Monitoring of friction-related failures using diffusion maps of acoustic time series

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Abstract

Friction-related failures are a frequent cause of catastrophic damage in mechanical systems, resulting in significant repair expenses and/or production delays in industry worldwide. Scuffing is one of the most concerning friction-related failures mechanisms. It is characterized by an abrupt failure mechanism, which makes its prediction unresolved today. Therefore, any in situ and real-time monitoring system of such failures is of great demand and this contribution is supplement to and enrichment of existing studies on this topic. We propose a probabilistic analysis of acoustic signals synchronously recorded with the coefficient of friction during real-life experiments. They, simulate a specific industrial operating condition of a cam, cylinder pairs and gear teeth. The experimental conditions are a reciprocating movement of a stainless steel cylinder (counter-body) against a grey cast iron body. The counter-body was under a constant load and had a line contact with the body. The acoustic signals from one-directional strokes are selected and substituted by their wavelet spectrograms. Then, diffusion maps are computed over several spectrograms. Their corresponding Fourier spectra are used as features of the surface state and so of the upcoming failures. The real surface state and damage were identified from the dynamical behavior of the coefficient of friction. Based on our approach, it was found that the friction behavior can be divided into four confidence intervals, three for the steady-state regime and one for the scuffing regime. The transients between the confidence intervals allow estimating the proximity of scuffing and this is the major achievement of this work. The first transient took place between 5 min 40 s. – 47 min 30 s. prior to scuffing. The second one took place between 50 s. and 13 min 40 s. The proposed in situ and real-time monitoring system has significant industrial potential since it is built as a client – server architecture and requires minimum modifications of existing commercial industrial machines.

Index Terms — in situ and real-time monitoring, scuffing monitoring, friction monitoring, acoustic emission, acoustic time series, diffusion maps.

1) Introduction

In the field of tribology, friction-related failure is a negative phenomenon that significantly affects key industries, including transportation, mining, energy production, etc. The financial cost is estimated at over half-billion or 0.056% of the gross national product (GNP) only in the UK during the year 2016. The cost includes repair, re-manufacturing of the failed parts and production delays [1-3]. One of the most concerning failure mechanisms in tribology for mechanical engineers is known as scuffing. It is a major threat, which occurs in lubricated sliding surfaces, working under extreme conditions of load and/or speed. These conditions are...
typical for semi-journal bearings, cylinder pairs, piston ring, and gear teeth [4-6]. Scuffing occurs suddenly and is usually accompanied by an unexpected increase in the coefficient of friction (COF) and, in some cases, contact temperature [7]. Due to its nature, scuffing is the least understood failure mechanism making it very challenging to predict [7]. Obviously, any in situ and real-time monitoring technique able to predict scuffing prior its occurrence is of a high industrial demand [8,9]. To address this issue, we propose an innovative probabilistic analysis of acoustic signals based on diffusion maps. If the proposed approach is successful, it will be easily transferred to other friction-related failure systems. The major benefit is that it would provide engineers and technicians some time to take decisions and actions to avoid failures and accidents with promising significant economic savings in a broader number of applications [2,7,10].

Most existing techniques in friction monitoring are only capable to detect severe failures [4-6,10]. In these studies, one of the most promising technique is acoustic emission (AE). AE is usually elastic waves that are generated from a micro-event within a solid state material (e.g. cyclic fatigue cracking, corrosion), by the interactions of solid–solid media (e.g. impact, friction or wear), or solid–fluid media (e.g. leakage or cavitation) [8]. Actually, the use of AE monitoring in tribology has been intensively studied in the last decade [8-21] and today it is one of the main monitoring technique [9,11]. The reasons are twofold. First, AE is known for its high sensitivity and good capability in failures detection [11,12]. Second, AE are produced from real physical state change of the material [9,11]. In tribology, AE is generated by the interactions of the surface non-uniformities, known as asperities [8,9,11]. These interactions cause many effects such as elastic and plastic deformations, abrasion, cracks, etc. This results in a continuous stochastic surface modification, causing a statistical diversity in the AE contents [9,11,20]. Consequently, establishing a robust correlation between the AE signal and their corresponding sources (events) is highly demanding. In other words, the challenge is to identify the origin of the events within the complex AE modulations. To achieve this objective, state-of-the-art signal processing of the AE is crucial. It gives the possibility not only to track the changes in the AE signals, but, potentially, also allows detecting the origin of the surface damage [4,9,11,20]. Furthermore, the low costs, commercial availability of hard/software, and a high spatially-temporal resolution make AE essential in many friction-related applications [9,11,13,14]. In the literature, several studies showed an efficient usage of AE for specific tribo-conditions [8-21]. The correspondence between AE and specific friction behavior was well investigated for several tribo-systems [9,11,13,18,20]. In particular, surface failures were characterized by the occurrence of higher frequency content in the AE Fourier spectra. This phenomenon was efficiently exploited for practical failure detection and wear monitoring by Hase et al. [15] and Rubtsov et al. [19]. The extension of the AE analysis to the time-frequency domain by Taura and Nakayama [16] and Wei [21] allowed detecting more specific friction events by using simultaneously the time-frequency and frequency domains. Towsyfyan et al. [14] unified the root mean square (RMS) with the time-frequency domain which tremendously improved the robustness in failures detection.

Unfortunately, at present, the detection of failure has still some practical limits. The main one is that when failure is detected, significant damage of the mechanical parts already occurred. It is too late to take any preventive measures. Hence, the development of an in situ and real-time monitoring system for early failure detection providing sufficient time for decisions and actions prior a catastrophic failure is of utmost importance for industries. Despite this fact, few works were found in the literature. The reason is that early failure predictions require higher data organization and involve different data driven models. The dynamic models by de Queiroz et al. [22] and Specker et al. [23] are based on methods from automatic control theory.
The models are described by a system of differential equations. The models are fed with the AE signals and provide an expected response to each particular friction regime, thus indicating the "near failure" surface state. de Queiroz et al. [22] proposed a non-linear model with an asymptotic analysis that potentially may provide the short-term failure predictions. In contrast, Specker et al. [23] described a linear dynamic model that follows more simple friction dynamics. The dynamic models may operate with complex friction systems, characterized by alternating loads, non-constant strokes, etc. However, these models have numerous disadvantages. They involve complex mathematics, require a deep prior knowledge of the specific tribological system, are sensitive to noises and require a precise tuning. The latter is very difficult if not impossible to achieve in real-life conditions.

