



## UNCERTAINTY AND SENSITIVITY ANALYSIS FOR THE OPTIMAL DESIGN OF DISTRIBUTED URBAN ENERGY SYSTEMS

G. Mavromatidis<sup>1,2\*</sup>, K. Orehounig<sup>1,2</sup>, J. Carmeliet<sup>1,3</sup>

<sup>1</sup> Chair of Building Physics, Swiss Federal Institute of Technology Zurich, Switzerland

<sup>2</sup> Laboratory for Urban Energy Systems, Swiss Federal Laboratories for Materials Science and Technology, EMPA, Duebendorf, Switzerland

<sup>3</sup> Laboratory for Multiscale Studies in Building Physics, Swiss Federal Laboratories for Materials Science and Technology, EMPA, Duebendorf, Switzerland

\*Corresponding author; e-mail: mavromatidis@arch.ethz.ch

### Abstract

Deterministic model-based design of energy systems assumes perfect knowledge for all the model input parameters. Such a practice entails the risk of sub-optimal designs due to uncertainty that could cause model parameters to deviate from their original values. Thus, the first step towards designing a robust energy system should be understanding uncertainty's effects and its main drivers. Such investigations are facilitated by using uncertainty and sensitivity analysis. In this paper, an uncertainty quantification workflow is presented using the energy hub concept as the computational model. Initially, a characterization of uncertain input parameters is outlined pertaining to the buildings' energy demands, and the hub's technical and economic aspects. Subsequently, Uncertainty Analysis (UA) is performed by propagating the inputs' uncertainty through the model in a Monte Carlo fashion to study how the model's output is affected. Finally, Sensitivity Analysis (SA) is performed using the *Morris method* to screen out unimportant parameters and *Variance-based SA* to quantify the contribution of input parameters to the output's variance. The framework is illustrated with a case study, a residential urban neighbourhood for which an energy system is designed. The output of this paper allows us to test the model's robustness, understand the extent to which uncertainty actually matters by examining how much the model output varies compared to the deterministic case, and understand the influence and the interactions between the parameters of the model.

### Keywords:

Uncertainty analysis; Global sensitivity analysis; Energy hub; Monte Carlo; Urban energy systems

## 1 INTRODUCTION

### 1.1 Distributed energy systems (DES) and uncertainty

DES are expected to be a core component of future urban energy systems incorporating a multitude of generation and storage technologies that supply the energy needs of multiple buildings. The challenge to optimally design and operate DES relies heavily on modelling; however, as with any numerical modelling effort, models for the optimal DES design are irrevocably affected by uncertainty. Human behaviour and the uncertain future global economic and energy outlook are a

subset of factors that can introduce uncertainty to a DES model.

Nonetheless, designers usually assume perfect knowledge of all the model input parameters, reducing an inherently uncertain problem to a deterministic one. This practice entails the risk of suboptimal designs leading to excess costs and/or carbon emissions, but also to more adverse consequences like disruptions of service and the failure to satisfy building energy demands.

In the literature, the issue of uncertainty in DES has been widely recognized, resulting in many studies applying uncertainty analysis (UA) and

sensitivity analysis (SA) to investigate uncertainty's impacts on design and operational aspects of DES, e.g. [1-8].

A shortfall of these studies, though, is that, in some cases, the focus is placed only on single uncertain parameter categories, like energy prices (e.g. [8]) or energy demands (e.g. [1]). Also, while other studies focus on more parameters, they exclude others that could influence the design and operation of a system. For instance, Ren et al. [5] investigate several factors excluding, though, energy demand uncertainty from their study.

An additional shortfall of previously published studies is that they employ local SA techniques (e.g. [2-7]). In these methods, one-at-a-time (OAT) sampling varies one uncertain parameter, while all the other parameters are fixed. Even though, such an approach is easy to implement and understand, it only explores a limited part of the input space. In contrast, global SA techniques investigate the effect of a parameter while all the other parameters are varied as well over their entire range, allowing the interactions between parameters to be studied.

Therefore, the goal of this paper is twofold: first, to present a structured procedure that considers and characterizes the uncertainty of all parameter types involved in the model, and second, to illustrate the use of UA and Global SA for the investigation of the influence of model parameter uncertainties on the model's behaviour and the identification of the most influencing parameters.

