Irreversible Cyclic Cohesive Zone Model for Prediction of Mode I Fatigue Crack Growth in CFRP-Strengthened Steel Plates

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ABSTRACT

Experimental studies on various strengthening systems for steel elements under fatigue loading showed that the use of CFRP strengthening system could significantly enhance the fatigue lifetime. Besides, more recently it was shown that the use of prestressed unbonded CFRP strengthening system results in an additional reduction of the fatigue crack propagation and promotes crack arrest. Different models have been proposed to evaluate the fatigue lifetime of CFRP-strengthened steel members (e.g. S-N curves and fracture mechanics-based models making use of Paris’ law or similar). As an alternative approach in this study, the numerical assessment of mode I (tensile mode) fatigue crack growth of an existing macrocrack in unstrengthened and CFRP-strengthened (nonprestressed bonded and prestressed unbonded) tensile steel members is investigated by using a cyclic CZM. The key advantage, compared to the above-mentioned methods, is that it introduces a constitutive relationship of the material, capable of being calibrated for different materials and being used for any geometry. In this way, the crack initiation, crack propagation and crack retardation (crack arrest) are the natural outcomes of the model. It is shown that the finite element model can be readily coupled with an
interface traction-separation law, to predict the damage evaluation in the steel-CFRP interface. The comparison between the numerical and experimental results validated the proposed finite element modelling, which has also been used to perform a parametric study with respect to the main design parameters.

**Keywords:** Traction-separation law; CFRP strengthening; Fatigue crack propagation; Irreversible cyclic cohesive zone; bond-slip models.

*List of acronyms (in alphabetic order)*

CFRP  Carbon Fiber Reinforced Polymer  
CZ  Cohesive Zone  
CZM  Cohesive Zone Model  
DIC  Digital Image Correlation  
EDM  Electrical Discharged Machine  
FE  Finite Element  
FEA  Finite Element Analysis  
MT  Middle Tension  
NM  Normal Modulus  
NPB  NonPrestressed Bonded  
PUR  Prestressed Unbonded Reinforcement  
SMA  Shape Memory Alloy  
TSL  Traction-Separation Law  
UEL  User-defined ELement  
UHM  Ultra High Modulus

**1. INTRODUCTION**

The demand to increase the service life of some existing fatigue-sensitive metallic structures (such as bridges, offshore structures and communication tower) requires the development of efficient and reliable strengthening
techniques. In the last twenty years, CFRP strengthening bonded to the steel substrate have been recognized as an effective alternative to conventional techniques, such as bolted or welded cover plates, for the fatigue strengthening of steel structures. This is mainly owing to the superior properties of CFRP composites, such as corrosion resistance, light weight, high strength and stiffness. To this end, after some pioneer works [1], a considerable number of experimental, theoretical and numerical studies were performed in the literature to investigate the performance of bonded CFRP strengthening systems to enhance the static performance, and in particular, the fatigue lifetime of steel structural elements [2]. As a result of all the research on this topic, the service lifetime of many different old (railway and roadway) steel bridges around the world have been substantially increased through strengthening using unbonded [3,4] and bonded CFRP composites.

1.1 Motivation

Several numerical models have been so far proposed to predict fatigue lifetime of CFRP-strengthened steel elements. They refer to both S-N curves and Miner’s rule [5] or to fatigue crack propagation models based on fracture mechanics concepts [6]. In the latter case, the presence of a physical crack is assumed and empirical fatigue crack propagation models [7] were adopted to estimate the fatigue crack growth for intermediate stress intensity factor range [8]. Fracture mechanics models were also extended to cover crack propagation in the low stress intensity factor range by introducing the concept of fatigue threshold [9]. A more general model for the fatigue crack propagation was finally introduced in [10] in order to also model fatigue crack propagation for high stress intensity factor range. Computational investigation of fatigue crack growth in CFRP-strengthened steel elements should be able to consider three main aspects. Firstly, the model should be able to consider crack-growth induced debonding since it plays a fundamental role in the analysis. Secondly, the model should be able to capture the main phenomena of the fatigue crack growth in the steel substrate, such as fatigue threshold, crack propagation and retardation. In particular, closure effect should be captured since it is emphasized by crack patching. Finally, in a preventing strengthening application, a crack has not yet been detected but critical fatigue-sensitive details have been identified. In this case a major part of the fatigue life is usually spent during crack initiation or propagation of physically short cracks. A reliable model in this case should be also capable of incorporating both the crack initiation and propagation phase as part of the whole fatigue lifetime. An attempt in this direction was performed in [11] where, based on the concept of equivalent
initial flaw size, a fracture mechanics based model was developed to investigate both the fatigue crack propagation of physically short and long cracks.