The advantages of statistical models for in situ and real-time monitoring are in their robustness to noises. Additionally, they can be relatively easy constructed from datasets that are straightforward to collect and do not require prior knowledge about the tribological system. Therefore, they are more suitable for real-life applications. Several successful statistical models have been reported in the literature. In Hao et al. [17], the statistical complexity of AE is analyzed within a fractal model. Potentially, this method allows detecting not only failures, but also other friction regimes and so provides more detailed information. The early signs of failures were recognized by Towsyfyan et al. [14], where the regression model differentiated friction regimes preceding failure. The combination of the time-frequency features with a statistical model based on support vector machines (SVM) was proposed by Saeidi et al. [8]. The algorithm successfully classified three friction regimes (steady-state, pre-scuffing and scuffing). An extension of this work was reported by Shevchik et al. [20], where higher-order statistics over the time-frequency domain was combined with machine learning regression. This work did not only improve the classification accuracy of Saeidi et al. [8], but also predicted scuffing several minutes prior its occurrence.

This study was motivated by the success of statistical approaches in preceding works [8,20], and aimed to improve the prediction of scuffing. The choice of the scuffing is threefold. It is the least understood failure mechanism, it is challenging to predict, and finally, it is one of the biggest concern for engineers [4-7]. Consequently, any method able to predict scuffing can be undoubtedly tailored to any other tribological systems and friction regimes.

The main innovation in this work is the representation of friction as a diffusion of the AE narrow bands energies, tracked with diffusion maps (DM) [24,25]. DM are a recent extension of the non-linear graph mappings [26] that are suitable for statistical representations [27], dimension reduction, and compartments of time series [25]. In this study, all listed advantages were fully exploited.

The manuscript includes five sections. Section 2 presents information about the experimental tribo-setup and gives an overview on the data acquisition of AE signals and signal processing architecture. Section 3 includes a description of the sparse signals representations with wavelet spectrograms, a description of the DM, the construction of DM Fourier spectra, and the DM computations for continuous data flow. Section 4 discusses the results of processing of the experimental data. Finally, the conclusion and discussion of the future work are presented in Section 5.
2) Experimental Setup and data acquisition

2.1 Experimental setup, tribo-test conditions and materials

The experimental setup in this study was analogous to the one described in Saeidi et al. [8] and Shevchik et al. [20]. The major difference lies in the fact that, in this work, we had a line contact between the moving parts instead of flat-on-flat. This setup reproduced the operating and failure conditions in heavily loaded lubricated sliding surfaces. It is typically the case of cam and its follower, cylinder pairs, and gear teeth. In total, twenty experiments were carried out until scuffing took place. The first ten experiments were performed to establish the optimal algorithmic and machining parameters and so those intermediate results were not included in the analysis. All experiments were conducted using a reciprocating tribometer (SRV III, Optimol Instruments Prueftechnik GmbH, Germany) equipped with a control unit and an atmospheric chamber in which the acoustic emission setup was installed. A general view of the setup is shown in Fig. 1, A. The experimental conditions inside the chamber were kept at a constant temperature and relative humidity (35°C, 30% RH). The tribometer had a moving head (Fig. 1, A, 1) on which a counter-body was mounted (Fig. 1, A, 2) using a specific fixing holder. The moving head has two degrees of freedom along x and y-axes to guarantee a constant contact between the counter-body and body during the entire experiment duration. The moving head provided a periodic reciprocating movement (stroke) of the counter-body relatively to the static body. The counter-body was oscillating at a frequency of 6 Hz (12 strokes/second) with a stroke length of 4 mm. The effective sliding duration for each stroke was 54 ms and the time difference is due to the holding time for direction changes and overcoming the static friction. The counter-bodies (Fig. 1, A, 2) were cylinders of 11 mm in diameters and a length of 15 mm. They were made of 42CrMo6 hardened steel with a hardness value of 600 HV. The bodies (Fig. 1, A, 3) were cylinders of 25 mm in diameters and a thickness of 8 mm. They were made of cast iron (EN-GJL300) with a hardness value of 200 HV. The contact was of type cylinder-on-flat-plane inducing a line contact; where the cylinder was the counter-body and the flat plane was the flat surface of the body. The counter-body was pushed against the body with a constant load of 600 N. To simulate starving conditions and provoke scuffing in a reasonable time, before each experiment, the surface of the body was homogeneously covered with a density of 0.4 μl/cm² of poly-alpha-olefin oil (PAO, viscosity 8 cSt) using a high precision spraying machine (AutoJets, Model2250, USA). To guarantee an identical amount of oil for all the tests, the sprayed oil quantity was always verified by an accurate weighing with a tolerance of ±0.1 mg.

2.2 acoustic emission acquisition system

During the experiments, the AE signals were recorded using a piezo sensor Pico (Acoustic Instruments, USA) that was tightly placed on the base of the body, as shown in Fig. 1, A, 4. The recording of the AE signals for each individual stroke was triggered when the signals amplitude exceeded a threshold value. Based on previous experience, the threshold value was fixed at 29 dB. The recording sampling rate was 1 MHz and a typical AE signal is shown in Fig. 1, C. This figure confirms that the effective duration of each stroke was 54 ms. Hence, we decided to recorded the AE signals during 65 ms after it exceeded the threshold value to reference the noise levels. The changes in the AE amplitude at the beginning and at the end of each stroke were caused by the acceleration and deceleration of the moving head. The sliding velocity in the middle of each stroke was constant at 20 mm/s. The stroke parameters were set as a machine function and were kept the same throughout all experiments.
The processing of the AE signals was distributed between two different computers that were unified together within a single client-server architecture. The local computer (client) was equipped with a data acquisition card (DAQ) Advantech 1840 (Advantech, USA) that operated with a sampling rate of 1 MHz and was connected directly to the acoustic sensor. The same PC provided a pre-processing of the AE signal via wavelets decomposition of the signal (See Section 3.1). Next, the sonograms with wavelet bands were transferred to a remote PC (server) that computed the diffusion maps (DM). Finally, based on the DM, predictions of the current friction state were made, and the computational results were sent back to the local computer for a visual representation and decision making. A home-made customized code in C# was used for the client computer and provided the low-level data acquisition and wavelet spectrograms computations. Additionally, a Python code was developed for the server. Finally, the DM computations were an adaption of a python library Pydiffmap, release 0.1.0 [28].

