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Optimal dispatch of large multi-carrier energy networks considering energy conversion functions

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Abstract

An integrated coordination of multi-carrier energy networks including gas, heating, cooling and electricity can increase the flexibility, efficiency and sustainability of energy systems. The optimal dispatch of such systems is complicated by the non-convex nature of their energy conversion processes. Although these processes can be represented in mixed-integer linear programmes, real-time constraints of an online dispatcher may not be satisfied. In this paper, two approaches for alleviating this problem are developed and compared: one is based on a relaxed mixed-integer linear formulation and the other on mathematical optimization with complementarity constraints. Simulation results on realistic systems demonstrate that both approaches solve large multi-carrier dispatch problems efficiently. The mathematical optimization with complementarity constraints is computationally less intensive but the relaxed mixed-integer linear formulation is numerically more robust.

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Keywords: Energy conversion; energy hub; multi-carrier energy systems; MILP; MPCC;

1. Introduction

The need for energy efficiency measures, carbon emission reductions and the growth of distributed energy systems have sparked research in the operational optimization of energy systems. In particular the dispatch (unit commitment) of multi-carrier energy systems or energy hubs has gained attention as many degrees of freedoms can be exploited to increase efficiency and to access new types of energy storage [1]. In this paper, the focus is on the combined dispatch of electrical distribution grids and decentralized district heat networks.

Energy conversion processes are crucial to this endeavour. In most recent work on multi-carrier energy networks, the energy conversion has been regarded as a constant efficiency. With the adoption of small distributed modulating

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heat-pumps and combined heat and power plants, the specific properties of energy conversion processes cannot be ignored. Generally, the models of these processes involve non-convex constraints and are therefore impossible to address using efficient convex optimisation methods such as linear programming.

The non-convex characteristics of the heating, ventilation and cooling equipment have been included in model predictive control schemes with a focus on single building control via relaxation methods [2] and sequential quadratic programming [3]. The works of [4] and [5] represent these processes with a piecewise affine representation and use mixed-integer linear programming (MILP) to find a solution. This approach works adequately only if the systems to be optimised are not very large.

In the case of control of multi-carrier networks with a multitude of components, the computational load of the underlying dispatching problem can exceed the real-time requirements and faster methods are needed. Decreasing the computational load of mixed-integer linear programming is a part of ongoing research. By showing that the energy conversion function is mostly concave, a simplified MILP model is presented that only requires a binary variable per time step. Furthermore, [6] has proposed a novel optimisation technique based on inverse parametric optimization (IPO) and mathematical programming with complementarity constraints (MPCC). Any piecewise affine function can be reformulated in the MPCC framework as shown in [7]. This property can be used to solve the energy dispatch as a MPCC by formulating the energy conversion processes as inverse parametric optimization problems. The two methods are compared for a realistic full-scale multi-carrier energy network.

2. Multi-carrier energy network models

2.1. Multi-carrier nodes

A multi-carrier network, such as the one depicted in Fig. 3, consists of a set of nodes $\mathcal{N}$ of any energy carrier (gas, electricity, heat, cooling, hydrogen), a set of network links, a set of grid feeders G, a set of energy conversion systems P that connect the multi-carrier nodes, a set of loads L and a set of storage systems S. Over a horizon $T \in \mathbb{Z}^+$, demand and supply are balanced at every node $i \in \mathcal{N}$ and for every time-step $k = \{1, 2, ..., T\}$ in the multi-carrier network:

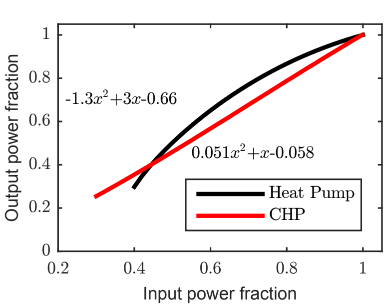
$$\sum_{p_{out} \in P_i} p_{out,k} - \sum_{p_{in} \in P_i} p_{in,k} + \sum_{s_{out} \in S_i} s_{out,k} - \sum_{s_{in} \in S_i} s_{in,k} + g_{i,k} - \sum_{l \in L_i} l_k = 0 \tag{1}$$

where $p_{in,k}$, $p_{out,k} \in \mathbb{R}$ are the input and output streams of a conversion system linked to node i, $s_{in,k}$, $s_{out,k} \in \mathbb{R}$ are the storage streams linked to node i, $g_{i,k} \in \mathbb{R}$ is a grid link and $l_k \in \mathbb{R}$ are loads at node i. P_i, S_i and L_i denote the sets of decision variables of the conversion devices, storages and loads connected to node i.

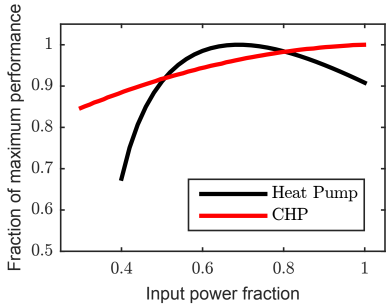
2.2. Energy conversion systems

Combined heat and power plants (CHP) and heat pumps (HP) are useful components of decentralised energy systems. The outputs of these energy systems can be modulated and controlled according to demand. Energy conversion systems are often subject to minimum load constraints and part-load efficiency variations. For simplicity, we consider only single input/output energy streams; generalisation to multiple input or output streams is possible. In this context, an energy conversion unit can be thought of as a function $f : \mathfrak{R} \to \mathfrak{R}$ mapping the input to the output energy stream. The efficiency (for CHPs) or coefficient of performance (for HPs) is then defined by $\epsilon = f(p_{in})/p_{in}$.

The minimum and maximum outputs of the energy system limit the conversion to a certain range. In the case of CHPs, the function in this range is often convex due to the increasing efficiency [4]. The electric power to heating power conversion of heat pumps is often concave [8], as shown in Fig. 1(a). The CHP curve shown is based on [9]. The heat pump coefficient of performance was calculated using the method from [10]. A polynomial fit of the conversion function reveals the degree of convexity. The second order coefficient of the polynomial fit indicates whether the function is convex (negative coefficient) or concave (positive coefficient) in the operating range of the system. It is important to note that a convex energy conversion function or a negative intercept of the polynomial fit can both lead to an efficiency that increases with the load factor (Fig. 1(b)). Sections 3.1 and 3.2 explain how the energy conversion process is integrated into the optimization framework.



(a) Energy conversion functions



(b) Efficiency and coefficient of performance

Fig. 1: Performance of energy conversion systems

2.3. Storage

For every storage system connected to a multi-carrier node, such as hot water tanks or ground source probes, and time step $k = \{1, ..., T\}$, a dynamic equation is formulated to represent the state of charge:

$$e_{k+1} = \alpha e_k + \beta_{in} s_{in,k} - \beta_{out} s_{out,k}, \quad 0 \leq e_k \leq \overline{e}, \quad 0 \leq s_{in,k} \leq \overline{s}, \quad 0 \leq s_{out,k} \leq \overline{s} \tag{2}$$

where $e_k \in \mathbb{R}$ is the state of charge, $s_{in,k}, s_{out,k} \in \mathbb{R}$ the charge and discharge rate, $\overline{e}$ is the storage capacity limit, $\overline{s}$ the discharge/charge rate limit and $\alpha, \beta \in \mathbb{R}$ are loss coefficients.

3. Optimization methods

3.1. Mixed-integer linear programming

Piecewise affine segments can approximate any conversion function, but require one binary variable per segment. This approximation is henceforth called the standard formulation. The resulting mixed-integer linear programme can be computationally intensive. In this section, a binary reduction is described to reduce the computational complexity of energy conversion constraints. The minimum load constraint cannot be modelled without a binary variable per time step. This single binary variable can be used to approximate the energy conversion in the case of concave functions. In the case of a convex function, the function can be approximated by a single PWA segment. The reduced number of binaries makes the search space of mixed-integer linear programme smaller. The problem becomes computationally less intensive.

