stratification during daytime, while the water column below remained weakly stratified due to nocturnal cooling-induced convection. Only the profiles collected during the stratified daytime were used for this study. An example of a temperature profile from Lake Neuchâtel and calculated Thorpe scales and dissipation rates are shown in Figure 3.

2.3. Müggelsee

[12] In Müggelsee, a small and shallow lake near Berlin (Germany), Lorke [1998] measured 90 microstructure profiles during varying wind-forcing. The maximum depth of this lake is only 7 m but, in contrast to the other study sites,

Figure 3. An example of a temperature microstructure profile measured in Lake Neuchâtel (left panel) and calculated Thorpe scales and turbulent kinetic energy dissipation rates (right panel). The inset on the left panel shows a blow-up of the fine and microstructure within the weakly stratified part of the profile.

the thermal stratification (typically $N^2 = 10^{-3} \text{ s}^{-2}$ and larger) was rather strong, compared to oceanic conditions.

[13] Whereas the data in Lake Baikal and in Lake Neuchâtel were measured using a modified Seabird SBE 11 [Gloor, 1995], the Müggelsee data were measured with a self-made microstructure profiler [Lorke, 1998]. Both probes used an FP07 thermistor (Thermometrics, New Jersey, USA) and were operated either freely sinking or slowly rising. The profiling speed of the Seabird probe was about 8 cm s^{-1} with a sampling frequency of 96 Hz. In addition to the temperature microstructure sensor, the self-made profiler was equipped with a shear probe enabling an independent estimate of the dissipation rate of turbulent kinetic energy [e.g., Oakey, 1982; Lorke, 1998]. The profiling speed was 30 cm s^{-1} and sampling rate was 1.6 kHz. Thus the vertical data resolution of the temperature profile was $\approx 1 \text{ mm}$ for Lake Baikal and Lake Neuchâtel and $\approx 0.2 \text{ mm}$ for Müggelsee. However, the physical resolution of temperature fluctuations, which is limited by the response time of the temperature sensor and its correction [Gregg and Meagher, 1980], was about 1 to 2 mm in both cases.

3. Data Processing

[14] Displacement length scales l were calculated by reordering the complete measured microstructure profiles. It can be expected that the histograms of displacement length scales are related to the level of turbulence as well as to the stratification. Both quantities are changing with depth for all data sets, as exemplified in Figure 3. Hence depth segmentation is inevitable. The profiles were segmented after reordering using a fixed segmentation length for each profile. The choice of the segmentation procedure and segmentation length can be crucial for all subsequent length scale estimations, as will be shown in section 7. For highly intermittent turbulence in the thermocline of the ocean or in deep lakes, the length scale analysis is usually performed for turbulent patches or even for individual overturns, which can be defined as a profile section for which $\sum_i l_i = 0$, while $l_i \neq 0$ for most i [Dillon 1982; Peters et al., 1995].

[15] Vertical displacements are more continuously created throughout the water column for the Lake Neuchâtel and Müggelsee data sets (e.g., Figure 3). Simon et al. (submitted manuscript, 2001) and Lorke [1998] found that the mean dissipation rates ε scaled at least qualitatively with the law-of-the-wall for the wind-forced surface boundary layer ($\varepsilon \sim 1/z$) in both data sets. The strong vertical gradient of the turbulence demands segmentation lengths as small as possible at least near the surface. The fixed segmentation lengths used throughout the following analyses were chosen individually in a way that they completely cover the largest overturns.

[16] For each segment, the root mean square L_T , the maximum value $L_{\max}$, and the histogram $P(l)$ [counts] of the displacement lengths l were calculated. Zero displacements were not included for these estimations in order to make the calculations more comparable to the patch-oriented analyses used in other studies (see above). However, a comparison showed that their influence on L_T is negligible (less than 10%). In order to cover the whole

Figure 4. Probability density functions of displacement length scales for different depth segments of the sample profile from Lake Neuchâtel shown in Figure 3. The given depth values correspond to the centers of the depth segments respectively. The segmentation length was 5 meters. Data points are shown for the first distribution to visualize the bin-size resolution. The inset plot shows the same data on a half-logarithmic scale in order to emphasize the exponential-like behavior.

range of overturns, the histograms of the absolute values of the displacement length scales were calculated using a logarithmic bin-size distribution with 136 bins between 10^{-5} and 900 m in size, being aware that these extreme limits have no physical meaning. The bin-size resolution can be estimated from the sample distributions shown in Figure 4.