2.3 Failure and failure characterization

The twenty experiments, with the aforementioned experimental conditions, provided the stochastic nature of failures due to scuffing. The time-to-failure ranged from 1,635 to 20,905 seconds (27 min. 15 s. to 5 hrs 48 min. 25 s.) resulting in the collection of more than 150,000 AE signals. The friction and failures dynamics can be observed from the behavior of the COF. It is important to note that no temperature rise of the body could be measure prior to failure [7,29]. Typical examples of COF behaviors for three experiments are shown in Fig. 1, D. Based on this figure, it is clearly visible that failures are characterized by a rapid increase of the COF until it reached a cut-off value of 0.5 to avoid tribometer damage [7]. These momentary values COF were used as ground truth characterization of the friction dynamics and were referenced to during the AE analyses. In addition, we would like to underline that the stochastic nature of friction, visible in Fig. 1, D, can be considered as a large variance of the time-to-failures for experiments operating under identical experimental conditions. More details on the COF are given in Section 3.

The surface damage after failure was additionally verified using an optical microscope. Figure 1, B is a typical example of a failed surface; where the direction of the reciprocating movement is indicated by the yellow arrow. In this figure, two zones defined as “damage” and “no damage” can be distinguished. Two zones with no damage are delimited by the green arrows on the left and right side of the body. In contrast, the damaged area, delimited by the red arrow, was characterized by material loss. The material loss is, actually, a material transfer from the surface of the body to the counter-body, which resulted in a significant increase of the roughness. Analysis of the failed surface confirmed that the mechanism behind the material transfer was scuffing. The origin and mechanisms leading to scuffing for the same experimental conditions and tribo-pairs have been investigated in previous works and more details can be found in Saeidi et al. [7,8,29,30]. At the same time, the non-damaged contact area preserved its initial surface roughness.
3) Signal processing

3.1) Wavelet spectrograms.

The collected AE signals were substituted by their corresponding wavelet spectrograms. The spectrograms were formed as the relative energies of the narrow frequency bands; extracted with wavelet packet transform (WPT) [31]. The efficiency of wavelets application in friction was demonstrated in previous studies [8,16,20]. Their advantages are in the original assignment of wavelets to operate with non-stationary signals that suits the origins of friction-related AE signals [8,9,20]. The tiling of the time-frequency domain using wavelets provides more information as compared to a standard Fourier analysis, which operates only in the frequency domain [8,20]. Additionally, for friction related real-life applications, the time-frequency domain and position of the moving head provide damage localizations as discussed in Shevchik et al. [20]. Simultaneously, wavelets are an efficient instrument for noise reduction [31], which is essential when “listening” real-life mechanical systems in motion. The application of wavelets results in the extraction of low and high frequency contents from the given discrete signal $x(n)$ [30]:

$$x_{vp}(n) = \sum_k h_v(k) \ast x(Mn - k)$$  \hspace{1cm} (1)

where $h_v$ are the filters of the wavelet channels and $v=1,..,M$ is a channel index of a wavelet basis, $k$ are the index of the wavelet filters coefficients, $x_{vp}$ is the low- narrow- and high-frequency band at $v=1$, $1<v<M$, and $v=M$, respectively [30]. In traditional wavelet decomposition, the multi-resolution is carried out with a recurrent split of only the high frequency content $x_{vM}$ according to Eq. (1). In WPT, this operation is applied to extract all

Fig. 1: A) Zoom on the experimental setup in the atmospheric chamber; where (1) is the moving head, (2) the counter-body, (3) the contact body, (4) the piezo-electric acoustic sensor; B) optical microscope image of a surface of the body after failure. The black background is the non-affected surface with an initial roughness of 0.8 μm. The green and red arrows delimit the “no damage” and “damage” areas, respectively, in the contact area; C) a typical raw AE signals from a single stroke recorded during 65 ms; D) typical COF vs time curves from three experiments ran until failure.
\( x_u \) components, where \( s \) in Eq. (1) indicates the order of recursion and is known as a decomposition level. The output of such operation is a pyramidal structure that incorporates the frequencies of different bandwidth (narrow ones at lower decomposition levels and broader at higher ones). The pyramidal structure of WPT for an AE signal from a single stroke is shown in Fig. 2, A. The decomposition level in the figure corresponds to \( s \) in Eq. (1). More details about this technique are described by Mallat [30], while its specific application in friction is explained in Saeidi et al. [8] and Shevchik et al. [20]. The ordered frequency bands in the time domain form the wavelet spectrogram. In our contribution, the relative energies of the frequency bands were analyzed and computed as: \( E_{rel,sn} = E_{sn}/E_{total} \), where \( E_{sn} = |x_{sn}(n)|^2 \) is the energy of the individual frequency band, \( x_{sn}() \) is the result obtained from Eq. (1), \( s \) is a decomposition level, \( n \) is a number of the band at level \( s \), \( E_{total} \) is the energy of all frequency bands \( x_{sn}(n) \) computed at all possible decomposition levels. WPT was carried out using a standard two channel \((M=2 \text{ in Eq. (1)}) \) Daubechies wavelet with 10 vanishing moments [30]. This choice was made after an exhaustive search in standard wavelet groups, including Symlets and Coiflets, and analyzing their respective performance. The selection criteria were the smallest approximation error for the given AE signals with the aforementioned wavelet bases [20].

### 3.2 Diffusion maps

In this study, the friction process was considered as a diffusion of the AE narrow bands energies, described by diffusion maps (DM) [24,25]. Diffusion maps are an embedding of the initial dataset into a lower-dimensional Euclidian space but preserving its original structure [24]. In this framework, the data structure is captured by investigating the local neighborhood of each data point \( u_i \) within a given dataset \( u \). The information about the neighborhood can be computed for different scales involving time dependent diffusion process. Technically, the possibilities are provided by random walks over the graphs, computed for given datasets [24]. In the graph \( G\{u,k(\cdot,\cdot)\} \), each pair of data point \( u_i, u_j, i \neq j \) within the dataset is linked to the parameter \( k(u_i,u_j) \), which is a weight for the neighborhood proximity for the specified pair \( u_i \) and \( u_j \). The function \( k(\cdot,\cdot) \) may be non-linear and is known as a kernel and provides the lower weights values as the distance between neibors increases [24]. According to the original work of Coifman and Lafon [24], DM are first built by introducing the density according to:

\[
m(u_i) = \sum_j k(u_i,u_j)
\]

where \( j = 1,\ldots,N \) and \( N \) is the size of the dataset. Then, the probability \( p(u_i,u_j) \) characterizes a transition from \( u_i \) to \( u_j \) and is computed by normalizing the weights by the density, specified in Eq. (2):

\[
p(u_i,u_j) = \frac{k(u_i,u_j)}{m(u_i)}
\]