## 2 MODELLING FRAMEWORK

### 2.1 Energy hub model

In this work, the energy hub concept [9] is applied at the urban level to design energy systems that meet buildings' thermal and electrical energy requirements. The model uses optimization techniques to select the installed components and their capacities, considering their optimal operating schedule, while optimizing for a desired objective. The minimisation criterion in this work is the equivalent annual cost (EAC) composed of the operating cost of the energy system plus the amortised investment cost of the units minus any revenue due to electricity exports.

The candidate technologies considered can be seen in Fig. 1 and include: a natural gas powered boiler and a CHP engine, an air-source heat pump (ASHP), photovoltaic (PV) panels, a sensible thermal storage tank and batteries. Grid imports and exports of surplus electricity are also enabled in the model. All technologies are assumed to be installed centrally, supplying energy to the buildings via small scale networks, except for PV panels that are distributed to the building roofs.

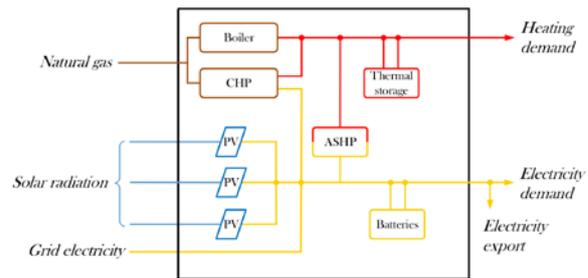


Fig. 1: Energy hub representation of the urban energy system.

The mathematical formulation of the energy hub model follows closely the ones used by preceding papers, such as [10-12], and it is a mixed-integer linear program (MILP). The MILP formulation is shown in pseudocode in Fig. 2.

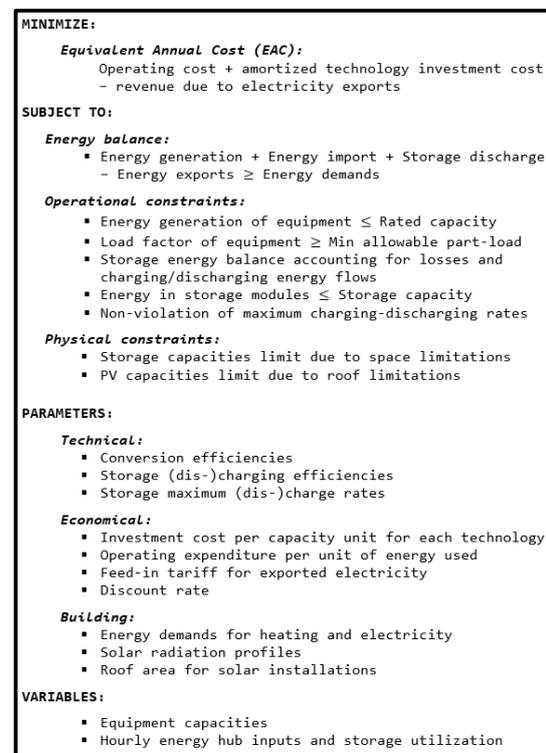


Fig. 2: MILP formulation for the energy hub.

### 2.2 Uncertainty characterisation (UC)

The first and most important step to investigate uncertainty is to characterise all its sources in the model and assign a mathematical representation to their uncertainty. In this work, sources from the literature are mainly used to perform this task.

In an energy hub model, uncertainty can affect economic and technical parameters as well as the solar radiation and energy demand profiles at the output of the hub. Regarding cost parameters, the uncertainty for the electricity and gas prices, as well as the feed-in tariff for exported electricity is assigned a uniform distribution allowing a  $\pm 15\%$  deviation from the nominal value [13], while for the investment costs, a similar distribution is selected with a range of  $\pm 30\%$  [13].

Deterministic design considers the technical characteristics of equipment constant throughout their lifetime. However, factors like poor commissioning, ineffective maintenance, and wear and tear can lead to performance deterioration, the extent of which is unknown. To represent, the probability of fainting performance, similarly to [14], a one-sided normal distribution is selected that enables random deviation of maximum -15% from the nominal characteristics.