[17] The histograms $P(l)$ were used to calculate $H(l)$ [m^{-1}], the probability density functions (pdf's) of the displacement length scales by dividing the histograms $P(l)$ by the nonuniform bin sizes Δ_i [m] and by the total number of nonzero displacements N_i within the respective segments. $H(l)$ can be nonzero only for $0 \leq l \leq L_{\max}$ and its integral is equal to unity:

$$\int_0^{L_{\max}} H(l) dl = 1 \quad (2)$$

The root mean square value of the displacement length scales L_T can be calculated from $H(l)$ by:

$$L_T = \sqrt{\int_0^{L_{\max}} l^2 \cdot H(l) dl} \quad (3)$$

[18] Because of the high temporal resolution of the Lake Neuchâtel data set (one profile every 15 min), the probability density functions were ensemble averaged over 3 subsequent profiles within the same depth bins. Figure 4 shows an example of the calculated pdf of displacement length scales

Figure 5. Schematic sketch of the structure of the probability distributions of displacement length scales for an ideal overturn. The left panel shows a linear temperature profile with two ideal overturns (inversions) and the corresponding displacement length scales as obtained by Thorpe reordering. The right panel shows the resulting probability distribution of displacement length scales for the corresponding temperature profile (not scaled for probability density).

for the example profile from Figure 3. Dissipation rates of turbulent kinetic energy ϵ were estimated from the spectra of temperature fluctuations using the Batchelor method following Gloor [1995] and Kocsis *et al.* [1999].

4. Overturns and Displacement Length Scales

[19] Displacement length scales calculated by Thorpe reordering contain some special features. Figure 5 shows two overturns, exactly upside down in an ideal, linear temperature profile. The according histograms of the absolute values of the displacement length scales are boxcar functions with an absolute height of two (one positive and one negative displacement) and with a length of the respective overturn height L . Therefore the content of the pdf $H(l)$ of such a histogram $P(l)$, which maps a profile segment containing several nonoverlapping overturns, will scale with the sum of those individual boxcar functions, as indicated in Figure 5. Since each overturn of size L is contributing to $H(l)$ uniformly at all scales $0 \leq l \leq L$ instead of giving a singular contribution at $l = L$, $H(l)$ cannot be interpreted as the pdf for the occurrence of overturns of the size L . The content of $H(l)$ for a certain displacement length l is biased by all overturns with heights $L > l$. The pdf of displacements for a profile section, which is dominated by a single, large overturn, will always look like a boxcar function, whereas the actual pdf for the occurrence of such overturns $H_N(l)$ would be a delta-function.

[20] The ratio $L_{\max}/L_T$ for a boxcar-like pdf can be calculated by using equation (3). Applying $H(l) = \text{const}$ leads to a constant ratio of $L_{\max}/L_T = \sqrt{3}$ and hence to the linear relationship

$$L_T = \frac{1}{\sqrt{3}} L_{\max}. \quad (4)$$

Continuously lined-up and nonoverlapping overturns with a uniform size distribution up to the maximum length $L_{\max}$ will result in an overlay of many stacked boxcar functions (Figure 5) leading to a linearly decreasing pdf of displacement length scales $H(l)$ with

$$H(l) = \frac{2}{L_{\max}} \left(1 - \frac{l}{L_{\max}} \right). \quad (5)$$

Integrating equation (3) with $H(l)$ according to equation (5) leads to the ratio $L_{\max}/L_T = \sqrt{6}$ for the stacked boxcar functions as indicated in Figure 5.

[21] Another constraint of the Thorpe method is that in real profiles there are many inversions that could be interpreted as smaller overturns within large overturning structures. This was demonstrated for instance by Peters *et al.* [1995] by obtaining a mean ratio between the vertical extent of overturns L_{ot} to the maximum displacements $L_{\max}$ within these overturns of $L_{ot}/L_{\max} = 1.5$ in the Pacific Equatorial Undercurrent. Overlapping and embedded overturns are not counted individually by applying the Thorpe reordering procedure but they will alter the boxcar-like pdf for the displacement lengths of the larger structures. Thus the embodiment of small-scale structures within a larger structure are not resolved by this method and will result in an increasing underestimation of $H(l)$ and $H_N(l)$ with decreasing l .