1.2 State-of-the-art studies

For fatigue sensitive steel elements, it was previously shown [12] that the use of prestressed CFRP strengthening system has several advantages over the passive (nonprestressed) ones. Although the use of passive CFRP strengthening system has been shown to effectively reduce fatigue crack propagation [13] [14], the application of prestressed system results, in fact, in an additional reduction of the crack opening displacement promoting the crack closure. A shortcoming associated with the use of prestressed bonded systems is related to the high interfacial stresses between the substrate and the strengthening strips. The release of prestress after the installation of the CFRP strips [15] results in high interfacial stresses in the end zones of the CFRP strips that promotes debonding and limits the level of the prestressing force that can be applied.

More recently, as an alternative technique, prestressed unbonded strengthening solutions were then proposed in the literature [16]. In this case mechanical end anchorages are used to install the CFRP strengthening strips instead of adhesively bonding them to the steel substrate. This technique was successfully applied to strengthen old steel bridges in Switzerland [3] and in Australia [4]. In all the above-mentioned studies, emphasis was on preventing crack initiation or on promoting crack arrest under both mode I [17] and mixed mode I/II [18] loading condition, resulting in the fatigue lifetime extension of old fatigue-sensitive steel structures. They proposed, through some conservative assumptions, suitable models to achieve a practical estimation of the fatigue crack threshold and the required prestressing level in the strengthening systems for crack arrest.

A different kind of strengthening system was recently developed making use of thermally activated SMA. Both bonded CFRP-based [11] and unbonded iron-based [19] strengthening systems were proposed. In both cases they develop compressive stresses in the substrate after the activation of the SMA strengthening system. This has a beneficial effect on the fatigue performance of the strengthening systems. While the implemented strengthening system in [19] is unbonded, debonding at the end of the strengthening patch was only observed in [11].
Recently, nonprestressed bonded CFRP strips were used to strengthen steel plates with inclined notches under fatigue loading [20]. The presence of a starter notch that is not orthogonal to the applied load direction leads to fatigue crack initiation and growth under combination of mode I and II loading. Experimental outcomes showed that the crack propagation of inclined crack starts as mixed mode but quickly turns to become almost mode I. They found also that fatigue crack growth curves for mixed mode can be derived from mode I crack growth ones through the definition of a modification factor. Note that no precrack was present at the beginning of the fatigue tests and this may significantly influence the fatigue lifetime. An experimental program was performed on central notched steel plate in [21]. Again, no precrack was created prior to starting of the fatigue tests. Experimental results revealed that fatigue lifetime is not sensitive to the load application angle, but the key parameter is the length of the projection of the initial notch size perpendicular to loading direction. This holds if the load application angle is in the range 0°-60°. Numerical simulation [22] was then performed to investigate inclination angle greater than 60°. The result was that the fatigue lifetime is insensitive to the load inclination angle up to 70° and 60° for bare specimens and double-sided repaired specimens, respectively. Note that the above results were not confirmed by the experimental study in [18] where it was shown that the mode mixity also plays a role on the fatigue lifetime. Fatigue crack growth in CFRP repaired four-point bend specimens was investigated in [10]. Various fatigue loadings (pure mode I, pure mode II and mixed mode I/II) were investigated by using finite element analysis. The effects of different shear-tension loading conditions and CFRP layer numbers on CFRP repair performance were investigated by a comparative analysis of numerical and experimental results.

Several studies proved the presence of crack-induced debonding, between the strengthening element and the steel substrate, during fatigue crack propagation [11]. These studies illustrate that debonding has a detrimental effect on the predicted value of the stress intensity factor and then on the crack growth rate. This holds for both CFRP-strengthened cracked steel plates [23] and beams [9]. Numerical and analytical models for the prediction of fatigue crack growth in CFRP-strengthened cracked steel beams were proposed by using a fracture mechanics approach in [24].