However, a different approach is needed for the uncertainty of the energy demands as, on the one hand, they are not scalar parameters but time series, and on the other hand, they are calculated as the output of a Building Performance Simulation (BPS) model. In this case, *the inputs of a BPS model* are deemed as uncertain and their uncertainty is propagated through the BPS model to obtain variable demand profiles. A description of these uncertainties is as follows:

Variations in the building material properties, like density, conductivity and thermal capacity are assigned a normal distribution [15], the indoor temperature set-point of each building is varied following a normal distribution, while a triangular distribution has been selected for the occupant density ( $\text{m}^2/\text{person}$ ) and the lighting and appliances density ( $\text{W}/\text{m}^2$ ) [16]. Additionally, a normal distribution is attributed to the per person ventilation rate of each building, as well as to the infiltration rate [15].

Variation to the nominal yearly schedules for building occupancy, lighting, and appliances usage is also introduced in two ways. First, each value of the yearly schedules is randomly varied by  $\pm 15\%$  around its nominal value (“vertical” variability). Then, we proceed to define blocks of hourly periods for the 24 h of each day (e.g. [00:00–06:00], [07:00–09:00], etc.) and within these blocks shuffling the schedules values with each other (“horizontal” variability). This approach allows us to maintain the order of actions causing specific energy patterns (e.g. processes happening in the morning versus processes happening at noon) while introducing some randomness within the blocks [17].

Finally, weather uncertainty is treated by using Actual Meteorological Years (AMY) for the period 1994–2013 built with data from Meteoschweiz [18]. This approach also allows us to characterise uncertainty of incoming solar radiation by using directly the solar patterns from the weather files.

In order to treat energy demands as a random variable, first, a series of 1000 demand profiles are calculated per building and aggregated to form the neighbourhood’s total demand. These aggregated profiles are then sorted based on their total yearly demand meaning that demand profile “1” is the one that corresponds to the smallest aggregated demand, while profile “1000” to the largest. These profiles are then treated like a discrete uniform

distribution at the range [1,1000] from which random demand profiles can be sampled.

### 2.3 Uncertainty analysis (UA)

Uncertainty analysis is the study of how the uncertainty in the model’s inputs affects the variability of the model’s outputs. In this work, a Monte Carlo method is used that starts by generating a sample from each input parameter distribution, e.g. of size  $N$ , and then evaluates the model  $N$  times, producing an *input-output map*. The  $N$  output values can be used to study the uncertainty in the output, create histograms and calculate statistical moments.

### 2.4 Sensitivity analysis (SA)

Sensitivity analysis (SA) is a related technique to UA, with a definition for it by Saltelli et al. [19] as: “*The study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input*”. The SA technique selected in this work is based on the decomposition of the model’s output variance. The goal is to calculate two quantitative sensitivity measures of the contribution of each input to the unconditional variance of the output [20]. The first, named the *first-order Sobol index*  $S_i$ , indicates the portion of the output variance  $V(Y)$  that can be attributed to the  $i$ th input  $X_i$  and is defined as:

$$S_i = V_{X_i}(E_{X_{-i}}(Y|X_i)) / V(Y) \quad (1)$$

where  $X_{-i}$  denotes the matrix of all input factors but  $X_i$ . However,  $S_i$  does not take into account the interaction between parameters. Two (or more) parameters are said to be interacting when their joint effect on the output is different from the sum of their individual effects. Therefore, the *total-order Sobol index*  $S_{Ti}$  is defined that corresponds to the portion of  $V(Y)$  attributable to all the effects of  $x_i$  and is defined as:

$$S_{Ti} = E_{X_{-i}}(V_{X_i}(Y|X_{-i})) / V(Y) \quad (2)$$

One advantage of variance-based methods is that they do not require any assumption for the model form such as linearity or monotonicity. Their drawback, though, is their high computational cost requiring  $n(p+2)$ ,  $n \cong 500\text{--}1000$  model runs [21] for  $p$  uncertain parameters. Therefore, it would be computationally prohibitive to consider a variance-based method with the complete portfolio of uncertain parameters due to their number, as in this case  $p$  is equal to 17.

A common approach in this case is to use a screening method to identify non-influential parameters with a small number of model calls [22]. The most popular screening technique is the one introduced by Morris [23–24]. The method of Morris is based on a discretisation of the input space and performs a series of OAT experiments with the variation direction being random. The main output metric of the Morris method for each uncertain parameter are the mean  $\mu$  which is a measure of the overall influence of the factor on the output, and the standard deviation  $\sigma$  which

indicates if the factor is interacting with other factors or if it has nonlinear effects on the output.