The probabilities for all pairs of data points in Eq. (3) may be unified within a row normalized diffusion matrix \( P \). In random walks, the transition probabilities in \( P \) characterize the shortest paths between any pair of data points, while the data structure is characterized by the paths with higher probabilities [24]. The data structure at other scales can be estimated by raising \( P \) to higher powers \( P^t \) [24]. In this case, the entries of \( P^t \) are \( p^t(u_i,u_j) \), which are conditional probabilities that sum all possible paths of length \( t \) between the pair of observations \( U_i \) and \( U_j \) [24]. By doing so, the DM are a collection of mappings of the observations \( U_i \) [24,25]:

\[
Y_i = [\lambda^T_1 \varphi_1(i), \lambda^T_2 \varphi_2(i), \ldots, \lambda^T_N \varphi_N(i)]^T
\]
where $\lambda_i^t$ and $\varphi_i$ are the eigenvalues and eigenvectors of the matrix $P^t$, respectively, while $\varphi_i(t)$ is $i^{th}$ element of a particular eigenvector. We have to mention that the eigenvectors of $P^t$ are equal for all $t$, thus forming the basis of the diffusion space. The structure of the data at fixed $t$ can be characterized by using the distance measured between the mapped observations. Their characteristics are $L^2$ which is defined as the distance between a pair $Y_i$ and $Y_j$ in a mapping, which can be approximated by a diffusion distance \cite{24,25} and can expressed through eigenvectors and eigenvalues of $P$:

$$\begin{align*}
D^2(U_i, U_j) &= \|Y_i - Y_j\|^2 = \sum_m (p^t(U_i, U_m) - p^t(U_j, U_m))^2 / \psi_1(m) = \sum_i \lambda_i^{2t} (\varphi_i(i) - \varphi_i(j))^2
\end{align*}$$

In Eq. (5), $\psi_1$ is the first left eigenvector of $P$, while $\psi_1(m)$ is its corresponding element, $m=1,...,N$. The same distance measure in diffusion and observation spaces in Eq. (5) indicates the preservation of initial data structure in the DM for any $t$.

As already mentioned, the basis of the diffusion space is defined by the eigenvectors $\varphi_i$, while their importance are specified with $\lambda_i^t$ \cite{24,25}. The acceptable accuracy may be achieved by using only $\varphi_i$ that corresponds to larger $\lambda_i^t$. In the present study, all $\varphi_i$ were used to preserve the process details that may affect the results. The time-dependent diffusion process in the framework can be understood as a gradual increase of $t$ that recovers the structure of the data at different scales \cite{24,25}. The lower $t$ values correspond to shorter paths with higher probabilities due to the dense neighboring of the data points. Any further increase of $t$ (e.g. increase of the path length) selects high probabilities paths following the original data structure. In this way, the data can be observed from the lowest scale of the local neighborhood (at low $t$) up to the global one (larger $t$) \cite{24}.

The structure of the data at a specific $t$ value can be described by a mutual distance measure, described in Eq. (5). In the present work, we employed a more general description to limit the dimensionality of the extracted features. The chosen parameter was the temporal information about the data dynamics through all $t$. This is captured by the DM Fourier spectra that can be expressed with the eigenvectors \cite{24,25,32} as: $\varphi_i(\omega) = \sum_k \varphi_i(k) e^{-i\omega t}$, where $l = \sqrt{-1}$ and $k=1,...,N$ is the dataset size, and $\varphi_i$ are the eigenvectors by analogy with the previous notation. The DM spectra $\varphi_i(\omega)$ can be used to measure the differences between two DM \cite{24,25}:

$$D_{XY} = \sum_i \lambda_i \|\varphi_x^i(\cdot) - \varphi_y^i(\cdot)\|^2 / \|\varphi_x^i(\cdot)\|^2$$

where $\varphi_x^i$ and $\varphi_y^i$ are the DM spectra obtained from different datasets. In this study, the signals flow was split into separate sequences of datasets, each included several wavelet spectrograms as described in Section 3.4. We computed the Fourier spectra for each dataset; hereafter referred to as DM spectra, and they were used as features to characterize the friction dynamics. The pairs of the neighbored datasets can be compared using Eq. (6).

In Equation (6), the eigenvectors of the DM remain the same independently from $t$. At the same time, the eigenvalues $\lambda_i^t$ of $P^t$ are the $t^{th}$ power of the eigenvalues $\lambda_i$ at $t=1$. Taking this into account, the power operator can be directly applied to the eigenvalues, thus saving computational time.

It is also important to note the importance of the choice of the kernel, which may tremendously affect the final results. Our choice of the heat kernel function was stipulated by its efficiency and popularity in practical tasks as demonstrated by Haghverdi et al. \cite{33}. The heat kernel function was defined as $k(u_i, u_j) = \exp(||u_i - u_j||/\sigma)$, where $\sigma$ is the neighborhood search parameter. It must be emphasized that the DM are
sensitive to the tuning of \( a \). In practice, \( a \) is adapted according to the prior knowledge of the dataset configuration. However, we decided to tune this parameter automatically using an algorithm from Berry et al. [34]. In this approach, additional optimizations of the kernel are possible and could potentially improve its performance; but these optimizations were left as future work.

### 3.3) Estimates of parameters for probability distributions

We described the friction dynamics as a drift of the DM spectra between the different distributions. The details and examples on real experimental data are presented in Section 4, while this section describes briefly the distribution estimates using a basic realization of Metropolis-Hastings (M-H) algorithm [35]. M-H is a special realization of Markov chain Monte-Carlo which, in the context of the present study, can be used in two ways with equal efficiencies. First, M-H may be used to estimate the distribution of parameters \( \theta \) over a given set of observations \( o_i \in \Omega \). Second, M-H allows selecting the observations \( o_i \) that fulfil the given parameters distributions \( \theta \) (e. g. subsampling problem), where \( o_i \) are the DM spectra. The former allows also finding the distribution parameter of the DM spectra groups that correspond to different friction regimes. The latter allows grouping the DM spectra according to their distributions. The application of both approaches to the experimental data is discussed in Section 4.3. One additional advantage of the M-H algorithm related to our study is its efficiency while operating with high dimensional data. Nevertheless, for such processing, alternative algorithms can be efficiently used as well, in particular the one developed by Forbes et al. [36].