1.3 Cohesive zone models advantages

In this study, the CZM proposed in [25,26], which has an irreversible behavior under unloading-reloading cycles, is implemented in order to study mode I fatigue crack growth in precracked unstrengthened,
nonprestressed bonded and prestressed unbonded CFRP-strengthened steel plates. Although a number of CZMs are available in the literature under monotonic loading, few models were proposed to investigate high cycle fatigue loading [25–27]. In these models, after each loading cycle, the maximum traction and stiffness of the material ahead of the crack tip is reduced by introducing a proper evolving damage variable.

The key advantage of using CZMs compared to the other common methods, such as the fracture mechanics one, is that they introduce a constitutive relationship of the material in order to describe the nonlinear processes associated to fatigue crack growth. Thus, they are only dependent on the material properties and not on the geometric features of the model. Consequently, they are capable of being calibrated to a wide range of materials and implemented for the desired geometry. Secondly, the crack initiation, crack propagation and crack retardation are the natural outcomes of the model, if the physical parameters are calibrated properly according to the material behavior. Finally, the finite element model can be readily endowed with an interface traction-separation model as well, in order to predict the size and shape of the debonded region for the current crack length. Note that the proposed methodology avoids the need to perform two separate FE analyses as in [11]: first to evaluate the size and shape of the debonded region between the CFRP plate and the substrate and, then, to evaluate the stress intensity factor based on the previously evaluated debonding. In this way, all the features of fatigue crack propagation of CFRP-strengthened steel elements are captured by the proposed model more efficiently. The drawbacks are eventually related to the computing time (however, related to the computing power at disposal) and to the calibration of the model parameters associated with damage under cyclic loading.

This study is organized as follows. First, the adopted damage-based cyclic CZM is described in Section 2. The implementation of the cyclic CZM in the Abaqus FEA is then introduced by defining a 2D interface user element (through UEL subroutine in Fortran). A detailed discussion of the peculiarities of the model follows and the Section closes with the description of the cohesive-based contact model for debonding of CFRP strengthening system. In Section 3, the experimental results used to validate the proposed model are briefly described, and in Section 4 the proposed numerical model is applied for the simulation of the fatigue crack growth in unstrengthened, nonprestressed bonded and prestressed unbonded CFRP-strengthened steel plates. Finally, in Section 5, the proposed numerical approach is validated with the experimental results and consequently parametric analyses are performed with a reference to a nonprestressed bonded CFRP-strengthened system.
2. NUMERICAL SIMULATION OF FATIGUE CRACK GROWTH

By adopting a CZM to investigate steel crack growth instead of classical fracture mechanics laws, there would be no need to compute in advance the fracture mechanics parameters (typically the stress intensity factor) in order to evaluate the fatigue crack growth rate. The evaluation of fracture mechanics parameters is not straightforward and often requires finite element analyses. Debonding always takes place close to the crack tip and reduces the effectiveness of the strengthening technique. While in the present approach, this feature is automatically taken into account by a proper interface law, defined between the CFRP plate and the steel substrate, in the literature a two-step technique is usually proposed [11] [28] [9].

2.1 The cyclic irreversible cohesive zone model for fatigue crack propagation

A damage-based cyclic CZM for fatigue-induced crack initiation and propagation, which allows for crack retardation and fatigue threshold, proposed originally by [25,26], is presented here, without loss of generality, within a two-dimensional space. Note that in the CZM context, “fatigue threshold” corresponds to a stop in the increase of the damage value, while a physical crack is arrested when its crack growth rate is less than a critical value [7]. It is then a threshold value for accumulation of damage under fatigue loading and it is regardless of presence of the crack or not. In cohesive zone modelling, at a selected time step, the presence of crack is realized only if some CZ elements in the back of the active CZ element with the highest damage value (potential crack tip) are completely failed, while the above-mentioned terms could be defined for all defined CZ elements in the model; no matter if their status is failed or active.

In addition, “crack retardation” refers to all mechanisms which could heal the crack by reducing the damage under cyclic loading, which from the physical point of view refers to plasticity-induced or roughness-induced crack closure [7].

The proposed model (see Figure 1b) presents a bilinear behavior under monotonic loading of the TSL and exhibits a nonlinear stiffness and degrading strength as functions of a properly defined damage variable, whose evolution in time is governed by a phenomenological equation, able to capture peculiar behaviors of a fatigue induced crack propagation, such as crack advance, threshold and retardation.