The method's main drawback is that its result can only be interpreted qualitatively. Nevertheless, the Morris method output allows us to fix non-influential parameters to their nominal values, and proceed to calculate the Sobol indices and obtain quantitative results only for the influential ones. Further details on the mathematical aspects of the SA methods can be found in [21-24].

### 3 CASE STUDY

To illustrate the developed framework, a case study is selected consisting of three residential buildings of various construction periods, for which an energy system is to be designed. The buildings' construction characteristics follow the ones in [25]. Nominal values for occupancy, internal gains, heating set points and the schedules for occupancy and appliances/lighting usage are taken from SIA 2024 [26]. The building energy demands are calculated using *EnergyPlus* with a Typical Meteorological Year (TMY) for the climate of Zurich, Switzerland used for the deterministic design. An illustration of the three buildings is shown in Fig. 3.

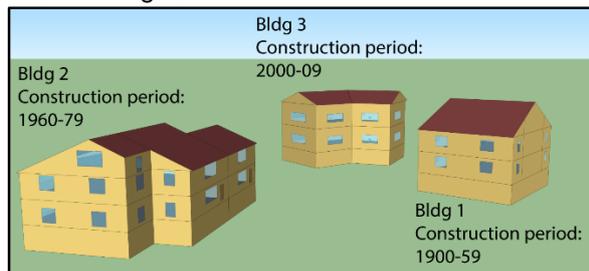


Fig. 3: Illustration of buildings composing the case study residential neighbourhood.

## 4 RESULTS

### 4.1 Deterministic model results

Before assessing the effects of uncertainty, the results of the deterministic version of the model are presented to be used as the comparison basis. The deterministic annual and peak building energy demands that are given as input to the model are presented in Table 1.

Bldg	Area (m <sup>2</sup> )	Annual demand (kWh/m <sup>2</sup> )		Peak demand (kW)	
		Heat	Elec.	Heat	Elec.
1	510	108	45	34	8
2	766	58	43	35	11
3	363	39	35	15	5
Sum	1639	69	42	79	24

Table 1: Annual total and peak energy demand for each building and for the whole neighbourhood.

The optimal energy hub design, in this case, consists of a heat pump, a CHP engine and a thermal storage module with their capacities shown in Table 2. The EAC in this case is equal to 26,280 CHF. The deterministic design excludes the installation of PV panels, batteries and boilers.

System components	Capacity
CHP	30 kW <sub>th</sub> /19 kW <sub>el</sub>
ASHP	22 kW
Thermal storage	88 kWh
Total cost	26,280 CHF

Table 2: Optimal energy hub design and EAC for the deterministic case.

### 4.2 Uncertainty analysis results

Similarly to the previous section, the first reported results are the variable energy demands of the buildings. As was mentioned in Section 2.2, variable demand profiles for each building are calculated by propagating the uncertain inputs of the BPS models through the model, with the results shown in Fig. 4

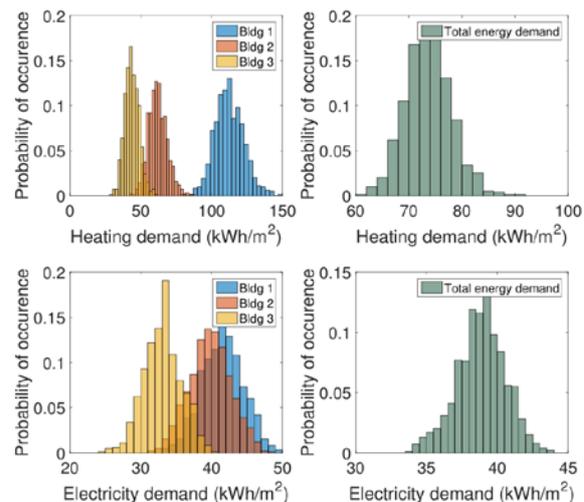


Fig. 4: Histograms of heating and electricity demand for the individual buildings and the whole neighbourhood.