5. Probability Density Functions of Displacement Length Scales $H(l)$

[22] The probability density functions $H(l)$ for five different depth-segments of the same temperature microstructure profile from Lake Neuchâtel are shown in Figure 4. In general $H(l)$ is decaying exponentially for increasing displacement length with a distinct cut-off shortly before the maximum displacement length $L_{\max}$. The cut-off becomes more abrupt for large maximum displacement lengths, especially when they approach the segmentation length. For a given profile and given segmentation, the absolute content of $H(l)$ at the smallest scales seems also to be a monotonic function of the maximum displacement length. For smaller $L_{\max}$, the pdf at small displacements is becoming greater (e.g., the near-surface distributions in Figure 4, where stratification limits the vertical extent of the overturns).

[23] A generalization of these features is shown in Figure 6, where sample pdf's from all data sets are compared. The four distributions cover more than three orders of magnitude of the maximum displacement length $L_{\max}$, ranging from 0.12 to 175 m. Also three orders of magnitude are covered by the content of the pdf $H(l)$ at the smallest displacements. The larger the maximum displacement, the more the distribution approaches the boxcar function, because the corresponding profile segment becomes dominated by a single large overturn, as explained in Figure 5. For smaller maximum displacements the decrease of $H(l)$ is more gradual.

[24] To estimate the contributions of the different displacement lengths to the Thorpe scale, a cumulative root mean square (*rms*) length l_{rms} as a function of a variably

Figure 6. Probability density functions of displacement length scales from all three data sets. The locations and the corresponding maximum displacement length scales are given for each curve. The chosen segmentation lengths are 70 m for Lake Baikal, 3 m for Lake Neuchâtel and 1.5 m for Müggelsee.

chosen maximum displacement length l_{max} was calculated using:

$$l_{rms}(l_{max}) = \sqrt{\frac{\int_0^{l_{max}} H(l) \cdot l^2 dl}{\int_0^{l_{max}} H(l) dl}}. \quad (6)$$

When the limit of integration l_{max} approaches the maximum displacement length scale L_{max} , the cumulative scale l_{rms} approaches the Thorpe scale L_T . For the single, ideal overturn, the integration of the corresponding boxcar-like pdf results in a linear relationship between l_{rms} and l_{max} with a slope of $1/\sqrt{3}$, as shown above. This linear relationship also becomes apparent in Figure 7, where l_{rms} is plotted as a function of l_{max} for the length scale distributions of Figure 6. For small values of l_{max} , all of the curves show the characteristic linear $1/\sqrt{3}$. With l_{max} approaching L_{max} the l_{rms} curves bend toward saturation and remain constant above L_{max} because, by definition, no more displacements contribute. The sharp bend-over is more pronounced for large maximum displacements, reflecting the more boxcar-like cut-off in the probability density distributions from Figure 6, while for smaller maximum displacements the transition to saturation is much more gradual. As a result of these apparent distribution features, the relationship between L_T and L_{max} becomes slightly nonlinear as indicated in Figure 7 and shown in greater detail in Figure 1.

[25] The general form of the pdf from Figure 6 and their integrals in Figure 7 suggest a simple way to transform these distributions into a dimensionless and universal form. It is straightforward to scale the displacement length by dividing it by the maximum displacement scale L_{max} , i.e., replace the ordinate l by l/L_{max} . For dimensional reasons, the probability density needs to be multiplied by L_{max} , i.e.:

$$l \rightarrow l/L_{max}; H(l) \rightarrow L_{max} \cdot H(l).$$

Figure 7. Cumulative root-mean square displacement length l_{rms} scale as a function of the upper limit of integration l_{max} according to equation (6) for the probability density functions of the data shown in Figure 6. The dots (■) represent the common estimates of L_T and L_{max} respectively and the thick line shows a linear relationship.

This scaling is also straightforward by reconsidering the special features of the estimation of the displacement length scales from section 4 and is implicitly given in equation (5).