The damaging traction versus separation model reads as:

$$ t = \begin{bmatrix} t_n \\ t_s \end{bmatrix} = F(k) \cdot \begin{bmatrix} \delta_n \\ \eta^2 \delta_s \end{bmatrix} $$  \hspace{1cm} (1)
where $t_n$ and $t_s$ represent the normal and shear traction components, respectively; $\delta_n$ and $\delta_s$ are the normal and shear components of the jump displacement vector along the interface, respectively; $\eta$ is a nondimensional parameter that couples the normal and shear effects through the definition of a scalar effective opening displacement:

$$\delta = \sqrt{\delta_n^2 + \eta^2 \delta_s^2}$$  \hspace{1cm} (2)

The nonlinear stiffness $F(k)$ is defined as a function of the damage variable, $k$ as per:

$$F(k) = \frac{\sigma_c (1-k)}{k(\delta_n - \delta_c) + \delta_c}$$  \hspace{1cm} (3)

In the above expression, $\sigma_c$ represents the traction peak of the TSL, while $\delta_c$ and $\delta_u$ are the critical displacement at which crack initiates and the failure displacement, respectively (see Figure 1b). A scalar effective cohesive traction measure $T$ is also defined as:

$$T = F(k) \delta$$  \hspace{1cm} (4)

which, for pure mode I crack propagation, reduces to the only existing traction component, $t_n$.

Fig. 1. (a) A large enough far-field cyclic mode I load/displacement history inducing mode I stresses, and (b) the corresponding TSL curve for an active CZ element.

Damage variable $k$ takes values between 0 and 1, corresponding to no damage (intact CZ element) and complete failure (failed CZ element), respectively. Note that Figure 1a represents the far-field applied load/displacement to the structural element while Figure 1b represents the traction-separation response in an active CZ element. The above expressions implicitly define ascending (O-A) and descending (A-B) linear
branches of the interface response in Figure 1b for monotonically increasing loading paths. Therefore, material softening starts at point \((\delta_\varepsilon, \sigma_r)\) at which the damage value is zero and begins to increase up to point \((\delta_\varepsilon, 0)\) where the damage variable is equal to 1 and the traction value becomes zero.

In particular, in this regard, the two following scenarios may occur:

- **Elastic branch O-A.** As long as the cohesive behaviour is within the initial elastic branch, \(k = 0\),
  \[
  F(k) = \sigma_\varepsilon / \delta_\varepsilon = \text{constant} ,
  \]
  and, therefore, Eq. (1) turns out to express a reversible behaviour.

- **Plastic monotonic descending branch A-B.** When the effective displacement \(\delta > \delta_\varepsilon\) and for monotonically increasing cracking path, the equality \(T = \sigma_r (1-k)\) holds, and therefore, \(k = \left(\frac{\delta - \delta_\varepsilon}{\delta_\varepsilon - \delta_\varepsilon}\right)\), and
  \[
  F(k) = \sigma_\varepsilon \left(\frac{\delta - \delta_\varepsilon}{\delta - \delta_\varepsilon}\right) .
  \]

The above two scenarios refer to a monotonic loading path. A third scenario occurs during all the unloading-reloading cycles, which plays a fundamental role for fatigue-induced crack propagation. In this context, the evolution of the damage variable \(k\) is governed by the following relationships:

\[
\begin{align*}
\dot{k} &= \dot{\alpha} k \left[ T - \beta \sigma_r (1-k) \right] \delta \quad \text{if} \quad \left[ T - \beta \sigma_r (1-k) \right] \delta > 0 \quad \left(\text{with} \quad \dot{\alpha} = \alpha \quad \text{for} \quad \delta > 0 \quad \text{and} \quad \dot{\alpha} = -\gamma \quad \text{for} \quad \delta < 0\right) \\
\dot{k} &= 0 \quad \text{if} \quad \left[ T - \beta \sigma_r (1-k) \right] \delta < 0
\end{align*}
\]

With respect to this third scenario (Unloading-reloading cycles) the following alternatives hold with reference to Figure 1b:

1. **Initial unloading stage (branch B-C):** \(\delta < 0\) and \(T > \beta \sigma_r (1-k)\); then \(\left[ T - \beta \sigma_r (1-k) \right] \delta < 0\) and \(\dot{k} = 0\), which means no damage evolution occurs. In the previous equation, the parameter \(\beta\) models the threshold for initiation of damage recovery.