Knowing the variation of the buildings' energy demands, the next step is to perform UA on the energy hub model by sampling from the set of uncertain parameter distributions. In total, 2'000 runs of the energy hub model are executed, and Fig. 5 shows the variation of the EAC in terms of its probability density function (PDF) and its cumulative distribution function (CDF). It can be seen that the energy system's optimal cost can vary from values that are lower than the deterministic EAC to values that are higher. In fact, there is approx. 60% probability that the cost will be exceeded as can be deduced from the CDF.

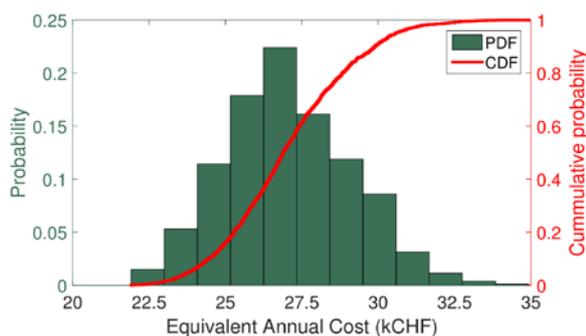


Fig. 5: Probability density function (PDF) and Cumulative distribution function (CDF) for the EAC of the energy hub under uncertainty.

Apart from the annualised cost, the variation of the optimal energy hub design for each of the 2,000 Monte Carlo runs is shown in Fig. 6. Each run is depicted as a semi-transparent line enabling a better visualisation of the areas where there is higher density of optimal designs.

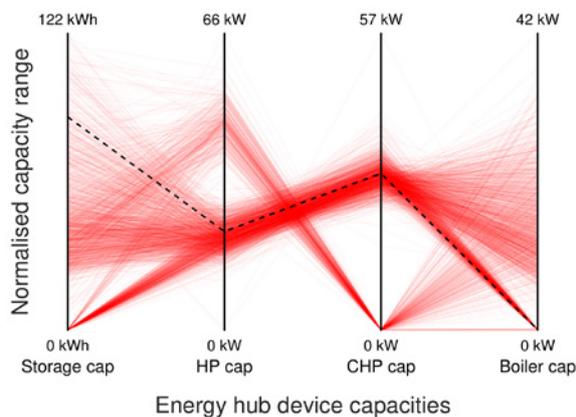


Fig. 6: Parallel coordinates plot showing the resulting optimal system capacities of 2000 runs of the energy hub model. The dashed line corresponds to the deterministic design.

PV and electrical storage are not included as in none of the 2'000 runs were they selected (due to the lack of any emission performance criterion), thus, the design was limited to a selection of four components. Starting from the thermal storage there is a number of designs that do not include it, while in the cases that do include it, the optimal capacity seems to vary considerably. On the other hand, it seems that for both the ASHP and the CHP engine there are two regions where the optimal capacities are clustered. Additionally, it is seen that when a high capacity for the HP is selected the design excludes the CHP engine completely, while when the low HP capacity is opted for, the CHP engine is present at a higher capacity. Finally, with regards to the boiler, a similar situation as with the thermal storage is observed with some designs omitting it completely, while when present its capacity is not clustered around a specific region. The implications of this analysis are twofold. On the

one hand, the optimal design of the energy system would be different depending on the actual realisation of the uncertain parameters; hence, the fact that uncertainty has an impact is indeed confirmed. In addition, for every Monte Carlo run that the optimal design is different than the deterministic one, it means that the latter design is suboptimal (otherwise the algorithm would have selected the deterministic design as the optimal one). This fact could mean that the deterministic design will lead to higher costs due to its suboptimality and that there is a probability that some portion of the energy demand will be unmet due to a shortage of essential generation capacity.

### 4.3 Sensitivity analysis results

The previous section revealed the effects of uncertainty on both the system's cost and its design aspects. In this section, we try to connect the output uncertainty to the inputs and determine which are the most responsible for this variability. First, a qualitative assessment is performed using the Morris method, with results shown in Fig. 7.