[26] By applying this relatively simple scaling, the density functions collapse into a universal form (especially at larger scales), as shown in Figure 8. For the lack of a better theory, the collapsed nondimensional curve was fit to an exponential decay law:

$$H(l) \cdot L_{max} = e^{b-a/l/L_{max}} \quad (7)$$

with $b = 2.0$ and $a = 7.3$ as the best fit parameters. The fit is also plotted in Figure 8 and resembles the data very well especially in the cut-off range, which provides the most contributions to L_T (see Figure 7). Some data from Lake

Figure 8. Nondimensionlized probability density functions $H(l) \cdot L_{max}$ and the fitted exponential decay law according to equation (7). The pdf's from 25 complete microstructure profiles are shown. Additionally the function $H(l) \cdot L_{max} \propto 1 - (l/L_{max})^{-2/3}$ is shown (see discussion).

Figure 9. The same data as in Figure 1 including the theoretical relationships between L_T and $L_{\max}$ according to the single overturn boxcar model ($L_{\max} = L_T \sqrt{3}$) and the exponential distribution model ($L_{\max} = L_T \cdot 7.3/\sqrt{2}$). Additionally, a fit to the function $L_{\max} = C \cdot L_T^{6/7}$ is shown (inertial subrange model; see discussion).

Baikal show systematic deviations (underestimations) from the exponential behavior at small $l/L_{\max}$. These points belong to profile sections with very large displacements (>100 m) so that even the largest overturns are not completely resolved within the chosen 70 m long profile segment. The systematic underestimation is not surprising in this case. The integral of the pdf (equation (2)) is invariant against the scaling proposed above and remains unity by definition. In order to maintain the normalization of the pdf according to equation (2), the two coefficients a and b in equation (7) are not independent but are related by $e^b = a/(1 - e^{-a})$.

[27] Integration of equation (7) according to equation (6) leads to a lengthy expression for $l_{rms}(l_{\max})$ which resembles the curves in Figure 8 very well (not shown) and converges in the limit $l_{\max} \rightarrow L_{\max}$ to:

$$L_T = \frac{\sqrt{2}}{a} \cdot L_{\max} \approx 0.194 \cdot L_{\max} \quad (8)$$

For the estimated $a = 7.3$, equation (8) provides a ratio $L_{\max}/L_T = 5.15$ which is about three times larger than the ratio $L_{\max}/L_T$ of equation (4) which applies to a boxcar-like pdf of a single overturn.

[28] The two calculated linear relationships between L_T and $L_{\max}$ (the one based on the boxcar-like pdf in equation (4) and the one based on the exponentially decaying pdf in equation (8)) are plotted together with the data from Figure 1 in Figure 9. Here, it becomes obvious that the two linear relationships act as boundaries for the data and the three different data sets have the individual trend to move from the lower boundary (exponential model) for small L_T to the upper boundary (boxcar model) for large L_T . These trends are indicated in Figure 9 and can be attributed to the increasing ratio between the profile segmentation length and $L_{\max}$ within these segments. All three data sets shown in Figure 9 were evaluated using different but fixed profile

segmentation lengths. When $L_{\max}$ is increasing and approaching the profile segmentation length, the segment portions, which are covered by large overturns, are also growing. In the limit, there is only one big overturn, covering the entire segment resulting in a boxcar-like pdf. At the other limit, if $L_{\max}$ becomes small compared to the profile segmentation length, the exponential-like pdf model applies. It should be noted, that this tendency is an artifact of the applied segmentation, which will be discussed in greater detail in section 7.

[29] In summary: The content of the probability density functions $H(l)$ at the displacement length l is the result of the accumulation of many different overturns with scales larger than l . $H(l)$ can be fitted empirically to an exponentially decaying function, described by only one parameter, e.g., L_T or $L_{\max}$. If the maximum displacement scale approaches or exceeds the segment length, the pdf $H(l)$ changes from an exponential-like to a boxcar-like form. In both cases the Thorpe scale is proportional to the maximum displacement length scale, i.e., $L_T \propto L_{\max}$, however the ratio $L_T/L_{\max}$ is about 3.0 times smaller in the second case as an artifact of the applied segmentation method. By using equation (1) for calculating the dissipation rate of turbulent kinetic energy ϵ from the Thorpe scale, the factor of three would produce an error of one order of magnitude in ϵ .

