Numerical modelling of in-plane behaviour of URM walls and an investigation into the aspect ratio, vertical and horizontal post-tensioning and head joint as a parametric study

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Masonry is a complex composite material with non-linear material properties, which make the numerical investigation of its structural behaviour a difficult task. In this paper, a micro non-linear model created using a general purpose finite element code, is discussed with respect to its ability to simulate the in-plane behaviour of unreinforced masonry walls. That is, the capability of model in predicting the cracking, crushing, and sliding phenomena, as well as the global-orthotropic behaviour of previously tested masonry walls are examined. The model is then used within a parametric study to investigate the effect of different aspect ratios, vertical and horizontal post-tensioning as well as the effect of head joints on the capacity of the masonry walls. It will be shown that, while enhancing the lateral in-plane strength of the masonry walls, vertical post-tensioning can also reduce their ductility. On the contrary, horizontal post-tensioning can cause a small reduction in the initial stiffness and yielding shear force of the masonry walls, but increases their integrity, thereby preventing them from sliding and shear failure. Furthermore, it will be shown that, in spite of having a significant influence on local behaviour, head joints do not influence the global behaviour of the masonry walls.

Keywords: *post-tensioning, in-plane behaviour, aspect ratio, masonry*

1. Introduction

There are many masonry structures throughout the world that have been built in the past decades and lack sufficient strength and enough ductility to resist strong ground motions and heavy live loads. To assess vulnerability of such structures, and to examine the efficiency of different retrofitting methods, a numerical tool is needed that can take into account non-linear behaviour and orthotropic characteristics of masonry. Various authors have carried out different numerical analyses on masonry buildings [e.g. 1–6]. An elastic homogenized macro model was used to study the

behaviour of different masonry structures [1–3]. The model did not deal with nonlinear characteristics of masonry materials. Luciano et al. [4] carried out both experimental and numerical studies on masonry arches under vertical loading before and after reinforcing by fibre composite materials. Four different material models were used for the masonry bricks: a linear-elastic model, a no-tension model, an elasto-plastic model and an isotropic damage model. The plastic hinges were specified by the tests and simulated by the brittle cut-off (no-tension) model in the numerical analyses. The linear-elastic model was able to simulate the behaviour of wall at very low field stress. The second model was not able to predict compressive failure. The elasto-plastic model was able to describe the behaviour in compression, but was not successful in reproducing the tensile response of the bricks. By applying the damage model to the bricks, the response of the reinforced masonry arch could be simulated satisfactorily.

Gambarotta and Lagomarsino [5, 6] proposed a model that took into account both mortar damage and brick-mortar debonding. The authors used this model in two different approaches, first [5], interfaces located in the bed joints and head joints were described by the damage model. Compressive failure was considered by assuming an elastic limit condition in the bricks based on Von Misses criterion that the parameter selecting the limit state surface is obtained with reference to the failure under simple compression. In the second approach [6], a continuum model based on an equivalent stratified medium made up of two typical layers was developed: one layer represented the mortar bed joint and another layer represented the brick units and head joints. The constitutive equations for masonry were obtained through a homogenization procedure involving the abovementioned damage model for the bed joints and simple damage constitutive equations for brick layers neglecting the head joints. In the latter approach only the vertical compressive strain in bricks was taken into account because the inelastic vertical extension in the masonry is localized in the bed joint due to their low strength in comparison with the tensile strength of the bricks. These simple assumptions for the brick constitutive model may be supported due to the fact that the collapse mechanisms of shear masonry walls are first characterized by the failure in mortar bed joints and successively, after a wide inelastic phase, by the brick failure [6]. Of the two approaches, the second one requires much less computations while having similar results relative to the first approach and to the experiments.

Despite valuable results, the above numerical models are generally either over-simplified, thereby ignoring some aspects of the complex nature of masonry, or too complex to be used as a general tool. In the present study, a numerical model is introduced that uses the general multi purpose finite element (FE) code, ANSYS [7], and takes into account the non-linear behaviour and complex nature of masonry walls such as cracking, sliding, and crushing. The numerical model is verified by two series of independent tests and its results will be compared to the predictions by the theoretical results. The model is then used to evaluate the effects of aspect ratio (ratio

of height to length), vertical and horizontal post-tensioning and head joints on the in-plane behaviour of masonry walls.

2. Failure mechanisms and material models for plain masonry

Masonry wall is a composite structure that consists of brick and mortar. The geometry and layout of joints play an essential role in the response of the wall and in the mechanism of failure. Figure 1 shows different in-plane failure modes for masonry walls.