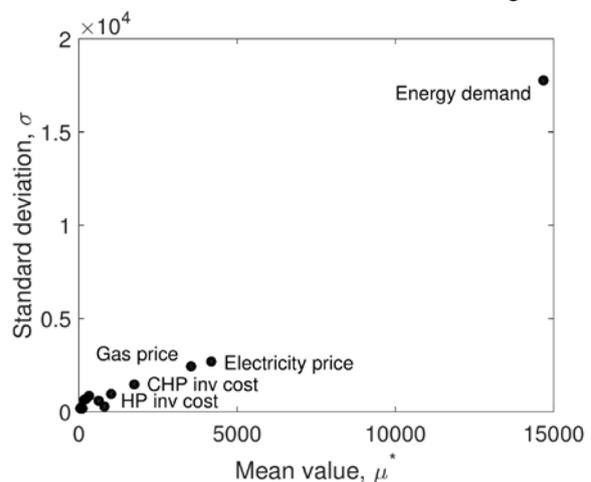


Fig. 7: Outputs from the Morris method showing  $\mu$  and  $\sigma$  for each uncertain parameter. The 5 most influencing parameters have been labelled.

The most important parameter is the energy demands of the buildings, followed by the operating cost parameters, for both gas and electricity. Finally, as the main dominating technologies that are installed in the energy hub, the investment cost for the HP and the CHP engine are also deemed important. The results would also imply that all the other parameters of the model, such as the remaining investment cost parameters and all the technical characteristics can be considered less important and, thus, can be fixed to their nominal values.

As the results of the Morris analysis can only be interpreted qualitatively, quantitative information can be obtained by calculating the Sobol indices for the 5 most important parameters. Having a set of 5 important parameters from the preceding section, the required number of computations is equal to  $1000 \times (5+2) = 7,000$  simulations, as

defined in Section 2.4. The results are shown in Fig. 8.

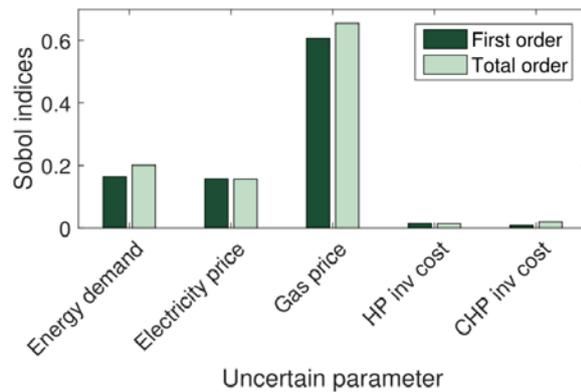


Fig. 8: First- and total-order Sobol indices for the most influencing parameters of the energy hub model.

The order of importance among the parameters, as evidenced by both Sobol indices includes the gas price as the most important parameter, followed by the energy demands and the electricity price in close proximity. On the other hand, investment cost parameters seem to be the least influencing, which is to be expected as over the lifetime of the project, operating expenditure tends to occupy a much higher portion of the total cost compared to the investment expenditure. Finally, the small differences between  $S_i$  and  $S_{Ti}$  show that the model is mainly dominated by first-order effects. More specifically, the sum of the first-order indices indicates that 95% of the output's variance is attributed to first-order effects, with only the remaining 5% being due to higher order effects.

As is obvious, the results of a SA are case specific and in this case the large influence of the gas price can be expected due to the dominance of gas powered technologies in the optimal energy system configurations. The results of such a study, though, are very useful in the planning phase of a system because they allow the designer to identify the most influencing parameters whose uncertainty if reduced will lead to the greatest reduction of the cost's variability.

## 5 CONCLUSIONS

In this paper, a multi-step workflow has been presented to investigate uncertainty in the context of urban DES design and its application is illustrated with the design of an energy system for a residential neighbourhood.

Starting from a deterministic energy hub model, a detailed characterisation of the uncertainty sources in the model has been performed. Performing UA in the planning phase of an energy system by propagating the input parameter uncertainty through the model has revealed the influence of uncertainty on the optimal system

cost, as well as the system's structure and size. Finally, further investigation using SA has narrowed down the most important input parameters, showing that the variation of the system's cost is predominantly due to the uncertainty in the gas prices.

In terms of future work, the first step will be to extend the framework to include additional objectives, such as the minimisation of carbon emissions and/or primary energy in order to investigate the variation with regards to these domains. Additionally, the paper's outcome will be used as the basis for studies that use stochastic programming seeking to obtain a *single system design* that will operate optimally under all realisations of uncertainty by showcasing the most important parameters to be used for the scenario generation.

## 6 ACKNOWLEDGEMENTS

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