Using a single binary and n segments, the approximated energy conversion function is formulated as follows:

$$p_{out} = f(p_{in}) = \sum_{i=1}^{n} a_i\, p_i + b\, d, \quad 0 \leq p_{out} \leq p_{max}\, d, \quad p_{in} = \sum_{i=1}^{n} p_i, \quad 0 \leq p_i \leq c_i\, d, \quad a_i \geq a_{i+1} \quad \forall i = \{1, ..., n\} \tag{3}$$

where $d \in \{0, 1\}$ is the binary on/off state variable, $p \in \mathbb{R}^n$ is the input stream vector, $a^T \in \mathbb{R}^n, b \in \mathbb{R}$ are parameters. The last condition makes the on-state operating range concave. The efficiency increase stems from the fact that the constant intercept b becomes less dominant in relation to $\sum_{i=1}^{n} a_i\, p_i$.

If the global objective of the energy system is cost reduction, formulation (3) is equivalent to the standard formulation but requires one binary per energy conversion constraint set and time step. Note that in this case, the terms $a_i\, p_i$ are selected by the optimizer in descending order of a_i.

3.2. Inverse parametric optimization

In this section, an inverse parametric optimization approach to the problem of multi-carrier energy dispatch is presented. It is based on [6] outlining a novel method for the optimal control of hybrid systems. The problem is stated as a mathematical programme with complementarity constraints (MPCC). An interior-point solver that is able to find a local solution can be applied to this MPCC. The interior-point solver potentially finds a local solution satisfying the real-time requirement of the application. The energy conversion functions are decomposed into two convex functions. Two parametric quadratic programmes can be found whose solutions are the decomposing convex functions. These parametric quadratic programmes reformulated as the Karush-Kuhn-Tucker (KKT) conditions are included as constraints in the dispatch problem. Based on [7], the energy conversion functions are represented as continuous PWA functions that are be reformulated as the difference of two convex PWA functions : $f(p_{in}) = \psi(p_{in}) - \phi(p_{in})$. The decomposed convex functions are defined on n_ψ and n_ϕ segments:

$$\psi_i(p_{in}) = a_{y,i}\ p_{in} + b_{y,i} \quad \forall i = 1,..,n_\psi, \quad \phi_i(p_{in}) = a_{z,i}\ p_{in} + b_{z,i} \quad \forall i = 1,..,n_\phi \tag{4}$$

where $a_y, b_y \in \mathbb{R}^{n_\psi}$ and $a_z, b_z \in \mathbb{R}^{n_\phi}$. In Fig. 2, the energy conversion function is approximated by a PWA function and decomposed into a convex and a concave PWA function. Their sum results in the PWA approximation of the energy conversion function. A convex parametric quadratic programme (PQP) can be found for a convex PWA function. As the energy conversion is composed of two PWA functions, two PQPs must be stated. (5) states the PWA functions used to express the energy conversion as an inverse optimization problem. $\bar{\psi}(p_{in}), \bar{\phi}(p_{in}) : \mathbb{R} \rightarrow \mathbb{R}$ are auxiliary functions of the form $\bar{\psi}(p_{in}) = a_\psi\ p_{in} + b_\psi$ and $\bar{\phi}(p_{in}) = a_\phi\ p_{in} + b_\phi$ that must satisfy $\bar{\psi}(p_{in}) < \psi(p_{in})$ and $\bar{\phi}(p_{in}) < \phi(p_{in}) \quad \forall p_{in}$. The auxiliary function is crucial because it ensures that the solution of the parametric optimization problem is the desired PWA function. The parametric optimization problem is equivalent to a projection of the auxiliary functions onto the decomposing functions [6].