Fig. 1. In-plane failure mechanisms of masonry walls

Several researchers have proposed failure criteria for masonry material [e.g. 8–14]. Malyszko [8] proposed three simple forms of the in-plane shear failure of masonry specimens. Accordingly several tests have been carried out in order to determine the strength parameters and failure surfaces. Ali and Page [9] suggested a micro finite element model for masonry considering elastic elements for bricks and link elements for horizontal mortar joints. An envelope of the shear and compressive strength was defined for bond failure of the joints based on experimental data. Although this model could present global nonlinear behaviour of masonry walls and crack distribution, but it could not predict the failure in bricks and mortar and the effect of multi axial stresses on the response. Assuming isotropic behaviour, Wawrzynek and Cincio [10] applied a brittle isotropic model to analyze masonry subjected to cyclic or dynamic loading. In the study, a biaxial envelope of the load capacity as a function of the principal stresses was used to define masonry failure. The model was fairly successful; however, the authors suggested that it should be modified for material with orthotropic properties like masonry. Ganz [11] also presented failure criteria for masonry under biaxial compressive stresses, neglecting the tensile strength (Figure 2). The failure criteria could be described by five mechanisms: tensile, compressive and shear failure of the bricks and tensile failure and sliding of the mortar joints. The failure criteria has been derived based on the tests and neglects the tensile strength of masonry [11].

In another study, Kumar and Bhandari [12] developed a two-dimensional, non-linear finite element model for the analysis of masonry arches. The model took into

account non-linearity in stress-strain relationship and cracking of masonry. The model was not able to replicate accurately the experimental behaviour, but could capture the crushing and cracking zones in the arches. Litewka and Szojda [13] supplied new experimental data for brittle material subjected to triaxial state of stress as well as presentation of the theoretical model capable to describe deformability and failure of initially isotropic brittle rock-like solids. In their study, the stress-strain curves and stress at failure were determined experimentally for cylindrical specimens of mortar under tri-axial state of stress. These experimental results showed a good agreement with the theoretical predictions obtained from the own theoretical model based on the methods of the damage mechanics.

Fig. 2. Masonry failure surfaces for compression proposed by various authors

Willam and Warnke [14] developed a constitutive model for tri-axial behaviour of concrete, which can be used to simulate the nonlinear response of brittle materials such as masonry. This theory has been used in the present work and is discussed in some details. The model considers multi axial stresses of brittle material and takes into account cracking, crushing and sliding phenomena by reflecting their effects on the stiffness matrix [7–14]. The failure criterion is expressed in the following form:

$$\frac{F}{f_c} - S > 0, \quad (1)$$

where:

- F – a function of the principal stresses,
- f_c – uni-axial compressive strength,
- S – expresses the failure surface.

The failure surface S is a function of principal stresses in addition to the tensile, uni-axial and biaxial compressive strengths (f_t , f_c and f_{cb}). The failure surface defined by

Equation (1) is formulated for each condition of principal stresses [7, 14]. The element considers a smeared crack analogy for cracking in tension zones and a plasticity algorithm to account for possibility of crushing in compression zones. Formation of a crack is achieved by modification of the stress-strain relationships of the element to introduce a plane of weakness perpendicular to stress direction. An amount between full shear transfer and no shear transfer may be used as the ability of a crack to transfer the shear stress across its plane. When crushing occurs in a section, any further application of load develops increasing strains at constant stress. The Willam and Warnke's failure surface developed for in-plane behaviour, that is, two-dimensional loading, is shown in Figure 2. A three dimensional eight noded solid isotropic element, Solid 65, has been provided in ANSYS [7] to model nonlinear behaviour of the brittle materials based on Willam and Warnke's theory.

3. Experimental studies

Two series of independent tests [15–16] which have been carried out on the masonry walls subjected to in-plane lateral load and their experimental data have been available in details, are used to validate the numerical model described in the next section.

Material specifications, geometry and loading details of the wall specimens are summarized in Table 1. In this table, thickness, height and length of the specimens are represented by b_w , h_w and l_w .

All specimens were cantilever and the vertical load has been applied uniformly on their upper edge using a concrete slab. In the experiments by Ganz and Thurlimann [15], the vertical load was first applied and kept constant, and then the lateral load was applied incrementally until the failure of the wall. Two of their specimens, W1 and W4, are used in the current study. The specimens tested by ElGawady [16], were subjected to dynamic excitations (i.e. shaking table tests) and axial (vertical) loading was applied to an upper concrete distributing beam using two external bars. Due to lateral deformations, the initial force in the vertical bars increased during the tests (shown by $P1$ (initial force) and $P2$ (force at end of test) in Table 1). Results from ElGawady's three unreinforced masonry specimens, consisting of two specimens with the aspect ratio (ratio of height to length) of 1.0 (L1 and L2) and one specimen with the aspect ratio of 0.4 (S2), will be used below.

4. Numerical modelling of plain masonry walls

A numerical micro model is created using the general purpose finite element code ANSYS [7]. All brick units and joints are considered in this model. Vertical joints (head joints) are represented by Solid 65 element with smeared cracks pattern of failure. Bed joints are modelled by Contact 52 element to simulate discrete cracking at failure. This may be justified by considering the low tensile strength of the bed joints and due to the fact that the shear strength of the masonry is defined as a combination of

low initial shear strength under zero compressive stress and increment in the strength induced by the compressive stress perpendicular to shear.