6. Probability Density Functions of Turbulent Overturns

[30] The pdf $H(l)$ quantify the length-scale distributions of the displacement scales as obtained by the Thorpe reordering procedure. As pointed out in the introduction $H(l)$ can not be regarded as the pdf for the effective eddy sizes, because each overturn biases the displacement pdf at smaller scales (Figure 5). This bias can be removed by taking the negative derivative of $H(l)$ with respect to l :

$$H_N(l) = -\frac{1}{C} \frac{\partial H(l)}{\partial l} \quad (9)$$

where $H_N(l)$ [m^{-1}] can be interpreted as the number pdf of turbulent overturns. The factor C [m^{-1}] normalizes the integral of $H_N(l)$ to 1 (equation (2)).

[31] For an ideal boxcar-like $H(l)$ of one single overturn, equation (9) gives the expected delta function $\delta(L_{\max})$ at the overturn scale $L_{\max}$. The artificial bias of the Thorpe reordering procedure on the pdf is removed in $H_N(l)$. The derivative of the empirical exponential decay model in $H(l)$ (equation (7)) would again lead to an exponential form in $H_N(l)$. However, by taking the numerical derivative of the estimated $H(l)$, the exponential form is less pronounced (Figure 10) and obviously only a first-order approximation. $H_N(l)$ follows an $l^{-5/3}$ power law rather than an exponential form over a wide range.

[32] Now $H_N(l)$ will be used to construct a more deterministic model for the pdf of the displacement length scales $H(l)$ than the empirical exponential decay. From Figure 10 it can be assumed that the number of overturns N_b , or better, their probability density $H_N(l)$, is proportional to $l^{-5/3}$. To incorporate the bias produced by the Thorpe reordering

Figure 10. Derivative of the scaled (nondimensional) probability density functions $H(l)$ from Figure 8 with respect to l plotted versus the scaled displacement length $l/l_{\max}$. As pointed out in the text, these curves give an estimate of the unbiased probability density function of turbulent overturns $H_N(l)$.

algorithm and hence calculate $H(l)$ by inverting equation (9), $H_N(l)$ must be integrated in the limits from l to $L_{\max}$:

$$H(l) \propto \int_l^{L_{\max}} H_N(l') dl' \propto \int_l^{L_{\max}} l'^{-5/3} dl' \propto 1 - (l/L_{\max})^{-2/3} \quad (10)$$

The result of equation (10) is plotted in its dimensionless form in Figure 8 together with the data and the empirical exponential function according to equation (7). The comparison indicates that equation (10) acts as an upper boundary for the data rather than resembling their mean. The deviations can again be attributed to the finite segment length. In general, the particular segment length as a sampling interval for $H(l)$ is not sufficient to cover a complete set of nonoverlapping overturns following the nonlinear probability distribution described by equation (10). When the segment length becomes small relative to the size of the largest overturns near $L_{\max}$, the chance of sampling a complete set of overturns following $H(l)$ decreases. However, for an infinitely long profile segment, the pdf of displacement scales $H(l)$ should follow equation (10), disregarding the effect of overlapping and embedded eddies.

[33] The functional form of $H_N(l)$ can be interpreted in the context of the inertial subrange in the spectrum of temperature and length scale fluctuations shown in Figure 2. The inertial subrange corresponds to the energy containing scales, which covers most of the observed displacement scales, before the effect of diffusion becomes important. The different size classes l of the distribution $H(l)$ can be interpreted as reciprocals of the wave number $k = 1/l$ and each temperature inversion as being a single realization of a turbulent eddy. As shown in the introduction, the spectral energy in the inertial subrange falls as a function of wave number k with $k^{-5/3}$ for both temperature

and length-scale fluctuations (Figure 2). The energy is transferred from larger to smaller scales (energy cascade), while the total turbulent energy per unit mass is conserved by definition. This energy conservation implies that the number of eddies per unit mass has to increase proportional to $k^{5/3}$ in the wave number domain or to decrease with $l^{-5/3}$ in the length-scale domain, i.e.

$$E \propto \int H_N(l) \cdot E(l) dl \propto \int l^{-5/3} \cdot l^{5/3} dl = \text{const.} \quad (11)$$

This conclusion directly confirms the observation that $H_N(l) \propto l^{-5/3}$ as shown in Figure 10.

