

Modeling the duration of intermittent ice cover on a lake for climate-change studies

David M. Livingstone^{a,*} and Rita Adrian^b

^aEawag, Swiss Federal Institute of Aquatic Science and Technology, Dübendorf, Switzerland

^bLeibniz Institute of Freshwater Ecology and Inland Fisheries, Berlin, Germany

Abstract

A generalized empirical approach to the problem of the prediction of intermittent lake ice cover for climate change studies is presented and applied to historical ice data from Müggelsee, a shallow lake in northern Germany that experiences either no ice cover, intermittent ice cover, or continuous ice cover, depending on the severity of the winter. The approach presented allows the total duration of ice cover in any winter to be estimated in terms of an air temperature probability function. Advantages over traditional empirical methods based on the mean air temperature during a fixed period or on the computed number of negative degree days are: (1) it is valid for both continuous and intermittent ice cover; (2) it is not necessary to specify a fixed time period for integration in advance; (3) the results emerge in time units, allowing them to be directly compared with the measured duration of ice cover without prior calibration; and (4) it is based on statistical properties of the ambient air temperature that are routinely forecast by climate models, making it easy to apply predictively. Regional climate model forecasts for northern Germany imply that for Müggelsee, the percentage of ice free winters will increase from ~2% to over 60% by the end of the current century.

The main meteorological forcing variables that affect lake ice cover, and that act as input variables driving physical models of lake ice cover, are air temperature, relative humidity, cloud cover, wind speed, and precipitation (Duguay et al. 2003). Of these, air temperature is acknowledged to be the most important (Vavrus et al. 1996; Williams and Stefan 2006). Air temperature is not only a key causal forcing variable in its own right, it is often correlated to some extent with the other meteorological forcing variables, implying that an air-temperature time series can contain more information on meteorological forcing than would be expected based on the causal role of air temperature alone. Thus, in modern physical models of lake ice cover, calculations of congelation ice growth are still based on the classical method of Stefan (1890), in which air temperature is the only meteorological forcing variable considered (Leppäranta 1991; Saloranta and Andersen 2007). Air-temperature data are generally more widely available than data on the other relevant forcing variables, and empirical models based solely on air temperature appear to be well-suited for predicting the timing of ice-on (Bilello 1964) and ice-off (Bilello 1961) when data on the full suite of forcing variables are not available. Statistically, air temperature is often able to explain 60–70% of the variance in the timing of ice-off (Palecki and Barry 1986; Livingstone 1997). Because predictions of the behavior of air temperature under various future climate scenarios are more reliable than predictions of the behavior of the other relevant forcing variables (Christensen et al. 2007; Meehl et al. 2007), empirical models based on air temperature, despite their evident drawbacks, are likely to be especially useful for making general predictions on the ways in which climate change might affect lake ice phenology.

Once frozen over, lakes in regions where air temperatures lie substantially below 0°C during much of the winter generally remain ice covered until the spring thaw sets in, allowing the limnological year to be divided into a well-defined period of ice cover and an equally well-defined period of open water. In such cases, the calendar dates of ice-on and ice-off are unambiguously defined and can be estimated quite well from the ambient air temperature alone using well-tested empirical models (Bilello 1964; Palecki and Barry 1986; Livingstone 1997). The duration of the period of ice cover is then given simply by the difference of the two estimates. In regions where the air temperature is close to 0°C during much of the winter, sometimes above and sometimes below, conceptual difficulties arise with this approach because the period of ice cover is often not continuous, but instead it is interrupted by periods of open water. In cases like this, with intermittent ice cover, it makes little sense to attempt to define two specific calendar dates of ice-on and ice-off as in the case of continuous ice cover. Instead, a better approach is to relate the total period of ice cover (i.e., the total number of individual days of ice cover during a winter) to a relevant characteristic property of the winter air temperature.

For Müggelsee, a lake in northern Germany that is a good example of a lake with alternating short periods of ice cover and open water during winter (Fig. 1), Adrian and Hintze (2000) showed this latter approach to function quite well: the total duration of ice cover was correlated with the December–February mean air temperature in the vicinity of the lake. However, while this temporally specific definition of a relevant mean air temperature may be applicable under the climate conditions currently pertaining at the latitude and altitude of Müggelsee, the same definition will not necessarily be applicable under past climate conditions, for instance, during the Little Ice Age, when the relevant time period is likely to have extended into November and March, or under future climate

* Corresponding author: living@eawag.ch

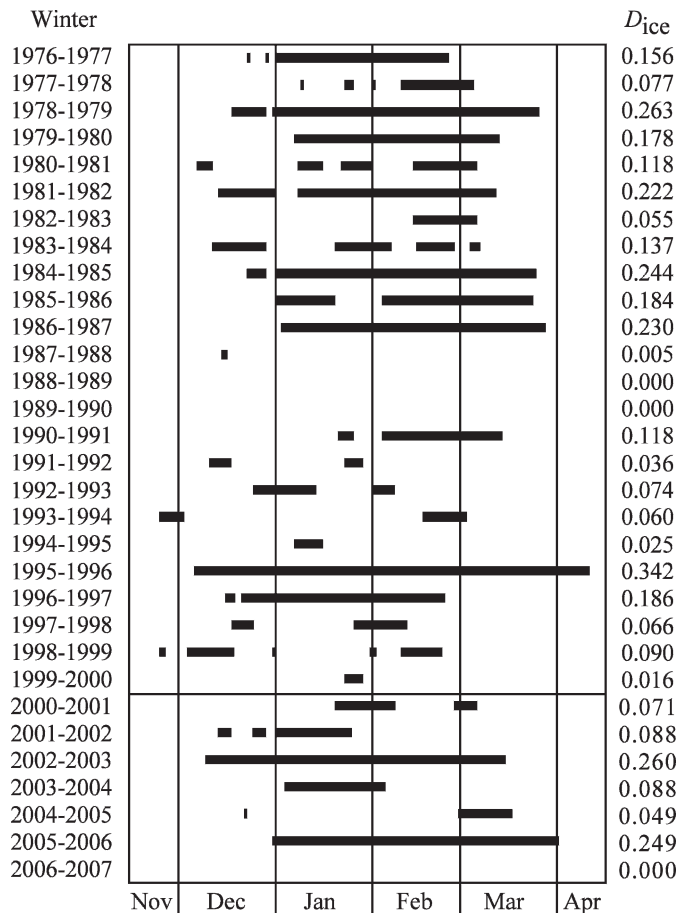


Fig. 1. Periods of ice cover on Müggelsee from 1976/1977 to 2006/2007, based on daily observations, and their duration, D_{ice} . A day of ice cover is defined as one on which at least 80% of the lake surface area was observed to be frozen over. The horizontal line separates the data used to formulate the model described in the text (1976/1977 to 1999/2000) from the data used to test the model (2000/2001 to 2006/2007). Adapted from Adrian and Hintze (2000) and extended to include additional data.

conditions, when the relevant time period is likely to contract substantially as a result of climate warming. In addition, the relevant time period will depend on latitude and altitude, being longer at higher latitudes or higher altitudes, and shorter at lower latitudes or lower altitudes. Thus, a more generally valid approach is necessary that does not depend on a specific, predefined time period.

In the present paper, the approach of Adrian and Hintze (2000) is taken further, and a generalized empirical approach to the problem of the prediction of intermittent lake ice cover is developed that allows the duration of ice cover in any winter to be estimated in terms of an air-temperature probability function. This approach allows the total duration of ice cover, D_{ice} , on a lake, whether continuous or intermittent, to be predicted simply and reliably from statistical properties of the ambient air temperature that are routinely forecast by climate models. This approach is then applied to make predictions about the future development of the duration of intermittent ice cover on Müggelsee from published regional climate

simulations (Räisänen et al. 2004) based on the Intergovernmental Panel on Climate Change (IPCC) global climate scenarios of Nakićenović and Swart (2000).

Study site and available data

Müggelsee is a highly eutrophic, polymictic lake located at 52°26'N, 13°39'E in Berlin, northern Germany; it has a surface area of 7.3 km² and a maximum depth of 8 m (Driescher et al. 1993). The mean elevation of the lake surface is 34 m above sea level. It is one of the intercalated lakes of the River Spree, and it has a mean water residence time of 8 weeks. The discharge of the River Spree is regulated by man-made impoundments that ameliorate the influence of floods and prevent the lake surface from fluctuating more than 15–35 cm about its mean elevation. The surrounding topography is flat, and the lake basin is oriented east west. Further details on lake morphometry and catchment area are given by Driescher et al. (1993). The climate of the region is temperate continental, and winter climate shows a high degree of interannual variability: monthly mean air temperatures in January, the coldest month, vary within the approximate range of 7°C to +5°C (Adrian and Hintze 2000). A reduction in the duration of winter ice cover on Müggelsee in recent decades has caused a substantial shift in the timing of the spring phytoplankton bloom, which now occurs much earlier than previously (Adrian et al. 2006).