2. **Final unloading stage (branch C-D):** \(\delta < 0\) and \(T < \beta \sigma_r (1-k)\); then \(\left[ T - \beta \sigma_r (1-k) \right] \delta > 0\) and \(\dot{k} = -\gamma k \left[ T - \beta \sigma_r (1-k) \right] \delta < 0\), which accounts for damage healing or crack retardation, captured by the parameter \(\beta\) (damage recovery threshold) and \(\gamma\) (rate of damage recovery).
3. Initial reloading stage (branch D-E): $\delta > 0$ and $T < \beta \sigma_r (1-k)$; then $[T - \beta \sigma_r (1-k)] \delta < 0$ and $\dot{k} = 0$, which accounts for no damage evolution.

4. Final reloading stage (branch E-G): $\delta > 0$ and $T > \beta \sigma_r (1-k)$; then $[T - \beta \sigma_r (1-k)] \delta > 0$ and $\dot{k} = \alpha k [T - \beta \sigma_r (1-k)] \delta > 0$, which means damage incrementation governed by the parameter $\alpha$. Note that depending on the input parameters, the location of point G could be either on the descending monotonic branch, as in the first reloading cycle in Figure 1b, or at the lower level as in the following reloading cycles. In addition, during this stage, the new maximum value of traction, which is less than the corresponding value to the previous cycle (e.g. point A), is reached at point F. This degradation continues to occur in the next cycles, as well.

While following the monotonic descending branch (Plastic monotonic descending branch) the evolution of damage $k$ is only a function of the current opening displacement $\delta$, during unloading-reloading cycles (as those typically induced by fatigue loading), damage evolution depends on the rate of deformation, previous damage accumulation, cohesive traction and crack retardation. In particular, when the traction $T < \beta \sigma_r (1-k)$ during unloading, damage is reduced in order to capture crack retardation effect (see point 2 above).

Restrictions on the model parameters $\alpha$, $\beta$ and $\gamma$ that are necessary for the model to be well posed from the mathematical point of view are not herein discussed in detail. Readers are referred to [25]. Obviously, one should have $\alpha \geq 0$, $0 < \beta < 1$ and $\gamma \geq 0$ while the following additional conditions govern the parameters $\alpha$ and $\gamma$:

$$\alpha < \frac{4}{\sigma_r (\delta_u - \delta_r)(1-\beta)}$$

$$\gamma \leq \frac{4}{\sigma_r \delta_u \beta^2}$$

(6)

2.2 Finite element implementation of the cyclic cohesive zone model

The cyclic CZM described in the previous section is implemented in the Abaqus FEA [29] by defining a 2D interface user element (through UEL subroutine in Fortran). These new defined interface elements have zero thickness and are generated by duplicating the nodes of the surrounding bulk elements. Each CZ element has 4 nodes, 2 Gauss integration points and linear shape functions. The bulk elements are assumed to have a linear-elastic, isotropic and homogeneous behavior under the hypothesis of small displacement and small rotations.
The terms referring to the interface elements are instead highly nonlinear and dependent on the nodal displacement vector at the interface.

2.3 Discussion of the proposed model

The first peculiarity of the model is its capability to simulate both crack initiation and propagation. The input of the finite element code is, in fact, the displacement histories at a critical point, no matter if it is identified as a physical existing crack (stable crack propagation problem) or a notch (crack initiation problem). This avoids the need to investigate separately crack propagation from crack initiation as in [11].

In order to better illustrate the potentialities of the model, let us consider a mode I cyclic displacement history being applied at potential crack tip, where a small number of separation cycles are used to approach the ultimate displacement $\delta_u$, computed by solving the equations of the CZM. The applied separation history is reported in Figure 2a together with the corresponding traction history and the evolution of the damage variable $k$. Note that the applied separation history (expressed as $\delta / \delta_u$) refers now to an active CZ element and the maximum separation displacement at each cycle is increasing due to the local stiffness reduction. Displacement and traction curves are normalized for comparison purposes with respect to their maximum value, $\delta_u$ and $\sigma_c$, respectively. The corresponding traction versus separation curve is visualized in Figure 2b. Both figures prove the capacity of this cyclic CZM to capture a decay in the stress transfer mechanism due to damage accumulation, caused by an increasing displacement value at an active CZ element regardless of presence of the crack or not. Moreover, note that in Figure 1b the final failure of the CZ element is achieved for $\delta / \delta_u$ less than one due to fatigue damage.