As mentioned, M-H is a special realization of a Markov chain Monte-Carlo that estimates the parameters distributions \( \theta \) analyzing some given observations. This algorithm requires a proposal distribution \( Q(\theta | \theta^*) \). It allows obtaining the necessary observations \( o_i \) from unknown posterior \( P(\theta^i = \theta | o) \) which affects the convergence of the algorithm. Having \( Q \) as an input, the estimates of \( \theta \) distributions are carried out with random walks over the distribution space (the part of Markov chain). Simultaneously, the jumps to update \( \theta^* \) are accepted or denied based on the fit of the observations \( o_i \). The acceptance decision is made by a specialized likelihood, based on Bayes formula [35]:

\[
P_{\text{accept/deny}} = \frac{\prod_{i=1}^{Z} f(\theta^i = \theta^*|o_i) P(\theta^*)}{\prod_{i=1}^{Z} f(\theta^i = \theta|o_i) P(\theta)}
\]

where \( f \) is the probability density which is proportional to the posterior and \( Z \) is the number of fitted observations. Using Eq. (7), the updates of \( \theta^* \) are accepted at higher \( P_{\text{accept/deny}} \) - e. g. if the conditions \( \prod_{i=1}^{Z} f(\theta^i = \theta^*|o_i) P(\theta^*) > \prod_{i=1}^{Z} f(\theta^i = \theta|o_i) P(\theta) \) are met. This implies that the new parameters \( \theta^* \) provide a better fit of the samples \( o_i \). More details related to this technique can be found in the original work of Forbes et al. [36].

After testing a wide range of distribution, our initial conditions for the H-M algorithm were the normal distributions for the probability density according to:

\[
f(\theta|o_i) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\left(\frac{(o_i-\mu)^2}{2\sigma^2}\right)}.
\]

where the mean was directly computed from the given samples \( o_i \) (e. g. the DM spectra), thus reducing the parameter space only to the unknown variance \( \theta = (\sigma^2) \). The proposal distribution was also chosen as a normal: \( Q(\theta) = Q(\mu^{\text{current}}, \sigma) \) with a fixed dispersion of the parameter space \( \sigma = 0.8 \). In this case, the observations \( o_i \) were the DM spectra of the individual frequency bands, computed according to the description in Section 3.4.
Processing of acoustic emission data flow in the experimental setup

The computation of the AE data flow in our setup is schematically presented in Fig. 2, B-C. Figure 2, B, shows the AE raw signals from the individual reciprocating (forth-back) movements. Figure 2, C is the corresponding wavelet spectrograms, computed according to Section 3.1. In order to follow the evolution of the surface state by the COF, only the AE signals from one-directional strokes were selected and their corresponding wavelet spectrograms were computed as shown in Fig. 2, B-C. The decision to select only one-directional strokes was to put in correspondence each frequency band with a specific surface contact area. This is very important since the stroke velocity is not constant, as described in Section 1. In other words, it takes into account the acceleration/deceleration at the beginning/end of each stroke and the constant speed in the middle the stroke. This allows linking each frequency band with the same surface area. In this setup, the changes in the frequency band energy with time corresponded to the modifications of a specific surface area. The practical application of this approach is discussed in Shevchik et al. [20]. From Figure 2, A, it is evident that the decomposition levels 1-3 contain a lot of noise such as from the environment. Hence, taking into account all decomposition levels can be detrimental to the results. To overcome this problem, a reduction of noise in wavelet spectrograms was performed by selecting the bands $x_{s,v}$ (see Eq. (1)) from decomposition levels 4-12. This is indicated by the red frame in Fig. 2, A. It is important to note that this range is specific for the present setup.

To track the diffusion process, several wavelet spectrograms were accumulated, thus splitting the signal flow into separate datasets as shown in Fig. 2, C. In this figure, the size of each dataset was $N \times M$, where $N$ specified the number of accumulated spectrograms and $M$ corresponded to the number of frequency bands $x_{s,v}$ in each spectrogram. The optimal size of the datasets was established empirically during exhaustive experimental trials. The optimal size of the datasets was found to be: $N = 90$ and $M = 510$, with $N = 90$ corresponds to 15 s. (frequency of 6 single strokes per second). After this time, a new accumulation of spectrograms takes place and the DM computations are repeated.

The DM were computed according to Eq. (4) for each individual frequency band inside the accumulated spectrograms tracking its diffusion with time. The computations input was $\{x_{s,v}\}$, where $x_{s,v}$ are the individual relative energies of the frequency bands from Eq. (1), and $n=1,..,N$ is the number of accumulated spectrograms. Then, the DM spectra were computed using Eqs (6) and (7) for each band $\{x_{s,v}\}$, resulting in the computations of $M$ values $P_{freq.band}$ hereinafter referred to as feature vectors. These feature vectors were referenced to the COF and the ones from the different friction regimes as well as failure were estimated. More details are given in the next section.
Fig. 2 The signal processing scheme: A) A typical example of a wavelet spectrogram build from an AE signal recorded from a single stroke. The red frame defines the selected range of decomposition levels, while the different colors define the values of the relative energy within a specified frequency band; B) the original sequence of AE signals with forth-back strokes; C) the corresponding wavelet spectrograms but only for the forth strokes. \( N \) defines the number of spectrograms included into separate dataset and the arrows indicate the single input for the diffusion maps computations.

4) Results and discussions

4.1) Friction dynamics in the time domain

Figure 3, A shows a typical COF curve of the friction behavior of a hardened steel counter-body reciprocating against a cast iron body. During the first minutes of sliding, a sharp increase of COF is observed, followed by a slower decrease. This behavior is defined as running-in, which is a common friction behavior for boundary lubricated metals [37]. Running-in is characterised by a redistribution of the lubricant on the contacted area during the mutual polishing of the contact surfaces [5,7,8,29]. The running-in is known for its complex behavior [5,7,8,29] and therefore was not considered in this work.

After the running-in, a stable friction behavior is observed and is known as steady-state. The transition from running-in to steady-state can be clearly seen in Figure 3, A, and is indicated by a line at 2,500 s. The steady-state is characterized by a maximum topological match between the two contact surfaces, leading to a low COF values with smooth changes over time. The steady-state continued until 20,550 s. In industry, this friction condition is wanted and corresponds to normal operation condition of lubricated systems [4,7]. Finally, a sharp increase of COF took place, as evident from the zoom in Fig. 3, A, and the experiments were automatically stopped when the COF reached a value of 0.5. This maximal COF value was decided based on previous experiences to avoid equipment damage. After, the surface of each body was observed using an optical microscope to be certain that significant damage and material loss took place. This is typical for the scuffing failure mechanism and an example of such damaged surface is observable in Fig. 1, B. For the experiment in Figure 3, A, scuffing starts at 20,550 s. and ends at 20,909 s. It is important to mention that no visible signs of the up-coming scuffing were present in COF characteristics during the steady-state. The complete scuffing process took six minutes which was relatively short as compared to the entire experiment duration.
4.2) Friction dynamics in the frequency domain

In this contribution, the friction dynamics was observed as smooth drifts of the DM spectra in the frequency domain and an example can be seen Fig. 3, B. In this figure, the collected DM spectra come from the experiment presented in Fig. 3, A, where the greater frequency numbers (x-axis) correspond to the DM spectra of narrower wavelet bands $x_{wm}$ (See Section 3.4 for details). The color gradient encodes the time-stamp of the acquisition of the individual DM spectrum. The known acquisition time stamps of the DM spectra permitted to associate them with the corresponding COF values and to estimate the proximity of each DM spectrum to scuffing.