Table 1. Specifications of masonry wall specimen

ElGawady (2004) [16]	ElGawady (2004) [16]	ElGawady (2004) [16]	Ganz (1984)[15]	Ganz (1984)[15]	Ref.	
S2	L2	L1	W4	W1	Wall specimen	
1.6×0.7×0.075 Aspect ratio 0.44	1.6×1.6×0.075 Aspect ratio 1.0	1.6×1.6×0.075 Aspect ratio 1.0	3.6×2.0×0.15 (Thickness of flanges is 0.9) Aspect ratio 0.56	3.6×2.0×0.15 (Thickness of flanges is 0.6) Aspect ratio 0.56	Dimensions of sample ($l_w \times h_w \times b_w$) (m)	
30	31	30	423	415	P_1	Axial load (kN)
38	44	90	423	415	P_2	
–	–	–	840	–	Eccentricity (mm)	
855	855	1044	2460	2460	$E_{X\text{-masonry}}$ (MPa)	
10	10	10	10	10	Thickness of joints (mm)	
160×100×75 Clay blocks	160×100×75 Clay blocks	160×100×75 Clay blocks	300×190×150 Concrete blocks	300×190×150 Concrete blocks	Size of bricks (mm)	
14.8	14.8	14.8	37.4	37.4	Brick	Compressive strength (MPa)
3.3	3.3	8.9	23.9	23.9	Mortar	
5.7	5.7	7.7	7.61	7.61	Masonry	
–	–	–	9.1	9.1	Brick	Tensile strength (MPa)
–	–	–	5.5 to 6.3	5.5 to 6.3	Mortar	
–	–	–	.05	.05	Masonry	
Dynamic (Time -history)	Dynamic (Time -history)	Dynamic (Time -history)	Monotonic	Monotonic	Type of lateral loading	

Contact 52 is defined by axial and sliding stiffness and supports tangential sliding based on Coulomb theory of friction, where sliding is eliminated when the element surface is under tensile axial force greater than the tensile strength of the mortar bed joints. Due to relatively low stress level, the brick units are simulated by an elastic solid element (the same procedure has been used by other authors [17]), Solid 45, and failure of brick units is examined manually based on Willam and Warnke's theory. Details of the meshing used in the model are illustrated in Figure 3.

Fig. 3. Details of meshing used in the numerical model

Material properties used in the model are driven from the test results and are given in Table 2. The compressive strength of the masonry constituents (f_{cb} and f_{cm}) were selected based on existing experimental measurements for all specimens (see Table 1). For Specimens L1, L2 and S2, due to lack of experimental measurements, the tensile strength (f_{tb} and f_{tm}) were assumed to be approximately 10 to 15 % of the compressive strength based on recommendations given in the literature [e.g. 18]. The sliding stiffness (KS) of the contact elements used in the bed joints can be calculated from the lateral drift observed in the tests. E modulus of mortar used in the head joints is then determined based on KS . Evaluating the E modulus of the masonry from the test results and considering the same stress in all components, the E modulus of the bricks is determined [16]:

$$E_m = E_{\text{masonry}} E_b / ((th_b / th_m)(E_b - E_{\text{masonry}}) + E_b), \quad (2)$$

where:

E_{masonry} , E_b and E_m are the elastic modulus of the masonry, brick and mortar,
 th_{brick} and th_{mortar} are the thickness of the bricks and mortar joints, respectively.

The axial stiffness can be evaluated based on vertical deformation observed in the tests. The coefficient of friction (μ) was estimated based on Coulomb theory through division of the base shear force of the yield point of the force-deformation curve by the corresponding axial (vertical) force, both recorded during the tests [15–16].

Table 2. FE model specifications

Specimen	Bricks			Head joints			Bed joints		
	f_{cb} (MPa)	f_{tb} (MPa)	E_b (MPa)	f_{cm} (MPa)	f_{tm} (MPa)	E_m (MPa)	KN (N/m)	KS (N/m)	μ
W1&W4	37.4	9.1	3600	23.9	2.5	400	400E6	120.E6	0.7
L1	14.8	2	2000	8.9	0.8	197	56E6	18E6	0.4
L2	14.8	2	2000	3.3	0.45	70	19E6	5.5E6	0.4
S2	14.8	2	2000	3.3	0.45	70	19E6	5.5E6	0.7

Nonlinear analyses are carried out using Newton Raphson approach. The simulation involves application of incremental and monotonic horizontal deformation to the top of the walls. Axial load is applied to different specimens in accordance to the tests. For Specimen W1, axial load is kept constant with uniform distribution. For Specimen W4, which has been subjected to an eccentric axial point load in the test (see Table 1), the upper concrete beam is also considered in the model. It is reiterated that in Specimens L1, L2 and S2, due to flexural cracking, the height of the walls increases and this causes significant fluctuations in the post-tensioning force [16] (compare $P1$ with $P2$ in Table 1). For recent specimens, axial load at each load step is adjusted based on experimental records [16]. Loading and boundary conditions used in the model are illustrated in Figure 4.