$$\begin{aligned} & g = \tilde{y} - \tilde{z} \\ & \tilde{y} \in \arg\min_{y \in \mathbb{R}} \frac{1}{2} \|y - \bar{\psi}(p_{in})\|^2 \\ \text{s.t.} \quad & y \geq a_{y,j}\ p_{in} + b_{y,j} \quad \forall j \in \{1, ..., n_\psi\} \\ & \tilde{z} \in \arg\min_{z \in \mathbb{R}} \frac{1}{2} \|z - \bar{\phi}(p_{in})\|^2 \\ \text{s.t.} \quad & z \geq a_{z,k}\ p_{in} + b_{z,k} \quad \forall k \in \{1, ..., n_\phi\} \end{aligned} \tag{5}$$

$$\begin{aligned} & g = y - z \\ & 0 = y - (a_\psi\ p_{in} + b_\psi) - \sum_{i=1}^{n_\psi} \lambda_i \\ & 0 = z - (a_\phi\ p_{in} + b_\phi) - \sum_{i=1}^{n_\phi} \theta_i \\ & 0 \leq y - (a_{y,i}\ p_{in} + b_{y,i}) \perp \lambda_i \geq 0 \quad \forall i \in \{1, ..., n_\psi\} \\ & 0 \leq z - (a_{z,j}\ p_{in} + b_{z,j}) \perp \theta_j \geq 0 \quad \forall j \in \{1, ..., n_\phi\} \end{aligned} \tag{6}$$

In order to include the two inverse parametric optimization problems in a higher-level optimization scheme, their KKT conditions are stated in (6). λ and θ are the Lagrange multipliers of the PQPs.

If the global objective is a cost minimisation that always results in the selection of the optimum efficiency by the optimizer, the PQP of the concave segments can be replaced by inequality constraints, thereby reducing the complexity, i.e. the number of complementarity constraints and Lagrange multipliers. In the case of energy conversion functions, the convex function can very often be constructed by two segments, so that only a limited number of complementarity constraints must be introduced to the optimization problem.

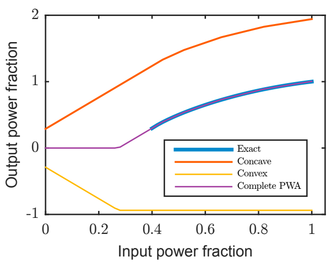


Fig. 2: Decomposition of energy conversion curves

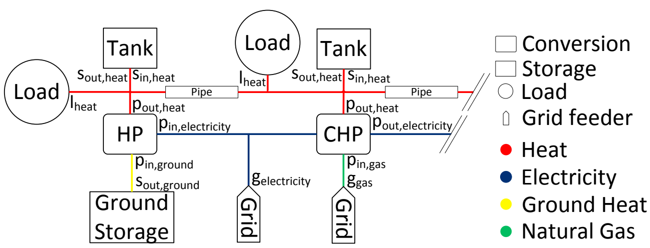


Fig. 3: Multi-carrier energy network

3.3. Optimization problem

The operational cost, the quantity to be minimised, is the summed costs of all the energy taken from all grid feeders N_G. Given costs information $c_{i,k}$ and grid connections G, the dispatch problem over the horizon T is stated as:

$$\min_{p,s,g,e} \sum_{k=1}^{T} \sum_{i=1}^{N_G} c_{i,k} g_{i,k} \quad \text{subj. to constraints sets (1), (2), ((3) or (6))} \tag{7}$$

4. Case study

The case of a 24-hour dispatch (7) of a realistic scale multi-carrier energy network is considered. The multi-carrier energy network consists of single branch network with heat pumps, thermal storage, CHPs and building heat loads distributed along the branch (Fig. 3). Five cases with an increasing number of conversion systems and loads are studied that cannot be solved in real-time using the standard formulation. The number of systems is fixed so that part-load conditions are maintained. Every heat pump is supplied by a ground borehole heat storage. Thermal storages are placed at the same nodes as the heat pumps. A thermal grid loss of 5% is assumed. A heat load at a certain node can be met by different components from different locations of the thermal grid. The residential heat load data is based on [11]. The heat pump base COP curve is taken from [10]. The CHP base efficiency curve is taken from [9]. It is assumed that the performance curves deviate according to a normal distribution from the base curves.