Daily observations of the presence or absence of ice cover on Müggelsee have been made since the winter of 1976/1977 (Fig. 1). Here, the lake is considered to have been ice-covered if at least 80% of the total lake surface was frozen over. Interannual variability in the total duration of winter ice cover is large, reflecting the interannual variability in winter climate. From Fig. 1, it can be seen that intermittent ice cover during winter is the rule, but that long periods of continuous ice cover can occur during extremely cold winters (e.g., 4 months during the winter of 1995/1996), while the lake can be completely ice-free during extremely mild winters (e.g., 1988/1989, 1989/1990, and 2006/2007). Based on personal observation, the thickness of the ice can be as much as 0.5 m in cold, long-lasting winters (such as that of 1995/1996). For the purpose of the analysis described here, the ice-data time series was divided into two subseries: from the winter of 1976/1977 to the winter of 1999/2000 (24 winters) and from the winter of 2000/2001 to the winter of 2006/2007 (7 winters). The first subseries was used to develop the model described herein, and the second was set aside to validate the model.

Daily minimum and daily maximum air temperatures were available from Tempelhof Airport, ~20 km west of Müggelsee, for each day without interruption from 1951 onward. Additionally, hourly air-temperature data were available from Tempelhof Airport from 1951 to 2000, but two blocks of these data were missing (01 January 1971–31 December 1980; 01–31 December 1997). Hourly air-temperature data were also available from Schönefeld Airport, ~10 km southwest of Müggelsee, from 1973 to 2000, again with one missing block of data (07–31 July

1994). Occasional individual missing data in both hourly data series (12 h in the Tempelhof series, 13 h in the Schönefeld series) were estimated by linear interpolation. The Schönefeld data were used to estimate the missing blocks of Tempelhof data by linear regression, based on all full days of data common to both stations during the period 1973–2000. Linear regression coefficients between the two data series were calculated for each month and for each hour of the day separately, giving 288 sets of regression coefficients. Correlation coefficients were all extremely highly significant ($p \ll 0.0001$), and the root mean square error (RMSE) between the measured and calculated Tempelhof series was low (0.84°C), indicating that the missing blocks of Tempelhof data could be estimated well from the Schönefeld data. Thus, the uninterrupted series of daily minimum and maximum air temperatures at Tempelhof from 1951 to 2000 could be supplemented by a series of hourly air temperature data covering the same period, with the exception of 1971–1973.

Methods

Empirical determinations of the thickness, timing, and duration of ice cover, both in polar marine areas and on inland waters, are traditionally based on some form of the accumulated degree-day method (Stefan 1890; Arnet 1897; Bilello 1961). This approach rests on the premise that freezing is governed by the integral over time of negative air temperatures only, and thawing is governed by the integral over time of positive air temperatures only. Thus, the timing of ice-on can be related to the number of negative degree-days accumulated during the freezing period, and the timing of ice-off can be related to the number of positive degree-days accumulated during the thawing period. Although a variant of the accumulated degree-day method will be touched on later for comparison purposes, the approach described here is different, and simpler. Its basic underlying premise is that the duration of ice cover D_{ice} , expressed here nondimensionally as the fraction of the annual cycle during which a lake is ice-covered (continuously or intermittently), can be estimated well by a simple functional dependence $g(D^-)$ on the duration of time during which the ambient air temperature is $<0^\circ\text{C}$ (D^-), also expressed here nondimensionally as a fraction of the annual cycle. Equivalently, D_{ice} can be related to the fraction D^+ of the annual cycle during which the ambient air temperature is $>0^\circ\text{C}$ ($D^- + D^+ = 1$, since the duration of the annual cycle is assumed to be constant):

$$D_{\text{ice}} = g(D^-) = g(1 - D^+) \quad (1)$$

It follows that the fraction of the annual cycle with open water, $D_{\text{open}} = 1 - D_{\text{ice}}$, is given by:

$$D_{\text{open}} = 1 - g(D^-) = 1 - g(1 - D^+) \quad (2)$$

The analysis conducted here is in terms of D_{ice} and D^- , but, as the above equations show, this does not imply that D^- is any more relevant than D^+ for the determination of D_{ice} and D_{open} .

In the specific case of Müggelsee, the validity of the basic premise embodied in Eq. 1 is borne out by Fig. 2A, which shows an approximately linear dependence of D_{ice} on D_h^- (defined as D^- computed from the Tempelhof hourly air temperature data), implying that $g(D^-)$ is approximately a first-order polynomial. The high degree of correlation between D_{ice} and D_h^- , with over 90% shared variance, allows D_{ice} to be estimated well from D_h^- by linear regression (Table 1A).

For comparison purposes with the more traditional accumulated degree-day approach, D_{ice} was also related to the number of negative degree-days, N_d (Table 1B), and the number of negative degree-hours, N_h (Table 1C), that occur during the annual cycle (essentially the integral over time of negative air temperatures only). Both these relationships are weaker than that between D_{ice} and D_h^- , although in the case of N_h the difference is slight, so that either D_h^- or N_h could be used to obtain an acceptable empirical estimate of D_{ice} . (That N_h is not a better predictor of D_{ice} than D_h^- results presumably from air-temperature fluctuations having a greater effect on ice growth and decay when the air temperature is close to 0°C , and ice and snow layers are thin or absent, than when it is well below 0°C , and ice and snow layers are thick and insulating.) The relationship between D_{ice} and D_h^- is much the simpler of the two relationships because only the signs, but not the magnitudes, of the air temperatures enter into Eq. 1. An additional conceptual advantage of Eq. 1 is the fact that D_{ice} and D^- share the same units and can both be expressed as nondimensional fractions of a year. On several grounds, it is therefore preferable to estimate D_{ice} from D_h^- rather than from N_h .

Because historical instrumental air-temperature data are not always available at subdaily intervals and because climate models do not normally generate air-temperature predictions on subdaily timescales, the relationship illustrated in Fig. 2A is of little practical use. However, the duration of time during which the hourly air temperatures are $<0^\circ\text{C}$ (D_h^-) is extremely tightly linked to the duration of time during which the daily mean air temperatures are $<0^\circ\text{C}$ (D_d^-), regardless of whether the daily mean air temperature is computed as the true mean of the 24 once-hourly measurements ($D_d^- = D_{d1}^-$; Table 1D) or, more simply, as the mean of the daily minimum and daily maximum air temperatures ($D_d^- = D_{d2}^-$; Table 1E; Fig. 2B). Thus, computation of D^- is robust with respect to timescale, and a knowledge of D_d^- therefore suffices to give an estimate of D_{ice} that is only slightly less accurate than the one obtained from D_h^- , again regardless of whether the daily mean air temperature is computed as the true mean of the 24 once-hourly measurements ($D_d^- = D_{d1}^-$; Table 1F) or as the mean of the daily minimum and daily maximum air temperatures ($D_d^- = D_{d2}^-$; Table 1G; Fig. 2C).

Because freezing and thawing are not instantaneous processes, short periods of freezing or thawing temperatures are unlikely to result in a change in the status of the ice cover. An improvement in the accuracy of the estimate might therefore be obtained by smoothing the air-temperature data before calculating D^- . To assess the

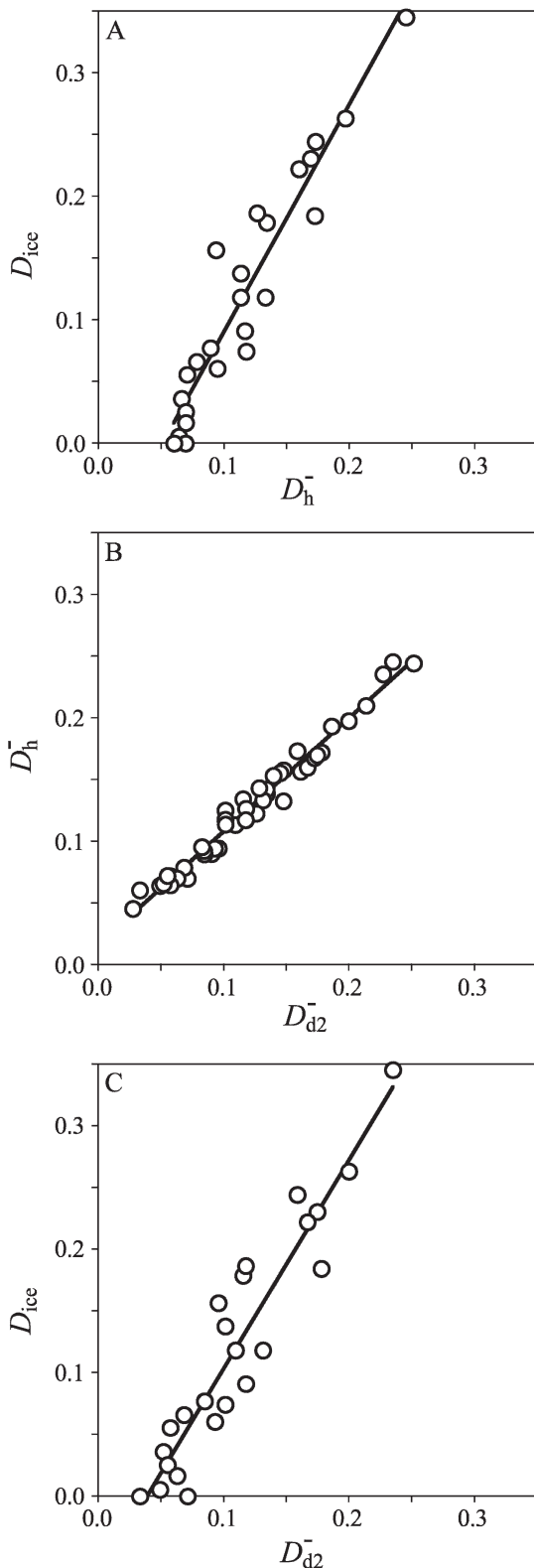


Fig. 2. Dependence of (A) D_{ice} on D_h^- (1976–1977 to 1999–2000), (B) D_h^- on D_{d2}^- (1951–1952 to 1999–2000), and (C) D_{ice} on D_{d2}^- (1976–1977 to 1999–2000), where D_{ice} = fraction of the annual cycle (01 July–31 June) during which Müggelsee was ice covered (see Fig. 1); D_h^- = fraction of each annual cycle during which the hourly measured air temperature was $<0^\circ\text{C}$; D_{d2}^- =