[34] Small eddies with associated length scales beyond the inertial subrange become rapidly dissipated and one should expect their pdf to decrease more pronounced below the diffusive cut-off. Such a decrease is, however, not observable in Figure 10. $H(l)$ in Figure 8 shows a wide scatter at small length scales and generally a constant mean absolute value. This scatter, which can be attributed to progressive overlapping of overturns and probably also to noise, explains that the diffusive cut-off can not be observed in Figure 10, where $H_N(l)$ was calculated according to equation (9) as the numerical derivative of $H(l)$. The constancy of $H(l)$ at small $l/L_{\max}$, however, indicates that no more overturns occurred and the constant content of $H(l)$ represents the accumulated bias from larger scales.

[35] The same argumentation can be used for the discussion of the nonlinear relationship between L_T and $L_{\max}$ as shown in Figures 1 and 9. The probability density distribution $H_N(l) \propto l^{-5/3}$ will result in a pdf of displacement length scales $H(l) \propto l^{-2/3}$ by inverting equation (9):

$$H(l) \propto C \cdot \int H_N(l) dl \propto C \cdot l^{-2/3} \quad (12)$$

Applying this result to equation (3) to calculate L_T as a function of $L_{\max}$ yields:

$$L_{\max} \propto C \cdot L_T^{6/7} \propto L_T^{0.85} \quad (13)$$

This is actually the slope of the lines in Figure 9 that represents the trend of the three individual data sets and agrees with the empirical relationship $L_{\max} = L_T^{0.85}$ observed by Lorke [1998]. However, there is an offset between the respective fits of the data sets in Figure 9 to equation (13), which is related to the normalization constant C (in equation (9)). C is mainly determined by the lower limit of integration in equation (12) and hence is related to the length scale l^* at which the transition from the inertial subrange to the viscous-convective subrange (Batchelor spectrum, see Figure 2) occurs. Kolmogorov [1941] estimated a lower bound of the transition length scale l^* with

$$l^* \gg \left(\frac{\nu^3}{\varepsilon} \right)^{1/4} \quad (14)$$

as a prerequisite for the existence of an inertial subrange in spectra of temperature fluctuations. Thus l^* should scale with $\varepsilon^{-1/4}$. With higher rates of dissipation, the transition

Table 1. Estimated Scaling Factors C^* and Mean Rates of Turbulent Kinetic Energy Dissipation ϵ for the Three Considered Data Sets

	C^*, m^{-1}	$\epsilon, \text{W kg}^{-1}$
Lake Baikal	5	$5 \cdot 10^{-11}$
Lake Neuchatel	2.9	$1 \cdot 10^{-9}$
Mueggelsee	2.3	$5 \cdot 10^{-8}$

C^* was estimated by fitting respective length-scale estimates from the entire data sets to the function $L_{\max} = C^* L_T^{6/7}$ (equation (12)).

occurs at smaller length scales and the integration in equation (12) starts at larger l leading to lower C . Estimates of the scaling factors C , which were obtained by fitting equation (13) to the three different data sets from Figure 9, are given in Table 1 together with averaged dissipation rates. Although there seems to be a negative correlation between both quantities, the limited number of data points and the large heterogeneity of ϵ within the particular data sets do not allow a detailed analysis at this point. But the results obtained so far encourage a more comprehensive analysis of $L_{\max}/L_T$ as a function of the dissipation rate ϵ .

7. How to Estimate Turbulent Length Scales Most Reliably

[36] From a practical viewpoint, the universal form of the probability density of the displacement length scales can be used to obtain statistically well defined estimates of L_T from less well-resolved temperature profiles, like common CTD profiles. Attempts to use CTD data for the application of the Thorpe method and thus for the estimation of the rate of turbulent kinetic energy dissipation or turbulent diffusivities were described by *Mudge and Lueck* [1994], *Galbraith and Kelley* [1996], and *Ferron et al.* [1998]. However, since the Thorpe scale is mainly determined by the larger overturns [*Stansfield et al.*, 2001] and since large overturns are not very frequent, the statistics are often limiting. Therefore it is extremely useful to have additional statistical means to determine the Thorpe scale based on at least part, if not the entire histogram of the displacement scales.