![Figure 2](image.png)

Figure 2. Solution of the cohesive law for a mode I cyclic displacement being applied at an active CZ element, in terms of (a) damage and stress evolution, and (b) traction vs separation curve.
One fundamental feature of this model is the different definitions for damage evolution through cyclic loading. Both reloading and unloading phases are divided into two steps. In the first step, the damage value \( k \) is kept constant while in the second step the damage value changes; either increases in reloading or decreases in unloading, with the latter effect being evidenced by the damage reduction shown in Figure 2a, occurring close to the end of the unloading part of each cycle. This peculiar behavior of the cohesive law is governed by the model parameters \( \beta \) and \( \gamma \), see Eq. (5), which allows for the modelling of crack retardation. In fracture mechanics models, crack retardation is usually modelled by introducing the so-called crack closure parameter \( U \) [7]. Research is still needed to compare the results from the adopted model and the well-known empirical formula for the \( U \) estimation.

Another important feature of this model is the possibility to simulate fatigue threshold. Figure 3a shows the response of the cohesive law when a fixed 0.02–0.4 \( \times \delta_u \) cycling displacement history is applied at an active CZ element. After a certain number of cycles, the damage variable \( k \) does not increase anymore due to a compensation between damage healing and damage accretion, occurring at the end of each unloading and reloading cycle, respectively. If the cycling displacement range is increased to 0.02–0.7 \( \times \delta_u \), the damage variable \( k \) increases instead to 1 and then no crack arrests take places (see Figure 3b).

Finally, this model is particularly useful to simulate high-cycle fatigue regime, as it is possible to perform an extrapolation on damage and stress evolution by properly scaling the number of applied cycles and the corresponding fatigue parameters \( \alpha \) and \( \gamma \) [25,26]. In particular, the same response can be achieved by reducing the number of applied cycles \( N_{\text{max}} \) by a factor, say \( \lambda \left( \frac{N_{\text{max}}}{\lambda} \right) \), and by increasing the model parameters with
the same factor \((\alpha \cdot \lambda, \gamma \cdot \lambda)\). To illustrate this peculiar feature of the model, Figure 4 visualizes damage and stress evolution for different cyclic displacement histories having the same amplitude but different number of cycles. The response in terms of damage evolution and stress decay, computed by the four analyses, is the same, except for the frequency characterizing each response.

![Graphs showing damage and stress evolution](image)

Figure 4. Cohesive law response, in terms of damage and stress evolution, for an increasing displacement history at an active CZ element and for different values of the scaling parameter \(\lambda\): (a) \(N_{\text{max}} = 30, \lambda = 1\) (b) \(N_{\text{max}} = 60, \lambda = 2\) (c) \(N_{\text{max}} = 90, \lambda = 3\), and (d) \(N_{\text{max}} = 120, \lambda = 4\).

### 2.4 The cohesive-based contact model for debonding

Debonding of the strengthening patch in the crack propagation zone was often observed in the literature. In particular, in [30] it was observed through DIC measurements that the near-crack-tip debonding at different levels of crack propagation is not influenced by the fatigue loading. In fact, fatigue loading produces crack propagation in the bulk material which, in turn, extends the debonded region at the CFRP-steel interface. The propagating crack in the substrate acts as a wedge at the interface promoting then monotonic debonding due to the high stress concentration in the crack tip region. As a result, there is no need to extend the cyclic cohesive zone model proposed in Section 2.1 for fatigue crack propagation in steel to cover also fatigue debonding propagation at the steel-CFRP interface.
Debonding of the CFRP plate from the steel substrate is also modelled using the standard surface-to-surface contact model available in Abaqus FEA [29] as in [24,30,31]. A brief review of the model is presented in the following. More details can be found in the above-mentioned studies. Two contact surfaces are defined in the finite element model, one on the strengthening plate and the other one on the substrate. The interaction of these two surfaces is modelled by using a master and slave cohesive damaged contact scheme. This means that the adhesive layer is not physically modelled, and its mechanical properties are considered in the calibration of the TSL parameters of the interface. A traction vector $t$ versus separation vector $\delta$, representing the normal and shear components is introduced.

The interface response is initially linear, and it is represented by an uncoupled traction-separation model. The relevant elastic stiffness in the normal and in the two shear directions are related to the thickness and mechanical properties of the adhesive layer as explained in [31].