Fig. 4. Loading scheme and boundary conditions used in the numerical model

Figures 5 to 7 illustrate the responses of different walls in terms of top lateral load versus top lateral deformation. All curves involve two main branches: before yield and post yield. A relatively sharp slope and almost linear shape of initial part indicate a relatively stiff and almost elastic response of the walls before yield. The yield point corresponds to the beginning of sliding deformation in the bed joints. After yield, the walls experience relatively sharp decline in stiffness but relatively stable and consistent behaviour up to the last load step. The last load step is that of the tests, which for Specimens W1 and W4, corresponds to the onset of crushing at the toes of walls. It should be stated that the deformation seen for the ElGawady's unreinforced specimens corresponds to a predefined degree of damage and not to wall failure. This was done in order to apply retrofitting measures for further testing. As can be observed, the initial and post-yield stiffnesses, the shear strength corresponding to the last load step, and the value of ductility are in close agreement with test results for all specimens. As given in Table 1, the specimens are different in such specifications as mechanical properties of the bricks and mortar, type and eccentricity of axial load (in Specimen W4, axial load is a point load with an eccentricity of 850 mm while in the other four specimens, axial load is uniformly distribution), sectional geometry (the section of specimens tested by ElGawady and by Ganz are I shape and rectangular, respectively) and aspect ratio. Despite various characteristics, Figures 5 to 7 demonstrate that the model has been successful in simulating the test results. For instance, Specimens L1 and L2 are similar in geometry, but, due to higher axial load and stiffer mortar, the post-yield stiffness and the shear strength are greater in Specimen L1. As another example, Specimen S2 has smaller aspect ratio (0.4) compared to Specimen L2 (1.00). This causes higher stiffness, and therefore, larger shear force in S2 relative to L2 under identical lateral deformation, as is seen in the figures.

In addition to behaviour, the numerical model has predicted the nature of damage at different load levels. A comparison between tests and numerical results is presented in Table 3. It is seen that the nature of cracks and pattern of damage are in agreement with the numerical simulation. For all specimens, the simulation indicates that flexural cracks form initially at the most bottom bed joint, and, as lateral load increases, the cracks propagate upwards (see Table 3). In parallel to flexural cracks, damage is accompanied by sliding deformation over a large length of the bed joints when lateral deformation reaches the yield point. Beyond this point, step pattern shear cracks are observed in the model, which start at the bottom corner of all walls and proceed upwards as lateral load increases. It can be seen that in Specimen W4, due to eccentric point axial load, flexural cracks are more extensive and the damage of the head joints is shifted to those located in the wall end close to the axial load, relative to W1. In the specimens with larger axial load (e.g. L1), flexural and shear cracks are not as extensive as L2. For Specimen S2, due to smaller aspect ratio, and therefore, smaller flexural moment, there are considerably fewer flexural and shear cracks relative to Specimen L2.

Fig. 5. Numerical and test results of Specimens W1 and W4

Fig. 6. Numerical and test results of Specimens L1 and L2

Fig. 7. Numerical and test results of Specimen S2

Manual control indicates that the brick units have remained undamaged throughout numerical simulation for Specimens L1, L2 and S2, while crushing occurs at the toes of W1 and W4 at the last load step. This can be confirmed by the test results.

Table 3. Pattern of damage in different specimens

Specimen	Test results	Numerical model results
W1
W4
L1
L2

5. Comparison with theoretical relationships

In this section, the numerical results obtained by the above finite element model has been compared to theoretical relationships. The analytical relations proposed in [19] have been used which are summarized in Table 4. Sliding shear strength (V_{sliding}) may be defined by Coulomb friction law and depends on the amount of axial load, aggregate interlock and the quality of the interface between the brick units and mortar in the horizontal bed joints. The shear strength corresponding to the first cracking (V_{cr}) may be evaluated simply when the tensile stress surpasses the tensile strength at a wall end (see Table 4). As this table shows, for shear strength corresponding to the nominal flexural strength of the wall (V_n), the ultimate limit state method is used by assuming a length of a for the equivalent rectangular stress block with a stress of $0.85f'_m$ (f'_m is the compressive strength of masonry) and an extreme fibre strain of $\epsilon u = 0.003$ for unconfined masonry. In Table 4, tensile strength, Poisson ratio, moment of inertia, area, and initial moment of the specimens are represented by f_t , ν , κ , I_w , A_w and M_1 . As recommended in [19], a reduction factor of 0.5 is used to estimate the effective moment of inertia and area ($I_{w,eff}$ and $A_{w,eff}$). The other parameters have been defined in the previous sections.