[37] As shown above, the pdf of the displacement scales, based on reordering of temperature-microstructure profiles, is quite universal and can be described by only one parameter. This parameter can be either $L_{\max}$ or L_T . Therefore, we address here the question, which of the two scales can be estimated with a higher statistical confidence from a finite temperature-profile segment. This statistical investigation was performed by using numerical simulations incorporating the universal length scale distribution. As another outcome of the simulations, the impact of the chosen segmentation length on the displacement statistics will be discussed.

7.1. Simulations

[38] Non-overlapping temperature inversions within a linear temperature profile (like those shown in Figure 5) were produced with randomly generated sizes l using a probability density function $H_N(l) \propto l^{-5/3}$ with $1 \text{ mm} \leq l \leq$

$L_{\max}^*$. $L_{\max}^*$ is the preset upper limit of the overturning eddies, which we call the master-length scale in the following and which represents the level of turbulence. Thus $L_{\max}^*$ defined the distribution of randomly generated overturns. Each segment had a vertical resolution of 1 mm and a segment length of 3 m resulting in a total of 3000 data points per segment.

[39] By definition, the master length scale corresponds to the maximum displacement length $L_{\max}$ in an infinite long profile segment. However, since its pdf is very low, the probability to sample it within a finite segment (or sampling interval) is also low. $L_{\max}^*$ was chosen to be the governing length scale for the displacement length scale distribution. Alternatively, the distribution could be described by a master length scale L_T^* , which is connected to $L_{\max}^*$ by equation (13). Estimation of these master length scales is the general goal of the length scale analysis. The only constraint considered here is the statistical uncertainty due to a finite segment length representing the sampling interval regardless of numerous other constraints as mentioned in the introduction, such as eddy overlapping or nonstationary or inhomogeneous turbulence conditions.

[40] The procedure was repeated for 1000 profile segments using varying $L_{\max}^*$ with $0 < L_{\max}^* < 0.3 \text{ m}$. The length scale analysis as described in section 3 was performed on every individual segment and L_T , $L_{\max}$ and $H(l)$ were estimated. The nondimensional form of the estimated pdf $H(l)$ reproduced the function $H(l) \cdot L_{\max} \propto 1 - (l/L_{\max})^{-2/3}$ from equation (10) very well (Figure 8).

7.2. L_T or $L_{\max}$?

[41] The results of the length scale analysis are summarized in Figure 11. The ratio of the measured maximum displacement length $L_{\max}$ to the master length $L_{\max}^*$ is a one-sided histogram with unity as the maximum value by definition (Figure 11b). Differently, the ratio of the estimated Thorpe scale to the corresponding master length scale L_T^* as estimated from $L_{\max}^*$ using equation (13) exhibits a Gaussian distribution (Figure 11b). The question addressed here is whether $L_{\max}^*$ or L_T^* can be estimated with a higher statistical confidence from a finite sampling interval by estimating $L_{\max}$ and L_T respectively. The answer to this question is not straightforward, because for the estimation of L_T^* as a representative root mean square, an entire range of the distribution has to be resolved properly. For $L_{\max}^*$, only the largest overturn has to be estimated but the probability to sample it within a finite segment is very low since $H_N(l)$ is decreasing strongly nonlinear with l .

[42] Analyzing the cumulative probabilities of both histograms (Figure 11a) demonstrates that $L_{\max}$ can be estimated with a higher statistical confidence than L_T over the entire range of scales. The cumulative probability to sample $L_{\max}^*$ with a given uncertainty, which is expressed by $L_{\max}/L_{\max}^*$, is always higher than that of the according Thorpe scales L_T/L_T^* . Therefore it is more likely to sample $L_{\max}^*$ with a given statistical error than it is for L_T^* . The practical conclusion is that the maximum displacement length scale $L_{\max}$ is statistically a more reliable parameter for describing a displacement length scale distribution than a direct estimate of L_T . However, one must be aware that

Figure 11. Results of the numerical simulations using a pdf of $H_N(l) \propto l^{-5/3}$, determined by the master-length scale $L_{\max}^*$. The lower panel (b) shows the histogram of the ratios of estimated length scales L_T and $L_{\max}$ to the according master length scales L_T^* and $L_{\max}^*$ respectively, and the upper panel (a) displays the according cumulative probabilities.