importance of this effect, several hundred smoothed air-temperature time series with different degrees of smoothing were obtained by low-pass filtering the time series of daily mean air temperatures at different cutoff frequencies distributed between 1 yr^{-1} and 0.25 d^{-1} , using standard Fourier techniques (Fourier transformation followed by the removal of high-frequency components and inverse Fourier transformation of the result). D was then calculated for each year from each smoothed data set, as in the case of the unsmoothed data. The values of D calculated from the smoothed air-temperature data were then compared with D_{ice} to determine whether smoothing the air temperature improved the degree of association between D_{ice} and D . The proportion of shared variance (r^2) exceeded 85% at all cutoff frequencies $>0.065\text{ d}^{-1}$ (i.e., at all cutoff periods $<15\text{ d}$). The strongest relationship between D_{ice} and D was obtained at a cutoff frequency of 0.23 d^{-1} (i.e., at a cutoff period of 4.3 d); however, the improvement with respect to the values of D obtained from the unsmoothed air-temperature data (i.e., with respect to D_d) was minimal and resulted in an increase in the proportion of shared variance of only 1.7% (to 89.8%). Hereafter, therefore, unsmoothed daily mean air temperatures (computed as the mean of the daily extrema) are employed throughout.

Having shown that the total duration D_{ice} of ice cover on Müggelsee during any one winter, whether continuous or intermittent, can be estimated well from D_d , we now show that D_d can be modeled using a probability approach. The advantage of such an approach over a more traditional approach based on some form of mean air temperature is that it relates D_{ice} ultimately to a relevant characteristic property of the winter air temperature without having to define the temporal extent of the winter in advance. In the method employed here, D (and hence D_{ice}) is related to an estimate \hat{T}_m of the daily mean air temperature T_m .

The arc cosine model Most of the seasonally related variance of the daily mean air temperature T_m is associated with its first Fourier component, a sinusoid of period 1 yr, denoted here by \hat{T}_m and expressed as the following cosine function of time t (Fig. 3):

$$\hat{T}_m(t) = \theta_m - \alpha_m \cos 2\pi f(t - \tau_m) \quad (3)$$

where θ_m is the annual mean air temperature, and α_m , f , and τ_m are the amplitude, frequency, and phase, respectively, of the first Fourier component of the annual cycle. The minimum (v_m) and maximum (ξ_m) of the sinusoid are given by $v_m = \theta_m - \alpha_m$ and $\xi_m = \theta_m + \alpha_m$, respectively. The form of Eq. 3 was chosen so that the phase τ_m gives the forward displacement of v_m from 00:00 h on 01 January (Fig. 3A). The actual daily mean air temperature, T_m , varies around

←

fraction of the annual cycle during which the daily mean air temperature (calculated as the mean of the daily minimum and daily maximum) was $<0^\circ\text{C}$. Air temperatures were measured at Tempelhof Airport. Details of the linear regressions illustrated are given in Table 1A,E,G.

Table 1. Parameters characterizing the linear regressions (A–J) referred to in the text. The following are listed: the dependent variable (y); the independent variable (x); the number of data points (n); the proportion of variance explained, i.e., the coefficient of determination (r^2); the standard error of the regression (σ_e), expressed in yr and d; the intercept (a); and the gradient (b). All regressions listed are significant at the $p < 0.0001$ level at least. (A) D_{ice} on D_{h}^- (Fig. 2A). (B) D_{ice} on N_{d}^- . (C) D_{ice} on N_{h}^- . (D) D_{h}^- on D_{d1}^- . (E) D_{h}^- on D_{d2}^- (Fig. 2B). (F) D_{ice} on D_{d1}^- . (G) D_{ice} on D_{d2}^- (Fig. 2C). (H) D_{d2}^- on D_{prob}^- , where the computation of D_{prob}^- is based on individual values of σ for each year. (I) D_{d2}^- on D_{prob}^- , where the computation of D_{prob}^- is based on one constant value of $\sigma = 3.56^\circ\text{C}$ (Fig. 4C). (J) D_{ice} on D_{prob}^- (Fig. 4D). D_{ice} = total duration of intermittent or continuous ice cover on Müggelsee during one year, where the year is defined as extending from 01 July to 30 June (29 June on leap years). D = total duration of time during one year with air temperature $< 0^\circ\text{C}$, based on hourly measured air temperature data (D_{h}^-), daily mean air temperature data calculated as the true mean of 24 once hourly air temperature measurements (D_{d1}^-), daily mean air temperature data calculated as the mean of the daily minimum and daily maximum air temperatures (D_{d2}^-), and the probability model (D_{prob}^-). N_{h}^- = No. of negative degree hours during the year, expressed for convenience and comparison purposes in units of degree years ($^\circ\text{C yr}$) and N_{d}^- = No. of negative degree days during the year, also expressed in $^\circ\text{C yr}$. Air temperature data were measured at Tempelhof Airport.

	y	x	n	r^2 (%)	σ_e (yr)	σ_e (d)	a	b
A	D_{ice}	D_{h}^-	24	90.5	0.0299	10.9	0.0939	1.838
B	D_{ice}	N_{d}^-	24	85.3	0.0371	13.6	0.0037	0.296 ($^\circ\text{C yr}$) ¹
C	D_{ice}	N_{h}^-	24	89.8	0.0310	11.3	0.0329	0.298 ($^\circ\text{C yr}$) ¹
D	D_{h}^-	D_{d1}^-	46	98.8	0.0058	2.1	0.0171	0.911
E	D_{h}^-	D_{d2}^-	46	97.9	0.0079	2.9	0.0168	0.912
F	D_{ice}	D_{d1}^-	24	88.5	0.0328	12.0	0.0650	1.690
G	D_{ice}	D_{d2}^-	24	88.1	0.0334	12.2	0.0642	1.681
H	D_{d2}^-	D_{prob}^-	49	91.7	0.0160	5.8	0.0054	0.955
I	D_{d2}^-	D_{prob}^-	49	90.2	0.0173	6.3	0.0093	0.978
J	D_{ice}	D_{prob}^-	24	93.2	0.0253	9.2	0.0882	1.753

\hat{T}_{m} . The parameters of the sinusoid (θ_{m} , α_{m} , and τ_{m}) can be obtained easily from measured data by sinusoidal regression (Güttinger 1980).

In high-altitude or high-latitude regions where daily mean air temperatures are below 0°C for a substantial period of time in winter, the duration of ice cover D_{ice} is related to the duration D_{arccos} of the period during which the sinusoid describing T_{m} lies below 0°C (Weyhenmeyer et al. 2004). This is given by the difference of the two roots of the equation $\theta_{\text{m}} - \alpha_{\text{m}} \cos 2\pi f(t - \tau_{\text{m}}) = 0$, but it is most easily obtained by solving the equivalent equation with zero phase, which is symmetrical about its minimum on the T_{m} axis and has the same value of D_{arccos} , viz.

$$D_{\text{arccos}} = \frac{1}{\pi} \arccos \frac{\theta_{\text{m}}}{\alpha_{\text{m}}} \quad (4)$$

The arc cosine function in Eq. 4 is defined only when its argument $\theta_{\text{m}}/\alpha_{\text{m}}$ lies within the closed interval $[-1, 1]$, i.e., when the sinusoid of Eq. 3 is intersected by the line $T_{\text{m}} = 0^\circ\text{C}$ (giving $0 < D_{\text{arccos}} < 1$) or touches it (giving $D_{\text{arccos}} = 0$ or 1). This method is therefore only applicable when $v_{\text{m}} \leq 0^\circ\text{C} \leq \xi_{\text{m}}$ (the situation illustrated in Fig. 3A). If $v_{\text{m}} > 0^\circ\text{C}$ (the situation illustrated in Fig. 3B), \hat{T}_{m} always exceeds 0°C , and $D_{\text{arccos}} = 0$. Thus, $D_{\text{arccos}} = 0$ arises when $v_{\text{m}} \geq 0^\circ\text{C}$. (Similarly, $D_{\text{arccos}} = 1$ arises when $\xi_{\text{m}} \leq 0^\circ\text{C}$. This situation, which would be relevant for perennially ice-covered lakes, can be ignored for temperate-zone lakes.) Hereafter, $v_{\text{m}} < 0^\circ\text{C}$ (i.e., $D_{\text{arccos}} > 0$) is considered to define a cold winter, and $v_{\text{m}} \geq 0^\circ\text{C}$ (i.e., $D_{\text{arccos}} = 0$) is considered to define a mild winter.