The multi-carrier energy system dispatch is implemented in YALMIP [12] and MATLAB$^{\text{TM}}$. The mixed-inter linear programmes are solved with CPLEX$^{\text{TM}}$. The inverse parametric optimization approach is tested with IPOPT [13]. The case studies are run on an Intel Xeon$^{\text{TM}}$ 2.5 GHz CPU with 12 cores that can be exploited by CPLEX$^{\text{TM}}$.

The simplified mixed-integer linear programme is compared to the inverse parametric optimization method. Both approaches are warm-started with the solution from the previous time step, assuming a receding horizon application. The IPO of the previous time step is warm-started with the MILP solution. The relaxed mixed-integer linear formulation uses equation set (3). Equation set (6) is used in the IPO approach. The initial solution of the multi-carrier network case used for the warm-start is terminated with a MIP-gap of 2%. Two MILP cases are shown: MILP(a) terminates when a relative optimality gap of 2% is achieved; MILP(b) returns the best solution obtained in the same time span as the IPO method. The results are summarized in Table 1. The sub-optimality is calculated with reference

Table 1: Computation time and suboptimality

Case	Loads	CHP/HP	MILP(a)	MILP(b)	IPO
Case 1	10	2/2	6.08s	202.9%	0.05%/0.65s
Case 2	20	3/3	19.3s	1.70%	0.31%/3.00s
Case 3	30	4/4	34.5s	5.00%	0.55%/8.39s
Case 4	40	4/10	26.36s	1.40%	1.51%/5.06s
Case 5	50	5/10	19s	303.86%	1.6%/4.57s

to MILP(a) and indicated for MILP(b) and IPO. The computational effort is reduced with the IPO approach with small loss of optimality. In the largest case, the computation time is reduced by a factor of 4, with a 1.6% optimality loss. Due to the high number of nodes, the largest case is visualised in terms of the operating points of CHPs and heat pumps. The box plots of Fig. 4 show the efficiency distributions of the MILP and IPO solutions. The operating points found by the IPO are in the same range as the MILP, as expected from the small difference in overall cost. Fig. 5 shows the difference in the heating power dispatch. MILP(a) selects a heat pump to provide power to distant loads, incurring grid losses. The IPO approach activates CHPs closer to the loads but with higher costs. In the fourth case, the MILP finds a better solution than the IPO after the same amount of time. Nevertheless, it is problematic to limit the CPU-time in order to meet time constraints because very suboptimal results, as in the first and last case, might be obtained. The major drawback of the IPO application is its robustness. The IPO can converge to a locally infeasible point, depending on the initial solution provided. To sum up, the inverse optimisation formulation (with a warm-start) can be much more efficient for larger systems, with only small sub-optimality cost.

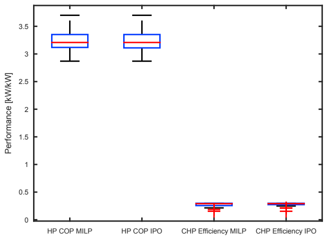


Fig. 4: Comparison of operation efficiency

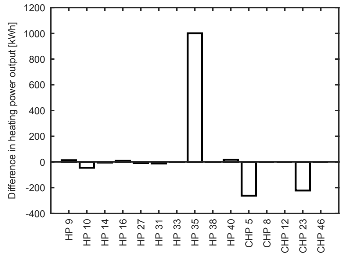


Fig. 5: Difference of heating output between MILP and IPO methods by device and node

5. CONCLUSIONS

Both approaches presented in this paper, the relaxed mixed-integer linear formulation and the inverse parametric optimization method, can be used to solve dispatch problems that consider the part-load behaviour of energy conversion systems in real-time. The inverse parametric optimization is more efficient but numerically less robust. The relaxed mixed-integer linear formulation is computationally more intensive but does not need a good warm-start point to find almost global solutions.

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