As a strength criterion in terms of the components of the traction vector $t$ is fulfilled, a single damage variable is introduced to model the evaluation of the normal and shear interface stresses. A discrete evolution of the damage variable is introduced in the finite element code in accordance with the adopted TSL. In this study, a bilinear TSL is adopted also for the surface contact between the CFRP plate and the steel substrate. Note that the proposed cyclic CZM could be also used for debonding simulation, although it is not necessarily needed. The required change should be applied on the configuration of CZ element through the definition of 3D CZ element [32] instead of 2D, which is outside the scope of this study.

3. EXPERIMENTAL EVIDENCE

Experimental results reported in [17] are used to highlight the potentials and also to validate the proposed model, that is to show its capability to capture the experimental evidence. The experimental tests are briefly summarized in this section, while more details can be found in [17]. An MT steel plate specimen with overall dimensions of $900 \times 150 \times 10$ mm$^3$ are considered (see Figure 5).
Figure 5. The three cases investigated in [17]: (a) bare steel plate, (b) a precracked steel plate strengthened with non prestressed bonded UHM CFRP plates, and (c) a precracked steel plate strengthened with prestressed bonded NM CFRP plates (all dimensions are in mm).

An EDM was used to cut a 15 mm long through-the-thickness notch at the centre of each specimen (see Figure 5a). Fatigue precracking was then performed to provide a sharp fatigue crack and minimize the adverse effects of residual stresses generated at notch tips through the EDM process. The dimension of the initial precrack was selected equal to approximately 1.0 mm on each side of the starter notch and it was created by applying a total of 200,000 fatigue cycles under $\Delta \sigma = 75$ MPa and a stress ratio $R=0.2$ for all the specimens. The three cases studied in [17] are: (a) a reference steel plate specimen without any CFRP strengthening, (b) a precracked steel plate strengthened with four strips of NPB-UHM CFRP with a measured thickness of 1.5 mm, and (c) a precracked steel plate strengthened with the PUR system consisting of mechanically anchored NM CFRP plates with a thickness of 1.4 mm. In the second and third cases, four CFRP plates with a width of 25 mm were
positioned symmetrically (on both sides of the steel plates) at a distance of 13.5 mm from the center of the specimen (see Figure 5). In the latter case, a total prestress force of 110.8 kN was generated in the CFRP plates prior to anchoring them to the precracked steel plate. The prestress force of 110.8 kN is equivalent to 791.4 MPa initial axial stress and 28% prestress level (i.e., ratio of CFRP stress to its tensile strength) in the CFRP plates. In this specimen, no fatigue crack growth was observed during 2.5 million load cycles with $\Delta \sigma = 75$ MPa, indicating a complete crack arrest. The cyclic load range was then increased to $\Delta \sigma = 91$ MPa, under which the precrack was arrested for another 2.5 million fatigue cycles. Upon increasing $\Delta \sigma$ to 105 MPa (40% more than the initial one) the fatigue precrack started to repopagate, resulting in the complete failure of the specimen after approximately 2.5 million additional fatigue cycles.

The mechanical properties of the steel plates along the loading direction (that was perpendicular to the rolling direction) are listed in Table 1 together with the mechanical properties of the CFRP plates utilized.

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic modulus (GPa)</th>
<th>Yield strength (MPa)</th>
<th>Nominal yield strength (MPa)</th>
<th>Ultimate strength (MPa)</th>
<th>Poisson’s ratio</th>
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<tr>
<td>Steel (S355J2+N)</td>
<td>205</td>
<td>430</td>
<td>355</td>
<td>527</td>
<td>0.3</td>
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<tr>
<td>UHM CFRP</td>
<td>435</td>
<td>-</td>
<td>-</td>
<td>1200</td>
<td>0.3</td>
</tr>
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<td>NM CFRP</td>
<td>156</td>
<td>-</td>
<td>-</td>
<td>2800</td>
<td>0.3</td>
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</tbody>
</table>

To strengthen the second precracked steel plate with nonprestressed UHM CFRP plates, a two-component epoxy adhesive was used. The average thickness of the adhesive was reported as 0.075 mm. The tensile strength, $\sigma_a$, of the adhesive was experimentally obtained as $\sigma_a = 26$ MPa, while the shear modulus $G_a$ was assumed equal to 730 MPa as reported in the technical datasheet (see [17] for further details).