The structure in the DM spectra dynamics in Fig. 3, B had the following characteristics. The start of steady-state was characterized by the presence of the higher frequency content (see dark red DM spectra, indicated with the arrow 2 in Fig. 3, B). Then, a gradual and smooth attenuation of the higher frequency content was observed from arrow 2 to arrow 3. As the experiments reached scuffing, the DM spectra were concentrated mainly in a region in the middle of the DM spectra and it is indicated by the arrow 4 in Fig. 3, B. This friction dynamics took place for all experiments. Although the dynamics of the DM spectra was gradually smooth, some sporadic short time fluctuations of individual DM spectra were observed. A typical example is indicated by the arrow 1. This single spectrum came from the steady-state and was characterized by the presence of low frequencies, as compared to the other spectra from the same time interval (see arrow 2 in Fig. 3, B). We believe they are artefact spectra coming from unknown short life surface phenomena taking place at the surface.

The gradual tracking of the friction dynamics from steady-state to scuffing (see Fig. 3, A) in the frequency domain was carried out by discretizing it into several confidence intervals. In this setup, the smooth changes of the real-time DM spectrum could be tracked according to its position within the specified confidence intervals and thus indicate its proximity to scuffing. The frequency space was divided into several confidence intervals as shown in Fig. 3, C. We have to mention that, in theory, a greater number of the predefined confidence intervals correspond to a more gradual tracking of the changes in the friction dynamics. Nevertheless, we have to taking into account that the larger the number confidence intervals, the larger the negative impact on the computational results. In contrast, a lower number of confidence intervals gave less information about the gradual changes of the friction process but provided better computational results. The choice of the optimum number of confidence intervals was experimentally determined through an exhaustive search and the optimal number of those was established as four. This optimal number may vary for other tribological systems. The final division of the frequency domain into the confidence intervals are represented in Fig. 3, C by the color code (green, light green, yellow and red).

The delimitation of the different confidence intervals in Fig. 3, C confirms the DM spectra dynamics in the frequency domains observed in Fig. 3, B. During the steady-state, the DM spectra starts with a high frequency content followed by a gradual attenuation of those until it raises back during scuffing. This is consistent with the literature [15,19] as well as with our previous results where the AE RMS has been investigated during running-in, steady-state, pre-scuffing and scuffing [30]. In this study, Saeidi showed that the AE RMS increases during running-in, then decreases during the steady-state and rise again during pre-scuffing. Based on our previous study [7], this behavior corresponds to the nature of the boundary lubrication conditions followed by starvation and finally the catastrophic failure/scuffing. Under the boundary lubrication condition, an oxide layer (so-called tribo-film) is formed to avoid metal-metal contact. However, if no more fresh lubricant is provided on the contact surface, the contact gradually reaches starvation. Under starvation
condition, numerous breakdown of the tribo-film takes place that results in micro-junction and micro-scuffing. This can explain the gradual changes seen in DM spectra. As starvation last longer, the tribo-film becomes more continuous and thicker and can act as an insulator, thus, the temperature of the contact surface may gradually increases. At a certain point, as explained in more details in our previous study [7], the oxide layer reduces to $\alpha$-Fe and results in metal-metal contact and, consequently, provokes strong adhesion and failure.

The acquisition time for the DM spectra from each specific confidence interval in Fig. 3, C is shown in Fig. 3, A, as a color map over the time axis (x-axis). From this mapping, it can be observed that the position of the DM spectra in the green confidence interval between 2,500 s-18,000 s. Next, the DM spectra moves to the light green confidence interval until 19,950 s. Then, the DM spectra are defined as the yellow confidence interval which lasted until 20,550 s. Based on the COF curves, these three confidence interval (green, light green, and yellow) can be attributed to the steady state regime. Finally, the red confidence interval was assigned to scuffing and so it included all DM spectra that were collected during scuffing between 20,550 - 20,909 s. Obviously, the propagation in real-time of the DM spectrum through these confidence intervals gives gradual information about the friction dynamics (and proximity to scuffing). Delimitating the frequency domain in Fig. 3, C was done once using the data from the experiment with the longest time-to-failure.

Fig. 3 The dynamics of the COF and the corresponding diffusion map spectra: A) A typical COF curve, where the blue frame defines a zoom which is shown in the box on its right. The color map along the time axis (x-axis) corresponds to the confidence intervals defined in C; B) diffusion maps spectra for the entire experiment duration in A. The numbered arrows indicate: (1) an artefact spectra coming from a sporadic short fluctuation during the steady-state friction regime, (2) start of the steady-state friction regime, (3) beginning of the yellow confidential interval, (4) start of the red confidential interval (scuffing); C) Delimitation of the frequency domain in the DM spectra space into discrete friction confidential intervals, where the color encodes the time intervals and corresponds to the color map in A.
Each confidence interval was described by a density probability from Eq. (8) with their individual variances and means. Both parameters were computed using the M-H algorithm (see Section 3.3). The position of the DM spectrum in a specific confidence interval was characterized by higher weights given by the probability density assigned to this interval.

We have to mention, that the choice of the probability density describing the confidence intervals was not obvious. The tests with normal, non-symmetric Gamma and Laplacian probability density functions were carried out, while the normal distribution showed the best results. The selection criterion was the “noise” in the final computations which is defined in Section 4.3. The smooth and fast decay in the symmetric Gamma and Laplacian distributions blurred the borders between the different confidence intervals, making impossible to track precisely the transient of the DM spectra from one confidence interval to another. In our study, the normal distribution allowed better parameters tuning that resulted in higher precision in determining the position of the DM spectrum and tracking of its transient between the different intervals.

4.3) Application of the algorithm to real tribological experiments

In this section, we exploited the division of the frequency domain into the confidence intervals, described in Section 4.2, and applied them for ten experiments. In these experiments, the friction dynamics was tracked as the propagation of the DM spectrum through the predefined confidence intervals in time. The results of two extreme test cases are presented in Fig. 4. They are characterized by a relatively long (≈ 8,095 s. ≈ 2h 13 min) and short (≈ 1,750 s. ≈ 29 min) time-to-failure. The curves of COF for both cases are in Fig. 4, A and C. The corresponding DM spectra in the frequency domain are presented in Fig. 4, B and D, where the borders of the predefined confidence intervals obtained from Fig. 3, C are delimited by their respective colors.