For each individual year from 1951–1952 to 2006–2007, a sinusoidal regression of the Tempelhof daily mean air-temperature data was conducted from 01 July to 30 June

(29 June on leap years) using the method of Güttinger (1980), and the parameters θ_{m} , α_{m} , and τ_{m} of Eq. 3 were determined. For cold winters ($v_{\text{m}} < 0^\circ\text{C}$; Fig. 3A), D_{arccos} was computed from Eq. 4; for mild winters ($v_{\text{m}} \geq 0^\circ\text{C}$; Fig. 3B), D_{arccos} was set to zero. The relationship between D_{d2} and the values of D_{arccos} obtained (based on the data from 1951–1952 to 1999–2000 only) is illustrated in Fig. 4A. Of the 49 winters from 1951–1952 to 1999–2000, 26 were cold (i.e., $v_{\text{m}} < 0^\circ\text{C}$ and $D_{\text{arccos}} > 0$). Based on these 26 years only, D_{d2} exhibits a curvilinear dependence on D_{arccos} . In the other 23 years, the winter was mild (i.e., $v_{\text{m}} \geq 0^\circ\text{C}$ and $D_{\text{arccos}} = 0$). Despite this however, D_{d2} was neither zero nor even approximately constant, but it varied considerably within the range 0.027 to 0.132, corresponding to a range of 10 d to 48 d.

The relationship between D_{ice} and D_{arccos} is shown in Fig. 4B. As would be expected from the linear relationship between D_{ice} and D_{d2} illustrated in Fig. 2C, the dependence of D_{ice} on D_{arccos} is qualitatively similar to that of D_{d2} on D_{arccos} . Of the 49 winters from 1951–1952 to 1999–2000, ice data are available from Müggelsee only for the last 24 (1976–1977 to 1999–2000). Of these winters, only 9 were cold (i.e., $v_{\text{m}} < 0^\circ\text{C}$ and $D_{\text{arccos}} > 0$). Based only on these 9 winters, D_{ice} , like D_{d2} , also exhibits a curvilinear dependence on D_{arccos} . In 13 of the 15 remaining mild winters (i.e., $v_{\text{m}} \geq 0^\circ\text{C}$ and $D_{\text{arccos}} = 0$), Müggelsee experienced ice cover of various durations up to $D_{\text{ice}} = 0.156$ (i.e., 57 d). Therefore, although the arc cosine model may function quite well in cold winters, it is unusable in mild winters, and it is generally unsuitable for shallow lakes in temperate regions at low altitudes, such as Müggelsee, that still experience ice cover in mild winters. This necessitates a modification to the above approach.

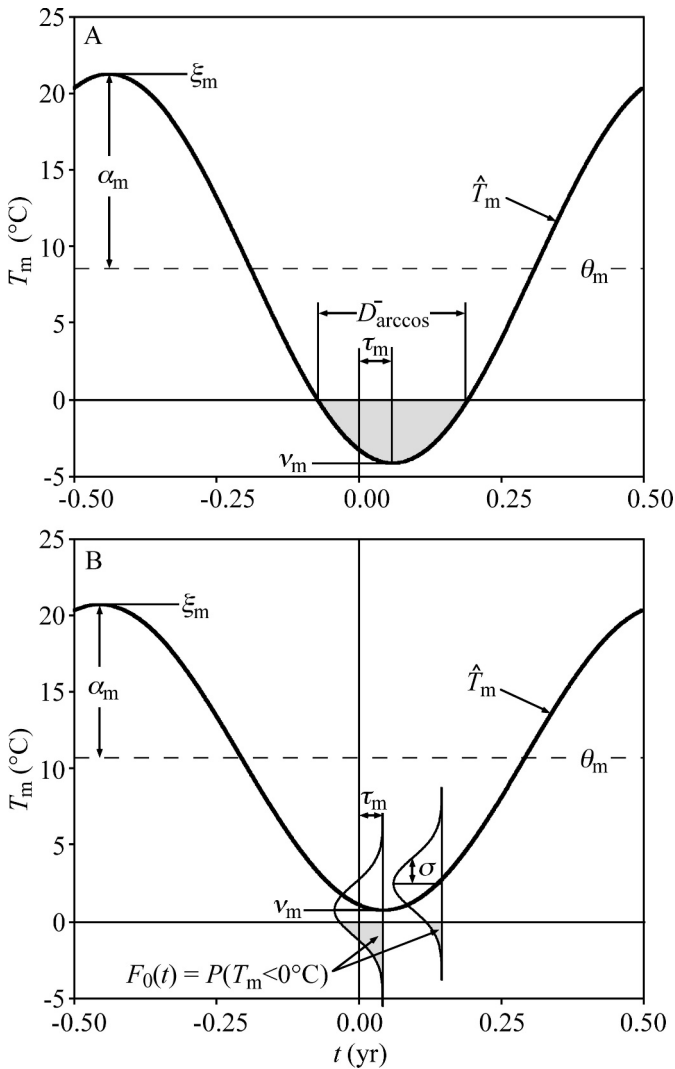


Fig. 3. Sinusoidal regressions of air temperature data from Tempelhof Airport, Berlin: (A) during the cold winter of 1995–1996, when Müggelsee was continuously ice covered for 4 months; and (B) during the mild winter of 1991–1992, when Müggelsee was only intermittently ice covered for a total of 2 weeks (cf. Fig. 1). \hat{T}_m = first Fourier component of the daily mean air temperature T_m (i.e., sinusoidal estimate of T_m ; Eq. 3); t = time; θ_m = annual mean air temperature; α_m = amplitude of \hat{T}_m ; τ_m = phase of \hat{T}_m (forward displacement from 01 January); ξ_m = annual maximum of \hat{T}_m ; v_m = annual minimum of \hat{T}_m ; D_{\arccos}^- = fraction of the annual cycle during which $\hat{T}_m < 0^\circ\text{C}$ (Eq. 4); σ = standard deviation of the actual daily mean air temperature T_m about the sinusoidal estimate \hat{T}_m ; $F_0(t) = P(\hat{T}_m(t) < 0^\circ\text{C})$ = probability that $\hat{T}_m(t) < 0^\circ\text{C}$ at any given time t (Eq. 8). This figure illustrates schematically that, while it is possible to employ D_{\arccos}^- to estimate the proportion of the annual cycle during which $T_m < 0$ when $v_m \leq 0$ (panel A), another approach in this case, a probability approach incorporating the probability distribution of T_m about \hat{T}_m must be employed when $v_m > 0$ (panel B).

The probability model Because actual daily mean air temperatures T_m vary around the estimate \hat{T}_m on both sides of the sinusoidal curve, daily mean air temperatures below 0°C are likely to occur in the vicinity of the minimum of \hat{T}_m , even when v_m is well above 0°C . A probability

approach (Fig. 3B) allows the total duration of the periods of negative mean air temperature to be estimated regardless of whether v_m lies above or below 0°C .

Assume that, at any given time t during the year, the daily mean air temperature T_m is normally distributed about a mean value \hat{T}_m with standard deviation σ . Given any value x , the probability $P(T_m < x)$ that $T_m < x$ at time t is then given by integrating the normal probability density function from $-\infty$ to x , yielding the cumulative normal distribution function F_x :

$$F_x = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{x - \hat{T}_m}{\sigma\sqrt{2}} \right) \right] \quad (5)$$

where the error function $\operatorname{erf}(z)$ is defined as:

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-u^2) du \quad (6)$$

and u is an arbitrary variable of integration (e.g., Gauschi 1970, eq. 7.1.22). Defining F_0 as the value of F_x when $x = 0^\circ\text{C}$, and making use of the fact that $\operatorname{erf}(-z) = -\operatorname{erf}(z)$:

$$F_0 = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\hat{T}_m}{\sigma\sqrt{2}} \right) \right] \quad (7)$$

Substituting Eq. 3 into Eq. 7 yields F_0 as a function of time:

$$F_0(t) = \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\theta_m - \alpha_m \cos 2\pi(t - \tau_m)}{\sigma\sqrt{2}} \right) \right] \quad (8)$$

The relative proportion of the year, D_{prob} , during which $T_m < 0^\circ\text{C}$ is then given by integrating F_0 over one annual cycle:

$$D_{\text{prob}} = \int_0^1 F_0(t) dt \quad (9)$$

Because the integration is carried out over one full cycle, the phase τ_m is irrelevant and can be set to zero for simplicity, giving:

$$D_{\text{prob}} = \frac{1}{2} \left[1 - \int_0^1 \operatorname{erf} \left(\frac{\theta_m - \alpha_m \cos 2\pi t}{\sigma\sqrt{2}} \right) dt \right] \quad (10)$$