Table 4. Analytical relations for masonry walls [19]

Shear sliding strength	$\tau = \tau_i + \mu \cdot \sigma_v$; $V = \tau \cdot A_w$; $0 < \tau_0 < 1.5MPa$; $0.3 < \mu < 1.2$; $\sigma_v = P / A_w$
Cracking flexural strength	$M_{cr} = [f_t + P/(A_w) - M_1(0.5 \cdot l_w) / I_w] \cdot I_w / (0.5 \cdot l_w)$; $V_{cr} = M_{cr} / h_w$
Nominal flexural strength	$P = \alpha \cdot f'_m \cdot b_w \cdot a$; ($\alpha = 0.85$); $M_n = P \cdot (l_w / 2 - a / 2)$; $V_n = M_n / h_w$
Corresponding deformation to cracking strength	$d_{cr} = V_{cr} \cdot [h_w^3 / (3E_{\text{masonry}} \cdot I_w) + 2(1 + \nu) \cdot \kappa \cdot h_w / (E_{\text{masonry}} \cdot A_w)] +$ $M_1 \cdot h_w^2 / (2 \cdot E_{\text{masonry}} \cdot I_w)$
Corresponding deformation to nominal strength	$d_n = V_n \cdot [h_w^3 / (3E_{\text{masonry}} \cdot I_{w,eff}) + 2(1 + \nu) \cdot \kappa \cdot h_w / (E_{\text{masonry}} \cdot A_{w,eff})] +$ $M_1 \cdot h_w^2 / (2 \cdot E_{\text{masonry}} \cdot I_{w,eff})$

Table 5 presents a comparison among numerical, test and theoretical predictions for strength and deformation corresponding to the cracking and ultimate states. Neglecting the tensile strength of masonry, geometry and other mechanical properties of the specimens needed for the analytical relationships are chosen based on the information given in Table 1. As mentioned in the previous sections, during the tests of Specimens L1, L2 and S2, axial load has increased by increasing in the lateral loading. In order to calculate the shear strength and deformation corresponding to the first cracking state, represented by V_{cr}^c and d_{cr}^c in Table 5, a constant axial load corresponding to the first cracking state (P_{cr}) was chosen from the experimental records [16] and used in the model and theoretical relations.

Table 5. Comparison among numerical and test results and predictions by analytical relations

S2	L2	L1	W4	W1	Specimen
					P_{cr}
40	40	42	423	415	
19.89	10.8	12.4	19.5	147.8	V_{cr} (kN)
14.215	7.433	8.123	19.5	147.8	V_{cr}^c (kN)
28.8	18.67	33.8	187	234	V_{max} (kN)
0.433	0.81	0.7	0.2	0.55	d_{cr} (mm)
0.27	0.53	0.4	0.2	0.55	d_{cr}^c (mm)
2.3	12.5	11.8	2.5–18	1.6–13.85	d_{max} (mm)
20	11	10	No data	No data	V_{cr} (kN)
29	17.3	31.5	180	237.4	V_{max} (kN)
0.4	0.9	0.8	No data	No data	d_{cr} (mm)
2.3	125	11.8	18	13.85	d_{max} (mm)
33.6	20.8	39.2	296	290	$V_{sliding}$ (kN)
15.24	6.7	7.0	14.85	168.25	V_{cr}^c (kN)
40.6	20.86	39.3	194.5	361.44	V_n (kN)
0.38	.70	0.60	0.29	0.76	d_{cr}^c (mm)
2.50	5.1	7.55	3.38	3.31	D_n (mm)
6.73	-10.94	-16.04	23.8	12.15	$[V_{cr}^c]_{theory} - V_{cr}^c]_{model}] / V_{cr}^c]_{theory}$ (%)
14.29	10.24	13.78	3.86	19.31	$[V_{max}]_{theory} - V_n]_{model}] / V_{max}]_{theory}$ (%)
28.9	24.28	33.3	31.03	27.63	$[d_{cr}^c]_{theory} - d_{cr}^c]_{model}] / d_{cr}^c]_{theory}$ (%)

(840 mm) of the axial load, the flexural strength is smaller than the sliding strength and therefore is predominant. This eccentricity causes the first cracking strength to be very small in Specimen W4. The table emphasizes a good agreement between the test and numerical results for all presented items (i.e. V_{cr} , V_{max} , d_{cr} and d_{max}). Furthermore, it can be seen that for all specimens the numerical results obtained for the cracking and maximum shear strength (V_{cr}^c , V_{max} , given in the column of numerical predictions) are in relatively good agreement with the predictions by the analytical relations (V_{cr}^c , V_n , given in the column of theoretical predictions). The maximum deviation occurs for Specimen W1 and is limited to 23.8% (see Table 5). A comparison between the numerical results and also predictions by the analytical relations for the deformation corresponding to the first cracking (d_{cr}^c) is presented in Table 5. It should be added that the shear force and deformation at first crack have not been measured during the experiments of W1 and W4.