The results of the propagation of the DM spectrum through the predefined confidence intervals with time is shown in the color maps above the time axis (x-axis) in Fig. 4, A and C. The dynamics observed of the DM spectra in the frequency domain was similar to the one in Fig. 3. The start of the steady-state was characterized by the presence of the DM spectrum in the green confidence interval, followed by a transient to light green, yellow and finally red (scuffing).

A careful investigation of the color maps in Fig. 4, A and C shows that the propagation of the DM spectrum through the confidential intervals was not perfectly smooth. Short time stochastic fluctuations in the frequency domain were observed, before returning to the DM spectrum to the original confidence interval. A typical example is visible in Fig. 4, C at 305 s, where a short fluctuation into the light green confidence interval took place from the green one. Such fluctuations in the color maps in Fig. 4, A and C, can be seen as non-uniformities, similar to impulse noise (also characterized by short time fluctuations). The nature of these fluctuations is unknown but has three potential origins. First, these fluctuations may be due to unknown short life surface phenomena taking place at the surface. As possible hypothesis could be in the occurrence of a micro-scuffing with an immediate formation of a tribo-film. Alternatively, they may be real occurrences of short life of the defined confidence intervals. Finally, they may come from possible computational artefacts. An investigation of the physical phenomena behind this behavior was outside the scope of this work, which was mainly concentrated on the statistical analysis. To understand the physical nature of these fluctuations, additional studies are required, where real-time computations using the developed algorithm will allow terminating the experiments and performing a complete material analysis. It is important to note that, at
present, no tribometers exist that allow carrying out such experiments which are required to investigate in real-time, the process dynamic of tribological systems.

The aforementioned fluctuations of the DM spectrum raised uncertainties in estimating the proximity of scuffing. Therefore, despite the fact that the nature of the fluctuation is not known, we considered them as noise and are hereafter referred to as “noise”. The median filtering of the computed color maps smoothed this “noise” so that the DM spectra dynamics from the different confidence interval are not mixed. The results of such filtering are indicated in Fig. 4, A and C with the colored arrows (green, light green, yellow and red) along the time axis (x-axis) on the top of the colored maps.

We also observed that the experiments with the shorter time-to-failure showed higher “noise” levels than the longer ones and the evidence of this is in Fig. 4, A and C. Another observation from the experiments in Fig. 4 was related to the time of the DM spectra in the different confidence intervals. For example, in Fig. 4, A and C, the time of the DM spectra in the yellow confidence interval was 820 s. and 75 s., respectively. The yellow confidence interval is the most interesting, as it just precedes the scuffing regime. It is in correspondence with the earlier works of Saeidi et al. [8] and Shevchik et al. [20] in which it was defined as pre-scuffing. In these works, the pre-scuffing regime was introduced without any physical basis. In the opposite, in the present study, the yellow confidence interval was determined experimentally as the positions of the DM spectra that are close to scuffing. Actually, knowing the proximity of yellow confidence interval to scuffing is crucial information for real industrial tribological systems as it allows a technician and/or engineer to stop the machine/process preventing its possible damage. At present, the physical background of this yellow confidence interval is still unknown and will be the focus of future work.

Finally, in all our experiments, the propagation of the DM spectrum through the confidence intervals followed the same order starting from green, followed by light green, yellow and red (scuffing). This allows a gradual tracking of the friction process and provides an estimate of the proximity to scuffing. It was found that the transient from the green to the light green confidence intervals preceded scuffing in the range of 340 s. – 2,850 s. (5 min 40 s. – 47 min 30 s.). The transient from the light green to the yellow confidence interval preceded scuffing in the range of 50 s. - 820 s. (50 s. and 13 min 40 s.). These results demonstrate the potential of using our approach and algorithm in industrial environment to predict scuffing in order to avoid catastrophic damage.
4.4) Evaluation of the algorithm accuracy

As defined in Section 4.3, the “noise” is characterized by the inconsistencies in the DM spectrum propagation through the predefined confidence intervals. Evidently, this “noise” brought uncertainties in estimating the proximity to scuffing. Therefore, the evaluation of our algorithm performance was characterized by two parameters: the “consistency” level and the “noise” level.

We defined the “consistency” level as the smoothness of the DM spectrum dynamics within a specified confidence interval. In other words, it is the ratio of the time of the DM spectrum of a confidence interval \( T_{\text{CI}} \) divided by the total time of the filtered DM spectrum within the same specified confidence interval \( T_{\text{Total CI}} \). \( T_{\text{Total CI}} \) was evaluated by applying a median filtering to the color maps above the time axis (x-axis) in Fig. 4, A and C, and is delimited by the color arrows. It can be also expressed as one minus the total time of all “noises” \( T_{\text{Total N CI}} \) within a specified confidence interval divided by \( T_{\text{Total CI}} \). Consequently, the “consistency” level can be expressed according to Eq. (9) as:

\[
\text{Consistency level} = \frac{T_{\text{CI}}}{T_{\text{Total CI}}} = 1 - \frac{T_{\text{Total N CI}}}{T_{\text{Total CI}}}
\]  

We defined the “noise” level as the ratio of the time of the stochastic short time fluctuations of the DM spectrum divided by \( T_{\text{Total CI}} \). Hence, the “noise” level can be expressed according to Eq. (10) as:

\[
\text{“Noise” level} = \frac{T_{\text{N CI}}}{T_{\text{Total CI}}}
\]  

where \( T_{\text{N CI}} \) is the time of a given “noise” within a specified confidence interval CI.
The “consistency” levels and “noise” levels are summarized in Table 1 for the experiments combined together. In this table, the diagonal cells (light blue) are the “consistency” levels, while non-diagonal ones indicate the “noise” levels. The computations are given for all predefined confidence intervals, marked with the colors. For example, for the green confidence interval, the “consistency” level was 78% and the “noise” levels were 13% for the light green, 6% for the yellow and 3% for the red (scuffing) confidence intervals. In other words, the DM spectrum stayed 78% of the time in the green confidence interval, 13% in the light green, 6% in the yellow and 3% in the red (scuffing) confidence intervals.