In addition to the parameters θ_m , α_m , and τ_m of Eq. 3, already determined, the standard deviation σ of the measured data around the sinusoid was determined for each year from 1951–1952 to 1999–2000. For each year, D_{prob} was then computed from Eq. 10. The relationship between D_{d2} and D_{prob} is strong and linear (Table 1H). The linear regression line of D_{d2} on D_{prob} has an intercept that is indistinguishable from zero (t -test: $t = 0.91$, $\text{df} = 48$, $p > 0.1$) and a gradient that is indistinguishable from unity (t -test: $t = 1.08$, $\text{df} = 48$, $p > 0.1$). Application of the Mann–Kendall test revealed no monotonic rising or falling temporal trend in σ at the $p < 0.05$ level, indicating that σ

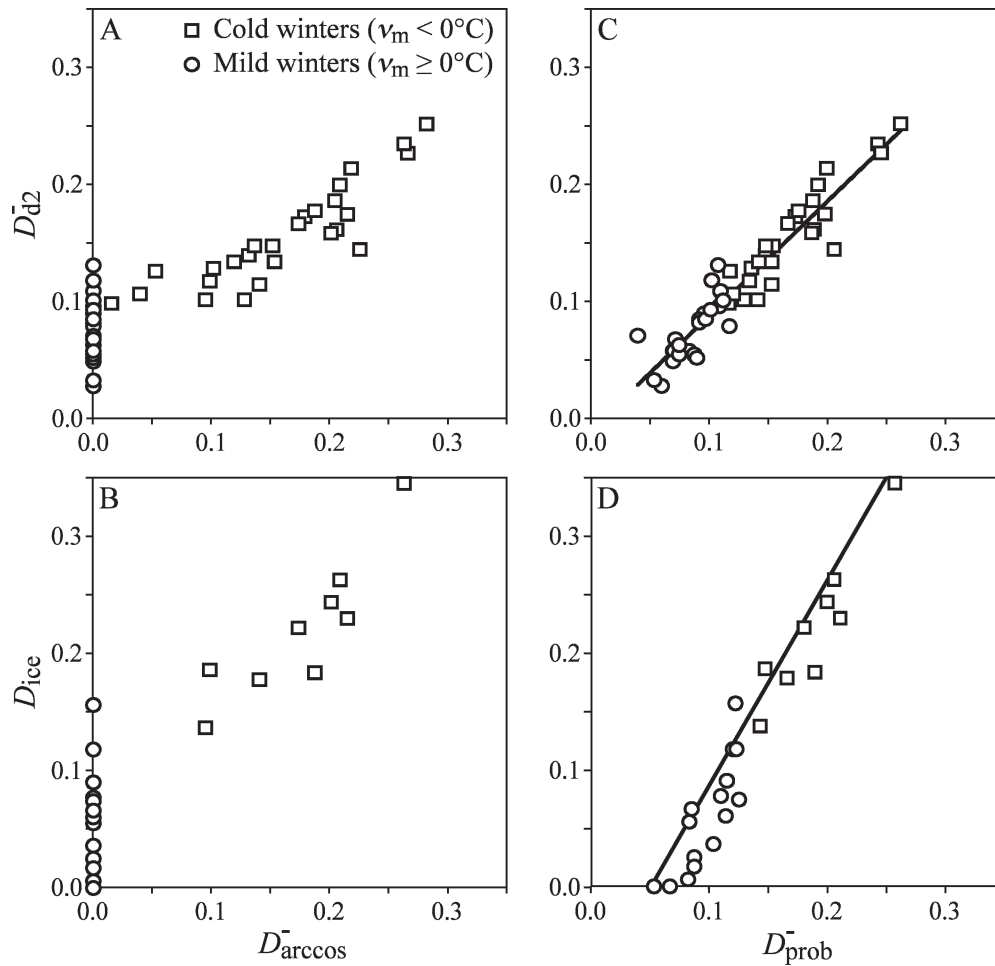


Fig. 4. Relationships (A) between D_{d2}^- and D_{arccos}^- (1951–1952 to 1999–2000), (B) between D_{ice} and D_{arccos}^- (1976–1977 to 1999–2000), (C) between D_{d2}^- and D_{prob}^- (1951–1952 to 1999–2000), and (D) between D_{ice} and D_{prob}^- (1976–1977 to 1999–2000), in cold winters (defined by $v_m < 0^\circ\text{C}$, where v_m is defined in Fig. 3) and mild winters (defined by $v_m \geq 0^\circ\text{C}$). D_{d2}^- = fraction of the annual cycle (01 July–31 June) during which the daily mean air temperature T_m (calculated as the mean of the daily minimum and daily maximum) was $< 0^\circ\text{C}$; $D_{arccos}^- = D_{d2}^-$ modeled using the arc cosine model (Eq. 4); D_{ice} = fraction of the annual cycle during which Müggelsee was ice covered (see Fig. 1); $D_{prob}^- = D_{d2}^-$ modeled using the probability model (Eq. 10) with constant $\sigma = 3.56^\circ\text{C}$. Air temperatures were measured at Tempelhof Airport. Details of the linear regressions illustrated in panels C and D are given in Table 1I,J, respectively.

can be considered stationary. Over all 49 yr, σ was confined to a relatively narrow range: $2.98^\circ\text{C} \leq \sigma \leq 4.12^\circ\text{C}$. Its mean value was 3.56°C , with a standard deviation of 0.32°C . Replacing the annually variable value of σ with a constant mean value of 3.56°C resulted in only a slight change in the regression line obtained (Table 1I); again, the intercept was indistinguishable from zero (t -test: $t = 1.41$, $df = 48$, $p > 0.1$), and the gradient was indistinguishable from unity (t -test: $t = 0.47$, $df = 48$, $p > 0.1$) (Fig. 4C). A general test of the sensitivity of the relationship between D_{d2} and D_{prob} to the value of σ used to compute D_{prob} revealed a low degree of sensitivity within the range $2.0^\circ\text{C} \leq \sigma \leq 4.0^\circ\text{C}$. Within this range, r^2 between D_{d2} and D_{prob} exceeded 90% and the RMSE between D_{d2} and D_{prob} was less than 10 d. For all further computations of D_{prob} , therefore, σ in Eq. 10 was set to its mean value of 3.56°C . Because the intercept and

gradient of the regression line illustrated in Fig. 4C are statistically indistinguishable from zero and unity, respectively, it also follows that D_{prob} gives a good estimate of D_{d2} , and hence D_h and D^- , without the necessity of conducting a linear transformation.

Based on the strength of the linear relationships existing between D_{ice} and D_{d2} (Fig. 2C) and between D_{d2} and D_{prob} (Fig. 4C), a strong linear dependence of D_{ice} on D_{prob} is also to be expected. This is confirmed by Fig. 4D, which shows that D_{ice} can be predicted well from D_{prob} , not only during the cold winters in which the arc cosine model is applicable, but also during the mild winters when it is not. A linear regression of D_{ice} on D_{prob} (Table 1J) explains 93.2% of the variance in D_{ice} , with a standard error of 0.0253 (i.e., 9.2 d). Thus, D_{ice} is more strongly related to D_{prob} than to the variables N_d (Table 1B) and D_{d2}

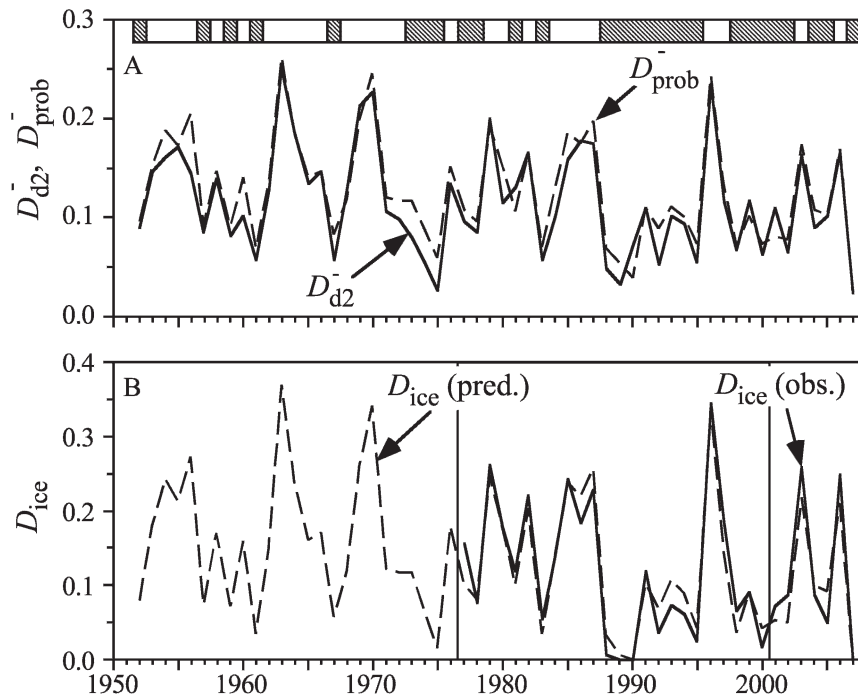


Fig. 5. (A) Comparison between time series of D_{d2} and D_{prob} , where D_{d2} = fraction of the annual cycle (01 July–31 June) during which the daily mean air temperature T_m (calculated as the mean of the daily minimum and daily maximum) was $< 0^\circ\text{C}$, and $D_{\text{prob}} = D_{d2}$ modeled using the probability model (Eq. 10) with constant $\sigma = 3.56^\circ\text{C}$. No calibration was employed to relate D_{d2} to D_{prob} . (B) Comparison of observed values of D_{ice} , the fraction of the annual cycle during which Muggelsee was ice covered (Fig. 1), with values predicted from D_{prob} using the linear regression of Table 1J illustrated in Fig. 4D. The part of the time series between the vertical lines (1976–1999) supplied the data from which the regression coefficients were obtained. The horizontal bar distinguishes mild winters ($v_m \geq 0^\circ\text{C}$, shaded) from cold winters ($v_m < 0^\circ\text{C}$, unshaded).