Since the masonry specimens experience a significant sliding deformation beyond the yield point of their force-deformation relationship (see Figures 5–7) as well as the flexural and shear deformations, therefore the maximum deformation observed in the model and tests (d_{max}) is larger than values predicted by the analytical relations (d_n). This is more significant for Specimens W1 and W4 which are subjected to a constant axial load whereas for Specimens L1, L2 and S2, the axial load and therefore the sliding strength are increasing by increasing in the lateral deformation. Indeed, the nominal flexural deformation predicted by the analytical relations is only based on the shear and flexural behaviour of the masonry walls and therefore give an underestimation of final deformation. Therefore a logical comparison between d_{max} and d_n is not possible.

6. Parametric study

Using the model, the influence of the aspect ratio, vertical and horizontal post-tensioning and head joints on the in-plane behaviour of masonry walls were investigated. Geometric and material characteristics of specimen L2 (see Table 1) were used for the reference wall. Table 6 presents a summery of characteristics of the specimens used for the parametric study. Each specimen is designated by a name that reflects its characteristics. For instance, L2-AR1.5 means specimen L2 with an aspect ratio of 1.5 or L2-AR1.5-PT0.2 refers to a specimen L2 with an aspect ratio of 1.5 post-tensioned by an axial force equal to 20 percent of its compressive strength.

Table 6. Wall specimens used for parametric study

Specimen	Description
L2	Reference specimen, see Tables 1 and 5
L2-AR0.44	Specimen L2 with aspect ratio of 0.44 (1600×700), other properties are the same as specimen L2.

L2-AR1.5	Specimen L2 with aspect ratio of 1.5 (1600×2400), other properties are the same as specimen L2.
L2-AR0.44-VPT0.2	Specimen L2 with aspect ratio of 0.44 (1600×700) and axial loading equal to 20% of its compressive strength (128.7 kN), other properties are the same as specimen L2.
L2-VPT0.2	Specimen L2 with aspect ratio of 1.0 (1600×1600) and axial loading equal to 20% of its compressive strength (128.7 kN), other properties are the same as specimen L2.
L2-AR1.5-VPT0.2	Specimen L2 with aspect ratio of 1.5 (1600×2400) and axial loading equal to 20% of its compressive strength (128.7 kN), other properties are the same as specimen L2.
L2-HPT0.2	Specimen L2 with horizontal post-tensioning equal to 20% of its compressive strength (128.7 kN), other properties are the same as specimen L2.
L2-VPT0.2-HPT0.2	Specimen L2 with vertical and horizontal post-tensioning both equal to 20% of its compressive strength (128.7 kN), other properties are the same as specimen L2.
L2-VPT0.2-HPT0.05	Specimen L2 post-tensioned in the vertical and horizontal directions by a force of 20% of its compressive strength (128.7 kN) and 5% of its compressive strength (32 kN) respectively. Other properties are the same as specimen L2.
L2-E-HJ	Specimen L2 with elastic behaviour for the head joints
L2-W-HJ	Specimen L2 without considering the head joints

6.1. Aspect ratio

Masonry walls with different aspect ratios exhibit different behaviour when subjected to lateral loading. In this section, the effects of the aspect ratio on the lateral behaviour of masonry walls and the ability of the model to predict this behaviour will be discussed. To do this, the aspect ratio of a reference specimen—wall L2 (see Tables 1 and 2) – was varied and the changes in behaviour due to this variation compared. Three aspect ratios were examined: 0.44 (1600 × 700 mm, specimen L2-AR0.44), 1.0 (1600 × 1600 mm, the actual size of wall L2), and 1.5 (1600 × 2400 mm, specimen L2-AR1.5). The effects of the aspect ratio on the global behaviour of wall L2 obtained from numerical simulations are shown in Figure 8 as a relationship between base shear force and drift. For all specimens, the maximum drift was selected based on the onset of predominant failure predicted by the model (i.e. sliding for L2-AR0.44 and crushing at the toe for L2 and L2-AR1.5).

Figure 8 emphasizes the higher stiffness and larger lateral strength of the masonry wall with a lower aspect ratio. That is, as can be expected, for walls with the same material properties, length and loading conditions, the shorter walls will possess larger lateral strength.

Fig. 8. Effects of the aspect ratio and vertical post-tensioning on the global behaviour of masonry walls predicted by the FE model

Table 7. Effects of the aspect ratio, vertical and horizontal post-tensioning on the crack pattern of masonry walls predicted by the FE model

Aspect ratio	Vertical post-tensioning	Horizontal post-tensioning
<p>Specimen L2-AR0.44 (last load step)</p>	<p>Specimen L2-AR0.44-VPT0.2 (last load step)</p>	<p>Specimen L2-HPT0.2 (first load step)</p>
<p>Specimen L2 (last load step)</p>	<p>Specimen L2-VPT0.2. (last load step)</p>	<p>Specimen L2-HPT0.2 (last load step)</p>

Table 7 shows the crack distribution at the last load step predicted by the model for specimens L2-AR0.44 and L2-AR1.5, respectively. In Table 7 it can be seen that flexural cracks are less prevalent in the squat wall (L2-AR0.44) and, sliding failure is the predominant failure mechanism. This behaviour is expected due to the low height and therefore large lateral stiffness of the wall and can be verified by the theoretical calculations using relations given Table 4. This is in comparison to the reference wall (Specimen L2) where failure was predicted by the model as a combination of shear and flexural cracking. In contrast to this, Table 7 shows that a more slender wall, L2-AR1.5, exhibits predominantly flexural and shear cracking in the bed and head joints. Furthermore, sliding movement, with maximum sliding in the first bed joints, occur in Specimens L2 and L2-AR1.5. In general, for walls with higher aspect ratios (e.g. greater than one), failure can occur due to a combination of shear and flexural cracking. As will be explained in the following section, if the axial load is high enough, flexural failure rather than shear failure can be expected to be the prominent failure mechanism.