$L_{\max}$ will be always underestimated whereas the estimate of L_T can be improved considerably by ensemble averaging since it is Gaussian distributed around the master length scale L_T^* .

7.3. Segmentation Length

[43] Simulations using different segmentation lengths revealed that the ratio of the segment length to $L_{\max}^*$ is crucial for a reliable estimation of $L_{\max}$ or L_T . As a rough guide, we recommend that the segment length should not be smaller than 10 times $L_{\max}$. When decreasing this ratio, e.g., from 10 to 1, the estimated value of $L_{\max}$ can be underestimated by an order of magnitude.

[44] This analysis showed that $L_{\max}$ is a more representative measure for the distribution of turbulent overturns following the distribution $H_N(l) \propto l^{-5/3}$ than the Thorpe scale L_T . Given that the characteristics of the turbulence can alternatively be described by the Ozmidov length scale L_O (equation (1)), one should expect a better correlation between L_O and $L_{\max}$ compared to L_T . Another advantage of $L_{\max}$ is that it is always greater than L_T and hence better detectable by instruments with a limited vertical or temperature resolution (CTD).

8. Concluding Summary

[45] Vertical overturns and displacements generated by shearing motions in stratified waters have been statistically analyzed based on temperature microstructure data sets from three different lakes with distinctly different stratification. The key conclusions from this analysis are:

1. The stability N^2 of the water columns of the three different systems varied by five orders of magnitude from $N^2 = 10^{-3} \text{ s}^{-2}$ (in a shallow lake) to 10^{-8} s^{-2} (in the deep and permanently stratified layers of Lake Baikal). As a result of this large variability in stratification the maximum overturning scale varied also by more than three orders of magnitude from 0.05 to 170 m and the Thorpe scale from 0.01 to 100 m.

2. Different authors have demonstrated that there is a strong correlation between the Ozmidov (energy-containing) scale L_O and the Thorpe scale L_T , which can be used to estimate rates of turbulent dissipation and vertical diffusivities. We showed that the maximum displacement scale $L_{\max}$ is even better correlated to the Thorpe scale L_T , implying that $L_{\max}$ can also be used for turbulence estimates.

3. As other authors previously demonstrated, we also found that most of the contributions for L_T (or L_O) stem from the largest few overturns.

4. The probability density functions (pdf's) of Thorpe displacements and overturning scales are distinguished and it is demonstrated that the inertial subrange model is the most appropriate for the occurrence probability of overturns. As a result, the pdf evolves proportional to $l^{-5/3}$ and the relation between L_T and $L_{\max}$ is $L_{\max} \propto C \cdot L_T^{6/7}$. The proportionality constant C is dependent on the level of turbulence, i.e., the rate of dissipation of turbulent kinetic energy. This observation is supported by the fact that the temperature, as well as the length-scale fluctuation spectra, show an identical form.

5. If the pdf's of the displacements follow an exponential-like or a box car-like function, both parameterized by a single length scale, then L_T is proportional to $L_{\max}$. All the data analyzed were between the relation for exponential-like pdf and the boxcar-like pdf, leaving a factor of 3 in between.

6. Based on the universal pdf of overturning scales we show with numerical simulations that $L_{\max}$ is statistically a more appropriate macroscopic length scale than L_T . This implies that it is well possible to determine $L_{\max}$ and consequently the level of turbulence with the given constraints even from not-very-well-resolved CTD data. Since not only the largest scales contribute to the results but also the shape of the entire displacement histogram, the attained values are statistically reasonably well established.

[46] Not considered in the above analysis was the effect of overlapping and embedded eddies. This issue together with a more comprehensive analysis of the dependency of the ratio $L_{\max}/L_T$ on ε should be addressed to future investigations. The results promise a high level of applicability to assess small-scale turbulence qualities from macroscopic length scale analysis of temperature fine structure.

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