(Table 1F,G), which were obtained directly from the daily mean air-temperature data, or even to N_h (Table 1C) and D_h (Table 1A), which were obtained directly from the hourly air-temperature data. The predictive ability of the probability model is therefore exceedingly good and, in this particular case, even surpasses that of any simple empirical approach based on measured air-temperature data.

A comparison of the time series of D_{d2} and D_{prob} (Fig. 5A) confirms that both the temporal structure and the absolute value of D_{d2} are given extremely well by D_{prob} during both cold and mild winters. Note that no calibration of any kind was conducted to fit the two curves illustrated in Fig. 5A. A comparison of the time series of the observed values of D_{ice} with the values predicted from D_{prob} using the linear regression of Table 1J (Fig. 5B) also confirmed that D_{ice} can be predicted extremely well from D_{prob} in both cold and mild winters. This is the case not only during the time period that was used in formulating the model (1976–1999), but also during the period 2000–2001 to 2006–2007, which was not included in determining the parameters of the model. Note that, in contrast to D_{d2} , the prediction of D_{ice} does require calibration, and that the regression coefficients of Table 1J are likely to be lake-specific.

From the Tempelhof air-temperature data (1951–1952 to 1999–2000), the mean value of α_m is 9.8°C , with a standard deviation of 1.2°C . Setting $\alpha_m = 9.8^\circ\text{C}$ and comparing D_{prob} with D_{arccos} (Fig. 6), it is clear that the probability and arc cosine models agree well when the air temperature $T < 0^\circ\text{C}$ for more than ~ 0.15 yr (~ 55 d), but the models diverge rapidly at lower durations, so that the probability model is preferable to the arc cosine model not only in mild winters with $D_{\text{arccos}} = 0$ and $v_m \geq 0$ (i.e., $\theta_m/\alpha_m \geq 1$), but also in colder (but not extremely cold) winters in which $D_{\text{arccos}} < 0.15$. From Eq. 4, this occurs when $\theta_m > \alpha_m \cos 0.15\pi$, i.e., when $\theta_m > 0.9\alpha_m$. For higher values of α_m , the point of marked divergence is lower (e.g., for $\alpha_m = 20^\circ\text{C}$, $D_{\text{arccos}} < 0.1$, $\theta_m > 0.95\alpha_m$), and for lower values of α_m , it is higher (e.g., for $\alpha_m = 5^\circ\text{C}$, $D_{\text{arccos}} < 0.25$, $\theta_m > 0.7\alpha_m$).

Results

Setting σ to its mean value of 3.56°C in Eq. 10, D_{prob} can be plotted as a function of θ_m and α_m in the form of a contour plot (Fig. 7), which can be employed as a nomogram to read off the value of D_{prob} in Berlin for any given value of θ_m and α_m . Because D_{prob} is not sensitive to σ unless it differs substantially from the value of 3.56°C

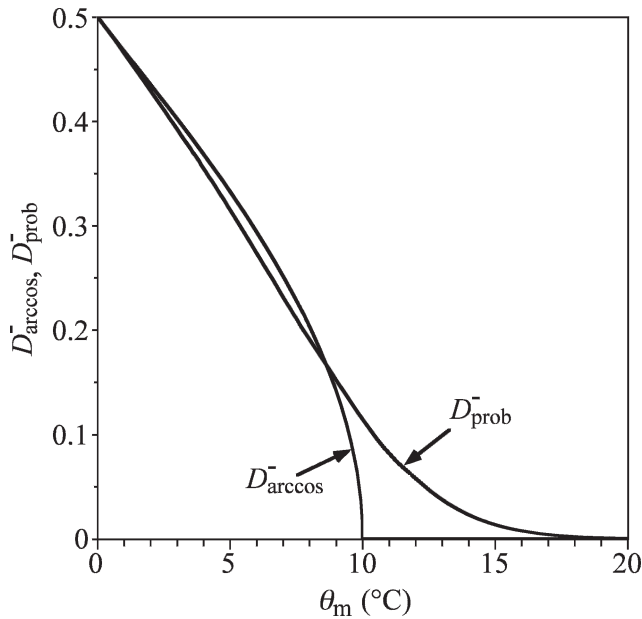


Fig. 6. Comparison of the results of the arc cosine model (D_{arccos}) and the probability model (D_{prob}) obtained when the amplitude α_m of the annual air temperature cycle is set to its mean value ($\alpha_m = 9.8^\circ\text{C}$).

used here, Fig. 7 also gives a good general estimate of the dependence of D_{prob} on θ_m and α_m globally. The values of θ_m and α_m for Berlin for each year from 1951–1952 to 2006–2007, which are included in Fig. 7, show that D_{prob} varied approximately between 0.03 and 0.26, i.e., between about 1 week and 14 weeks, during this time period.

Figure 8A shows the part of Fig. 7 that is most relevant to the Berlin data in more detail, including the predicted values of D_{ice} based on the regression relationship between D_{ice} and D_{prob} listed in Table 1J and illustrated in Fig. 4D.

Given future values of the annual mean and amplitude of the air temperature in Berlin predicted by regional climate models, Fig. 8A can be employed to predict the future duration of ice cover on Müggelsee. Specifically, it can be used to predict whether a given combination of θ_m and α_m will result in zero ice cover. Over the relevant range of values of θ_m and α_m (approximately $9^\circ\text{C} \leq \theta_m \leq 12^\circ\text{C}$ and $6^\circ\text{C} \leq \alpha_m \leq 10^\circ\text{C}$), the $D_{\text{ice}} = 0$ contour, which coincides essentially with the $D_{\text{prob}} = 0.05$ contour, is approximately a straight line with the following equation:

$$\alpha_m = 1.19\theta_m - 4.55^\circ\text{C} \quad (11)$$

Thus, an approximate criterion for Müggelsee to be ice-free during an entire winter is given by $\alpha_m - 1.19\theta_m + 4.55^\circ\text{C} < 0$. This criterion was satisfied for only one of the 49 winters during the period 1951–1952 to 1999–2000; viz. 1989–1990. However, it came close to being satisfied for the winter of 1988–1989. Based on actual observations of ice cover from 1976–1977 to 1999–2000, these two winters were the only winters in this period during which Müggelsee was ice-free (Fig. 1). The same criterion was applied to the winters of 2000–2001 to 2006–2007, which were not used in formulating the model, to test whether the model was capable of predicting the occurrence or nonoccurrence of ice cover on Müggelsee in these winters. Ice cover was predicted to have occurred in the winters of 2000–2001 to 2005–2006, but not in the winter of 2006–2007. These predictions are correct (Fig. 1). Application of the same criterion to the winters of 1951–1952 to 1975–1976 (for which data are available on air temperature but not on ice cover) resulted in a hindcast of no ice-free winters during this period.

Räsänen et al. (2004) performed regional climate change simulations for Europe using a high-resolution regional climate model (RCM) with the output from the HadAM3

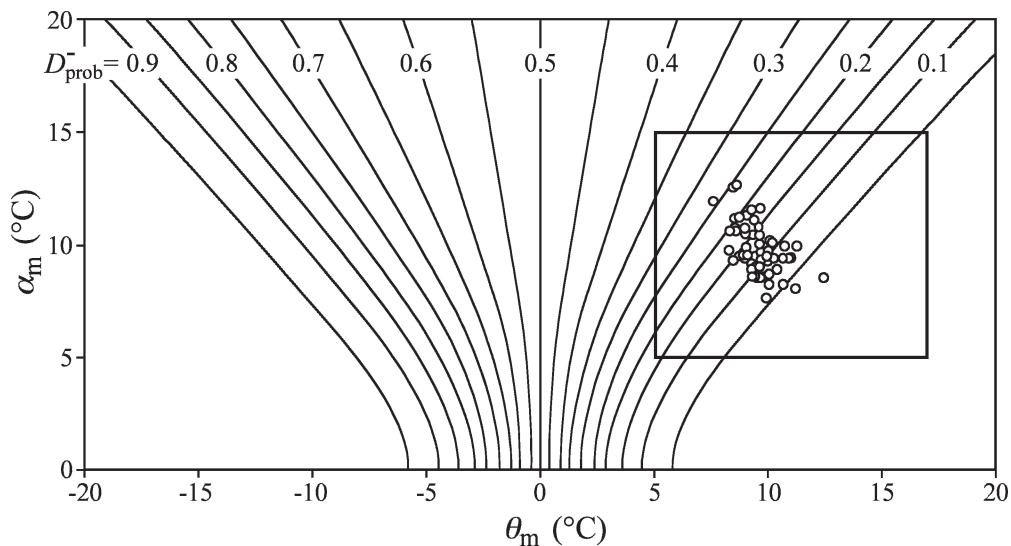


Fig. 7. Contour plot of the relative proportion of the annual cycle with air temperatures below 0°C (D_{prob}), based on the probability model (Eq. 10) for Müggelsee as a function of the mean (θ_m) and amplitude (α_m) of the annual air temperature cycle in Berlin. Measured values of θ_m and α_m (1951–1952 to 1999–2000) are shown as open circles; the corresponding values of D_{prob} are those obtained from Eq. 10 based on these measured values. The portion of the plot within the rectangle is illustrated in detail in Fig. 8A.