6.2. Axial loading

The lateral load carrying capacity of a masonry wall may be increased by post-tensioning, that is, through the application of additional axial (vertical) load to the wall. In order to investigate the effect of this axial load, the model was used again to simulate the behaviour of wall L2 with an additional post-tensioning force of 128.7 kN applied uniformly to the upper edge of the wall. This axial force is approximately 20% of the compressive strength of wall L2 and has been recommended for post-tensioning in “VSL Technical Reports, Series 2” [20]. It should be reiterated that ElGawady tested his un-reinforced specimens (e.g. Specimen L2) to a predefined degree of damage and not to failure in order to apply retrofitting measures for further testing. Therefore, the final deformation and ductility of Specimen L2 given in this section,

has been predicted by the model (are not taken from test results). Analyses were performed for the post-tensioned specimen L2 with varying aspect ratios of 0.44, 1.0 and 1.5. The characteristics of the post-tensioned specimens (referred to as L2-AR0.44-VPT0.2, L2-VPT0.2, and L2-AR1.5-VPT0.2) are given in Table 6.

The effects of post-tensioning on the lateral capacity and strength (global behaviour) of these specimens are also shown in Figure 8. The maximum drift has been selected based on the onset of predominant failure predicted by the model (i.e. crushing in the toe for all post-tensioned specimens). For all specimens, it can be seen that the post-tensioning causes the lateral strength to increase almost twofold in terms of applied base shear, while their ductility decreases about 25 to 30%. The increase is more than double for squat specimen (L2-AR0.44-VPT0.2). It can be said that despite the increase in lateral strength, failure occurs as crushing at the toes at the lower deformations and therefore the ductility is undermined in the post-tensioned walls.

The crack pattern under monotonically increasing lateral load at the last load step is shown in Table 7 for all specimens. When compared with Specimen L2-AR0.44, it is seen that the post-tensioning has caused the failure mechanism to shift from sliding to shear failure. For Specimens L2-AR1.5 and L2-AR1.5-VPT0.2, considerably fewer shear and flexural cracks were seen than were predicted for their reference walls, L2 and L2-AR1.5. Through the decrease in shear cracks, it can be said that the rocking mechanism (see Figure 1c) becomes a more likely failure mode in post-tensioned walls with large aspect ratio. Due to high axial loading, sliding movement decreases for all post-tensioned specimens.

In general, the study on the effect of increased axial load for walls subjected to lateral loads has shown this method can improve the behaviour of the masonry wall and preserve wall integrity. However, in order to augment ductility as well as lateral strength, the toes of post-tensioned specimens should be strengthened appropriately. To do this, the application of fibre reinforcement polymer (FRP) sheets has been suggested by other authors [16, 21].

6.3. Horizontal post-tensioning

Post-tensioning in the horizontal direction is thought to help with wall integrity under lateral loading [22]. In order to examine the effects of horizontal post-tensioning on the in-plane behaviour of the wall, a horizontal load of 128.7 KN, uniformly distributed along the sides of the wall, was applied to wall L2 (specimen L2-HPT0.2), without changing any other model parameters. The boundary conditions of specimen L2-HPT0.2 are given in Figure 9.

The effects of horizontal post-tensioning on the global behaviour are demonstrated in Figure 10. It can be seen that the initial stiffness of L2-HPT0.2 decreases relative to the reference specimen L2. This is due to the fact that the horizontal post tensioning causes vertical deformation (swelling) in the wall and therefore some initial cracks in the bed joints (see Table 7). Furthermore, according to Coulomb's friction law, the

initial swelling negates the effects of the normal compressive stress and therefore the yielding shear strength of L2-HPT0.2 decreases relative to L2. With increasing in the lateral loading, the initial cracks disappear and the flexural cracks and sliding in the bed joints remain as the predominant wall damage (Table 7). For this reason, at the last step of loading, both Specimens L2-HPT0.2 and L2 exhibit the same global behaviour. Furthermore, the shear cracks in the head joints do not occur for L2-HPT0.2 and the shear failure is therefore not as predominant a failure mechanism for this wall (see Table 7). Also, the model shows the maximum sliding movement is remarkably small compared to Specimen L2 and therefore is not significant for L2-HPT0.2. By inhibiting sliding as well as the shear failure mechanisms, it is found that horizontal post-tensioning can increase wall integrity.