Based on the results in Table 1, it is seen that the red confidence interval (scuffing) has the highest “consistency” with 86%, followed by light green (83%), green (78%) and finally yellow (68%). There are two possible reasons for the low “consistency”, and so the highest "noise" level, of the yellow confidence interval. First, it may be due to the few DM spectra available for this confidence interval, in particular for the short time-to-failure experiments (see Fig. 4, C). Second, the yellow confidence interval may be a transition state between steady-state and scuffing that is too small to be detected by the COF sensor. This transition state may be due to changes of the contact surface state (e.g. a small increase in roughness, small wear debris being trapped in the contact area, etc.) or changes taking place below the surface (e.g. cracks, phase transformation, etc.). At present, not only we are not able to distinguish between those possibilities but there is no such tribometer existing. Actually, as mentioned in Section 4.3, investigating the physical phenomena behind this behavior will be carried out as future work. This will be performed by using the real-time computations of this algorithm so that we will be able to stop the experiments at the appropriate time to perform a complete material analysis.

It is interesting to note, that the highest “noise” comes from the neighbored confidence intervals. The “noise” level of the green confidence interval is highest with the light green (13%), the one of the light green is with the yellow (11%), the yellow with the light green (14%), and the red (scuffing) with the yellow (9%). It is also observed that the “noise” level decreases as the time between the non-neighbored confidence intervals increases and evidence of this is in Table 1 for the green and red confidence intervals. This is a typical behavior of processes going through smooth transients between categories (or in this case confidential intervals) resulting in an overlap of the AE features [8,20,38-40]. This indicates that the origin of the “noise” level is certainly not due to computational artefacts.

Table 1. The “consistency” levels and the “noise” levels of different friction regimes

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To conclude, we would like to emphasize once more that, based on the COF behavior, the green, light green and yellow confidence intervals are categorized as a steady-state regime of a lubricated mechanical systems. Such steady-state regime is wanted and corresponds to normal industrial operation condition of lubricated systems [4,7]. During the experiments, the steady-state regimes showed no visible sign of failure in neither the COF curves, See Fig. 3, A and 4, A, and C, nor temperature measurements. On the contrary, the proposed method is capable of tracking the changes in the friction dynamics, thus providing sufficient time for decisions and actions to avoid catastrophic failures.

In practice, the number of the predefined confidence intervals can be reduced, giving priority to increase the “consistency” in the computational results. The estimation of the optimal number of these confidence intervals requires a better understanding of the scuffing mechanisms and contact mechanic, in particular the surface and sub-surface phenomena. This will be carried out in a future work where the gradual tracking of the friction dynamics using our approach will be used to stop experiments at specific moments for complete material analysis.

Finally, the approach presented allows multiple modifications and tunings to enhance the results. They are: i) the tuning parameters and ii) the probability distributions, assigned to confidence intervals, iii) the number of accumulated spectrograms for computations of a single DM spectrum, iv) the internal parameters of the DM algorithm, such as the kernel, distance measures, etc. For example, the normal distribution may be replaced by some specialized ones, providing a better transient tracking. Additionally, the self-tuning of all parameters can be realized as well.

5) Conclusions

On the one hand, predicting friction-related failures, e. g. scuffing, is a real defy for engineers. On the other hand, AE is a popular tool often used to monitor tribological systems, although its data processing is not a trivial task [9-19]. The challenge in data processing of friction-related AE is in its intricate content that stochastically changes even under the same operating conditions. These changes make the friction process highly complex and so is a major obstacle limiting the development of a universal monitoring methods [4,9,15,17,19]. Under these circumstances, the application of statistical methods towards the friction monitoring problem gives some promising advantages. First, it easily allows designing generalizations at different scales, skipping noisy details and choosing the most informative range. Second, a rich collection of statistical tools allows observing a variety of hidden friction phenomena within a single model. Last but not least, a statistical instrumentation allows comparing, embedding and adding models from different friction processes.

This contribution is supplement to and enrichment of existing studies on this topic. We present a statistical approach for an in situ and real-time friction monitoring system using AE signals of a specific tribology system. The tribo-conditions were a reciprocating movement of a stainless steel cylinder having a line contact with a grey cast iron counter body under a constant load. This simulates industrial systems such as cam, cylinder pairs, and gear teeth. During the experiments, the AE signals were recorded using a contact piezo electric sensor.

The experimental results showed that the dynamics of the COF contains stochastic fluctuations even in experiments with similar conditions, and thus complicating the process monitoring. Also, the AE signals included some changes related to friction process dynamics. It was also observed that in our experimental
results, during the steady-state, no sign of failure could be visible in neither the COF curves nor the temperature measurements of the samples.

In this study, the monitoring of the surface state was carried out tracking the changes in AE signals. Wavelet spectrograms of the AE were extracted in real-time providing detailed information about the time-frequency content from each stroke. The changes of the relative energies of the individual frequency bands were tracked by computing the Fourier spectra of the diffusion maps (DM). The behavior of the DM spectra in the frequency domain was put in correspondence with the COF so that its dynamics could be investigated, in particular, the upcoming friction related failure known as **scuffing**.

It was found that during the steady-state regime, the DM spectra starts with a high frequency content followed by a gradual attenuation of those until it raises back during the scuffing regime. At the same time, the proximity to scuffing was carried out by defining several confidence intervals in the frequency domain and observing the propagation of the real-time DM spectrum through those. The optimum number of confidential intervals was found to be four. The first three confidential intervals were related to the steady-state friction regime. We characterized the start of the steady-state by a green confidence interval, followed by a transient to light green and the end of the steady-state by a yellow confidence interval. Although no changes in the COF was visible during the yellow confidence interval, it is believed that it is a transition state between the steady-state and scuffing regimes. The last confidential interval corresponded to scuffing (red confidential interval). We tested our approach on ten independent experiments which demonstrated the repeatability of the DM spectrum propagation through the predefined confidential intervals independently from the COF and time-to-failure characteristics.

These promising results are affected by the presence of a "noise" level. The nature of the "noise" will be investigated in the future using the developed algorithm to stop the experiments at specific time and performing a complete material analysis. In addition, to improve the algorithm performance, multiple tuning is possible for statistical methods involved. In particular, the changes of feature dimensions (e.g. the range of selected frequency bands and/or the number of the spectrograms, from which DM are computed).

Based on the results presented, we can conclude that the DM spectra in the frequency domain of the AE signals allow tracking the surface state changes during tribological experiments. Certainly, the biggest achievement of our approach is to be able to estimates of the proximity of scuffing before its occurrence. In our experiments, two possibilities exist to estimate the proximity of scuffing. The first one is the transient from the green confidence interval to the light green which took place between 5 min 40 s. – 47 min 30 s. prior to scuffing. The second one is the transient from the light green confidence interval to the yellow which took place between 50 s. and 13 min 40 s. prior to scuffing. Finally, the flexibility of the proposed method is in the possibility of multiple parameters tuning without significant modifications of the algorithm. This is a major advantages, in contrary to machine learning [4,20], where any slight changes of the features, process parameters, etc. require a retraining with an updated training dataset.
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