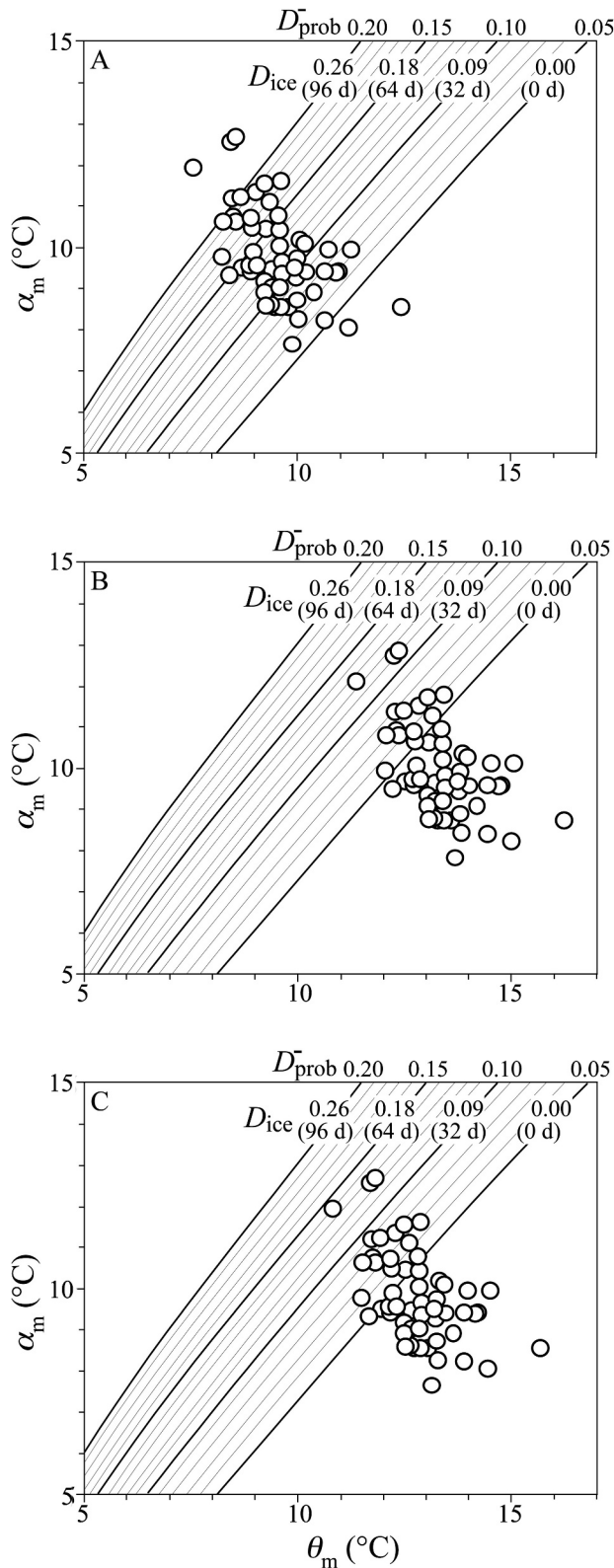


Fig. 8. Detailed section of part of Fig. 7 showing the contours of D_{prob} from Eq. 10 and the equivalent values of D_{ice} (in nondimensional fractions of a year and in days) from the regression equation of Table 1J. (A) Measured values of the mean (θ_m) and amplitude (α_m) of the annual air temperature cycle in Berlin during the period 1951–1952 to 1999–2000. (B) As panel

and ECHAM4/OPYC3 general circulation models (GCMs) as boundary conditions. The underlying climate forcing scenarios were the IPCC scenarios A2 (with high, continuously increasing greenhouse-gas emissions) and B2 (with greenhouse-gas emissions increasing at a lower rate) (Nakićenović and Swart 2000). According to Räisänen et al. (2004), the A2 scenario predicts a rise in global mean temperature from 1961–1990 to 2071–2100 by 3.2°C based on the HadAM3 GCM, and by 3.4°C based on the ECHAM4/OPYC3 GCM; the equivalent figures for the B2 scenario are 2.3°C and 2.6°C . These values lie in the middle of the uncertainty range for such global mean temperature predictions given by Cubasch et al. (2001) and are therefore neither exaggerated nor understated. The results of the simulations of Räisänen et al. (2004) included contour plots of the expected increase in air temperature between the IPCC reference period 1961–1990 and the period 2071–2100. Based on these plots, and taking the average estimate resulting from the two GCMs, for the A2 scenario, the annual mean air temperature in northern Germany is predicted to increase by $\sim 3.7^\circ\text{C}$ on average from the period 1961–1990 to the period 2071–2100. In addition, the difference between summer (JJA) and winter (DJF) air temperatures (T_{diff}) is predicted to increase by $\sim 0.8^\circ\text{C}$. For the Tempelhof data (1951–1952 to 1999–2000), α_m is highly correlated with T_{diff} ($n = 48$, $p < 0.0001$, $r^2 = 0.68$), and it can be estimated well as $0.56T_{\text{diff}}$. For the A2 scenario, this implies that α_m will increase by $\sim 0.4^\circ\text{C}$ from the period 1961–1990 to the period 2071–2100. For the B2 scenario, the corresponding increases are $\sim 3.2^\circ\text{C}$ for θ_m , $\sim 0.5^\circ\text{C}$ for T_{diff} , and $\sim 0.3^\circ\text{C}$ for α_m . It follows that for northern Germany, the main determinant of future changes in D_{prob} , and hence in D_d and D_{ice} , will be θ_m , whereas α_m is likely to play a comparatively minor role. (Note, however, that this will not necessarily be the case in northern Europe, where air temperatures are predicted to increase more in winter than in summer, or in southern and central Europe, where the reverse is the case; Räisänen et al. 2004). From the Tempelhof air-temperature data, the mean values of θ_m and α_m for the IPCC reference period 1961–1990 were 9.4°C and 9.7°C , respectively, which differ only slightly from the mean values computed for the period 1951–1952 to 1999–2000 (Table 2). The corresponding predictions for 2071–2100 are therefore $\theta_m = \sim 13.1^\circ\text{C}$ and $\alpha_m = \sim 10.1^\circ\text{C}$ for the A2 scenario, and $\theta_m = \sim 12.6^\circ\text{C}$ and $\alpha_m = \sim 10.0^\circ\text{C}$ for the B2 scenario (Table 2). If all annual air-temperature data for the period 1951–1952 to 1999–2000 are shifted by the predicted amounts, we get the distribution of θ_m and α_m near the end of the current century expected under scenario A2 (Fig. 8B) and scenario B2 (Fig. 8C).

←

(A), but measured values of θ_m and α_m shifted by amounts corresponding to the predictions of the regional climate model of Räisänen et al. (2004) for Berlin during the period 2071–2100, based on IPCC climate scenario A2. (C) As panel (B), but based on IPCC climate scenario B2.

Table 2. Comparison of parameters characterizing the annual cycle of air temperature in Berlin (at the Tempelhof Airport meteorological station) and on the Plain of Lombardy in northern Italy (at meteorological stations in Bergamo, Piacenza, Brescia, and Verona). For each air temperature time series, the following are listed: the mean (θ_m) and amplitude (α_m) of the sinusoid representing the annual cycle (Eq. 3), the standard deviation of the daily mean air temperature about the sinusoid (σ), and the duration of time during which the daily mean air temperature is $<0^\circ\text{C}$ (D_{prob}^-), computed from θ_m , α_m , and σ using Eq. 10 and expressed both as a fraction of the annual cycle and in d. For the time series covering 1952–2000, 1961–1990, and 1972–1999, the values listed for θ_m , α_m , and σ were obtained by conducting a sinusoidal regression of daily mean air temperature separately for each year of the given time series, and computing the mean of the results. In the case of the Berlin time series covering 2071–2100, the values listed for θ_m and α_m were obtained by increasing the values obtained for the original Berlin time series for 1961–1990 (a) and 1973–1999 (b) by an amount corresponding to the regional climate predictions of Räisänen et al. (2004) for northern Germany based on the IPCC A2 and B2 global climate change scenarios (the value of σ was assumed in all cases to be the same as in the original time series).