The in-plane behaviour of specimen L2 subjected to combined vertical (axial) and horizontal post-tensioning was also investigated and is summarized briefly here. To do this, Specimen L2-VPT0.2 was assumed to be also post-tensioned in horizontal direction in two different cases: horizontal post-tensioning force is equal to 128.7 kN (Specimen L2-VPT0.2-HPT0.2) and 32 kN (Specimen L2-VPT0.2-HPT0.05). The same trend was observed for the global and local behaviour of these specimens. However, as Figure 10 shows due to smaller horizontal post-tensioning force, reduction in the initial stiffness and shear strength of L2-VPT0.2-HPT0.05 is not significant. Therefore it can be said that in order to avoid the above-mentioned deficiencies, the horizontal post-tensioning force should not be chosen very high. Application of larger axial post-tensioning may be used for this. It should be emphasized that the local and global behaviour predicted by the model for the above horizontal post-tensioned specimen are in good agreement with the results reported in [22].

Fig. 9. Loading and boundary conditions used for horizontal post-tensioned specimens

Fig. 10. Effects of horizontal post-tensioning on the global behaviour of Specimen L2 predicted by the FE model

6.4. Head joints

As was discussed above, the head joints in masonry walls play a crucial role in their local in-plane behaviour, in particular, in the case of shear failure. Based on results given in Gambarotta and Lagomarsino [6], the effects of the head joints on the global behaviour of masonry walls are negligible in most standard cases. To investigate this in more detail, specimen L2 was reanalyzed applying the following two conditions: assuming elastic behaviour for the head joints (L2-E-HJ in Table 6) and omitting the head joints (L2-W-HJ in Table 6). For the second case, the head joints were replaced by the brick material. Other specifications of the model were identical to specimen L2.

Fig. 11. Effect of the head joints on the global behaviour of Specimens L2 predicted by the FE model

Results obtained by the model are shown in Figure 11. Generally speaking, when compared to results of Specimen L2, the difference for both cases is negligible (less than 10%), but is larger for the case where the head joints are neglected (L2-W-HJ). As can be expected, before the onset of cracking in the head joints, the base shear force-deformation curve obtained by model of L2-E-HJ is identical to that of specimen L2. According to the results of this section, it may be concluded that the head joints do not have a remarkable influence on the global behaviour of the in-plane walls though

the results should be verified for the walls with different aspect ratios or thickness of the joints. That is, the head joints may be omitted or considered elastic without a noticeable reduction in accuracy, when only an overall idea of the wall behaviour is required, for example, for cyclic analyses or practical applications.

7. Conclusions

In this study, a micro model for in-plane behaviour of plain masonry walls has been introduced using the general purpose finite element code, ANSYS. The model accounts for such characteristics as cracking, crushing, and sliding, and incorporates the bed joints, head joints and brick units. The model was used to simulate the results from existing masonry wall tests. In addition, a parametric study was carried out investigating the effects of the aspect ratio, post-tensioning and the head joints. It was shown that the model was successful in reflecting the global in-plane behaviour of masonry walls. Although the crack pattern predicted by the model was not in exact agreement with the test results, its results were consistent with the expected nature of masonry walls subjected to in-plane loading and their known failure modes. Also a good agreement was found for the shear strength observed in the model and tests and predicted by the analytical relations. In a parametric study using the model, it was shown that post-tensioning, a type of retrofitting method, can improve the in-plane behaviour of masonry walls remarkably irrespective of their aspect ratios. However, wall corners should be strengthened in order to preserve wall ductility. It was also showed that horizontal post-tensioning decreases the initial stiffness and yielding shear strength of the masonry walls. Nevertheless, it does not have noticeable effect on the ultimate shear strength of the wall and increases the wall integrity by preventing it from the sliding and shear failure. Higher vertical (axial) post-tensioning may be used to avoid the above reduction in the initial stiffness and yielding shear strength of a horizontally post-tensioned masonry wall. Although the head joints play a crucial role in the local wall behaviour, the parametric study showed that they do not influence remarkably the global behaviour of the wall.

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Numeryczne modelowanie płaskiego zagadnienia pracy ścian URM oraz analiza ich nośności z uwzględnieniem wpływu pionowych i poziomych stężeń oraz kotew czołowych

Mur jest złożonym materiałem kompozytowym o nieliniowych właściwościach, które powodują to, że ich analiza numeryczna jest złożonym i trudnym zadaniem. W artykule przedstawiono nieliniowy mikro-model stworzony przy użyciu uniwersalnego kodu elementów skończonych służący symulacji zachowania się niewzmocnionego muru w jego płaszczyźnie. Zbadano zdolność modelu do przewidywania występowania takich zjawisk jak pękanie, kruszenie, poślizg, tak jak we wcześniejszych badaniach ortotropowego zachowania się murów. Następnie model użyty został do badań parametrycznych wpływu pionowego i poziomego sprężania oraz wpływu kotew czołowych na nośność murów.