Air temperature time series	θ_m ($^\circ\text{C}$)	α_m ($^\circ\text{C}$)	σ ($^\circ\text{C}$)	D_{prob}^- ()	D_{prob}^- (d)
Berlin, 1952–2000	9.4	9.8	3.6	0.128	47
Berlin, 1961–1990 (a)	9.4	9.7	3.6	0.130	47
Berlin, 1973–1999 (b)	9.6	9.5	3.6	0.114	42
Berlin, 2071–2100 (a, A2)	13.1	10.1	3.6	0.038	14
Berlin, 2071–2100 (b, A2)	13.3	9.9	3.6	0.032	12
Berlin, 2071–2100 (a, B2)	12.6	10.0	3.6	0.046	17
Berlin, 2071–2100 (b, B2)	12.8	9.8	3.6	0.038	14
Bergamo, 1973–1999	12.6	10.5	2.7	0.036	13
Piacenza, 1973–1999	12.7	11.2	2.8	0.048	18
Brescia, 1973–1999	12.5	11.3	2.7	0.054	20
Verona, 1973–1999	12.8	11.0	2.7	0.040	14

When the criterion derived from Eq. 11 is applied to the shifted air-temperature data, the percentage of ice-free winters is predicted to increase from 2% to 73% based on scenario A2, and to 59% based on scenario B2. Müggelsee can therefore be expected to be completely ice-free during the entire winter in more than half of all winters during the period 2071–2100.

The availability of air-temperature data from many meteorological stations throughout Europe allows the situation predicted for Berlin at the end of the current century to be compared with the current situation in more southerly regions of Europe that experience higher mean annual air temperatures. Although such space-for-time substitutions cannot express the more complex effects of climate change, they can provide useful and illustrative examples. Table 2 shows such a comparison of Berlin with four meteorological stations (Bergamo, Piacenza, Brescia, and Verona) located on the Plain of Lombardy in northern Italy, 800 km south of Berlin. These stations are located 237, 138, 97, and 67 m above sea level, respectively. For these stations, air-temperature data are available uninterruptedly from 1973 to 1999. Based on the data from this period, θ_m at the Italian stations is comparable to future values of θ_m for Berlin predicted under the B2 scenario and is slightly less than those predicted under the A2 scenario. The values of α_m at the Italian stations are somewhat higher than the future values of α_m for Berlin predicted under both scenarios, but, as shown previously, the role played by α_m in determining D_{prob}^- (and hence D_{ice}) is less critical than that played by θ_m . For 1973 to 1999, the mean value of D_{prob}^- for Berlin is calculated to be 0.114 (42 d), implying an estimated future value for D_{prob}^- of 0.032 (12 d) based on the A2 scenario, and 0.038 (14 d) based on the B2 scenario. The equivalent values of D_{prob}^- for the stations in northern

Italy for the period 1973–1999 range from 13 d to 20 d (Table 2). Thus, the predicted effect of climate change on D_{prob}^- , and hence on the duration of ice cover on Müggelsee, can be compared to the effect of shifting the lake hypothetically 800 km to the south.

Discussion

Modeling the timing, thickness, and duration of lake ice is important because of the ecological effects of lake ice, and because lake ice phenology is undergoing large-scale, long-term change, with ice-on becoming later and ice-off earlier as the global climate warms (Magnuson et al. 2000). The formation of lake ice depends on a multitude of processes, both external and internal to the lake, that result in the lake's surface temperature falling to 0°C . These include the five major heat exchange processes (three radiative and two nonradiative) that determine the lake's heat content and its surface equilibrium temperature (Edinger et al. 1968), but also include the many more processes that determine the seasonally variable distribution of heat within the lake (Imboden and Wüest 1995). The main variables that determine the net heat exchange between a lake and the atmosphere are air temperature, relative humidity, cloud cover (which reduces incident short-wave solar radiation and enhances incident long-wave atmospheric radiation), and wind speed (Edinger et al. 1968; Sweers 1976; Livingstone and Imboden 1989). These same variables suffice to drive many of the heat distribution processes, although additional driving variables such as wind direction and precipitation also play a role. The decay and thawing of lake ice depend on the thickness of the ice layer, its condition, and on the wind stress acting on its surface (Williams 1965). The thickness

and condition of the ice depend on many processes, each of which is forced by several external variables. These processes include: the melting of ice by short-wave solar radiation, by long-wave atmospheric radiation, and by the exchange of sensible heat with the atmosphere; changes in ice structure, decay, and melting, resulting from the penetration of rain, melted snow, or runoff; variations in surface albedo caused by snowfall, rainfall, melting, solid deposition, and the presence or absence of air bubbles in black ice; variations in the insulating properties of the snow cover as a result of new snowfall, rainfall, compaction, melting at the air snow interface, and surficial and internal melting and refreezing to form snow ice layers; ice fracturing due to snow load; and ice growth and decay at the ice water interface (Gu and Stefan 1990; Leppäranta 1991; Duguay et al. 2003). Because of the multiplicity and complexity of the processes involved in determining the formation and thawing of lake ice, it is evident that it is impossible to capture their full effect on the total duration of winter ice cover in just one variable, i.e., air temperature. Like any empirical model of ice cover, the probability model described here therefore has limitations that are rooted in the physics of the problem. The effects of important forcing variables, such as cloud cover and snowfall, which usually appear explicitly in physical lake ice models (Duguay et al. 2003; Saloranta and Andersen 2007), appear in the probability model only implicitly, in the form of hidden correlations with air temperature. Changes in these correlations, perhaps as a result of climate change, are likely to result in a deterioration of the predictive ability of the probability model. Thus, when employing this model, one needs to be conscious of its inherent limitations, especially when applying it to another location or another climate.

For a number of lakes, ice duration, acting via the timing of ice-out, has been shown to be a major driver of spring plankton phenology, particularly of the timing of the phytoplankton spring bloom (Adrian et al. 1995; Weyhenmeyer et al. 1999; Gerten and Adrian 2000). Specifically, this is known to be the case in Müggelsee (Adrian et al. 1999; Gerten and Adrian 2000). During the winters of 1988–1989 and 1989–1990, Müggelsee was ice-free, and during the winter of 1987–1988, it was ice-covered for only 2 d (Fig. 1). After each of these three winters, the spring phytoplankton bloom in Müggelsee developed particularly early compared to the other winters with ice cover (Adrian et al. 2006). Given the strong relationship that exists between plankton phenology in spring and the duration of ice cover during the previous winter, it can be surmised that the extreme increase in the number of ice-free years projected for Müggelsee by the end of the current century will have a substantial effect on food web interactions in the lake. If the changes in phenology that occur are not the same at all trophic levels, mistimings in predator-prey relationships are very likely to occur. In more northerly lakes in which ice cover is usually continuous, oxygen conditions will likely improve as a result of a decrease in the duration of ice cover (Livingstone 1993). In such lakes, the frequency of winter fish-kills, which can be a consequence of long periods of ice cover (Greenbank 1945; Barica and

Mathias 1979), may diminish. Because many fish species spawn in spring, thus affecting subsequent predation on the zooplankton community, this change will contribute to the complexity of potential changes in food-web interactions.

Given the differences in the ways in which different trophic levels respond to winter warming, at the moment, any attempt to describe the long-term overall response of plankton communities to predicted changes in the duration of ice cover would be mere speculation. However, a criterion for a given lake to be completely ice-free during winter derived from Fig. 7 could be easily incorporated into plankton phenology models, thus providing an important module in models developed to predict the effect of future climate scenarios on food web interactions in winter and spring. Moreover, the probability model proposed here may be useful in forecasting the duration of ice cover on more northerly lakes, such as those of southern Scandinavia, which in future are likely to experience climatic conditions similar to those found in northern Germany at present. A one-to-one match of the ecological implications is, however, not to be expected because other relevant factors (e.g., day length) will not change (both water temperature and day length are important species-specific cues for the emergence of plankton resting stages; Cáceres 1998; Hairston 1998).

As in the case of any empirical model, the validity of the model presented here for Müggelsee is theoretically limited to the range of data employed in its formulation. It should not therefore be assumed that it will function well for other lakes under different meteorological conditions without appropriate further testing and calibration. However, given the success and general applicability of the suite of empirical models based solely on air temperature that are routinely employed to predict various aspects of ice cover such as ice thickness (Stefan 1890; Assel 1976) and the calendar dates of ice-on (Bilello 1964) and ice-off (Palecki and Barry 1986; Livingstone 1997), the empirical probability method proposed here certainly appears promising at the very least. To be able to make reliable predictions of the responses of lacustrine ecosystems to currently increasing winter air temperatures, it is necessary to couple biological lake models to models that are capable of forecasting important physical environmental boundary conditions, such as ice cover. The probability model proposed here would be ideal for this purpose: it is applicable regardless of whether the lake ice cover is continuous, intermittent, or absent; it is not dependent on a fixed definition of “winter”; the results emerge in time units, simplifying calibration; and it is based on statistical properties of the ambient air temperature that are routinely predicted by regional climate models. It is therefore straightforward to apply and provides a direct output that could easily serve as an input to relevant biological models.

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