4. SIMULATION OF MODE I FATIGUE CRACK GROWTH IN UNSTRENGTHENED AND STRENGTHENED STEEL PLATES

The UEL subroutine in ABAQUS FEA, based on the proposed cyclic CZM, is developed and used to simulate mode I fatigue crack propagation in unstrengthened and CFRP-strengthened steel plates described in the
previous Section. It is also worth mentioning that in the case of mode I fatigue crack propagation, the crack path is a priori known, and therefore, cohesive elements are only inserted along the expected crack growth direction. Note that in this study, the focus is on propagating cracks, not on non-initiating ones since the specimens were precracked before testing.

4.1 Fatigue crack growth in unstrengthened steel plate

As a first step, the fatigue crack growth in the unstrengthened specimen is simulated. Steel is defined as a linear-elastic material with the mechanical properties defined in Table 1. Material parameters of the TSL are set according to the steel properties, as the cohesive zone elements represent the separation process between steel surfaces. In particular, \( \sigma_c \) was assumed equal to the yield stress of material listed in Table 1. The other parameters of the CZM, such as the initial slope \( \sigma_c / \delta_c \), the ultimate displacement \( \delta_u \), and those governing the fatigue behavior, namely \( \alpha, \beta, \) and \( \gamma \), which do not possess a clear physical meaning, are calibrated by trial and adjustment approach, by comparing the numerical and experimental crack growth curves in term of rate of crack propagation and final crack length. Additional research is needed in order to define a more efficient calibration procedure. The selected set for the numerical study of mode I fatigue crack growth in steel plates made of the steel S355J2+N is listed in Table 2.

Table 2. The input parameters of the CZM for the numerical simulations of the mode I fatigue crack growth in the steel S355J2+N (used for unstrengthened and strengthened specimens).

<table>
<thead>
<tr>
<th>Cohesive strength, ( \sigma_c ) (MPa)</th>
<th>Initial slope, ( \sigma_c / \delta_c ) (N/mm³)</th>
<th>Fracture energy, ( G_f ) (N/mm)</th>
<th>Fatigue parameter, ( \alpha ) (mm/kN)</th>
<th>Fatigue parameter, ( \beta ) (-)</th>
<th>Fatigue parameter, ( \gamma ) (mm/kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>430</td>
<td>53,750</td>
<td>30.1</td>
<td>0.06</td>
<td>0.14</td>
<td>1.8</td>
</tr>
</tbody>
</table>

A typical FE mesh is reported in Figure 6a while the results of the simulated fatigue crack propagation curve is reported in Figure 6b and compared with the experimental results in terms of half-crack length (due to the symmetric model) versus number of elapsed fatigue cycles. To be consistent with the following two models of nonprestressd bonded and prestressed unbonded CFRP-strengthened steel plates, the finer mesh is also considered in the reference model where the CFRP is added later (see Figure. 6a, 7a and 8a). In this symmetric
model, a total number of 135 four-node linear cohesive zone elements with the size of 0.5 mm, being governed by the proposed cyclic CZM in Section 2, are located along the mode I crack path, in front of the crack. They are shown as green crosses in Figure 6a. There are totally 11612 continuum solid elements which consist of 4-node reduced-integration plane stress bilinear elements (CPS4R) and 3-node plane stress linear elements (CPS3). Plane stress elements were used since the steel plate thickness is small compared to the width and length. The symmetric boundary conditions are also defined on the nodes of right and bottom edges of the model. As it can be seen in Figure 6b, the agreement between the experimental and numerical results is rather good.

Figure 6. (a) Typical FE mesh used in the reference specimen analyses and (b) the results of the fatigue crack growth simulation compared to the experimental results from [17].
4.2 Fatigue crack growth in nonprestressed bonded CFRP-strengthened steel plate

A typical FE mesh of a quarter of the nonprestressed bonded CFRP-strengthened steel plates is shown in Figure 7a. Same as the unstrengthened model, the displacement history is applied to the reference point which is kinematically coupled to the upper surface of the model. This feature allows us to exclude the top 150 mm of the steel plate to decrease computational effort. To further reduce the computational cost due to the existence of the two CFRP plates and consequently two surface-to-surface contacts representing the adhesive layers in this model, symmetry is exploited. Total number of 43,664 shell elements consisting of both four-node with reduced-integration points (S4R) and three-node (S3) elements are implemented to model steel plate, and 20,000 S4R elements for each CFRP plate. The symmetric boundary conditions are also defined on the nodes of right and bottom edges of the model. Same as the reference specimen, 135 four-node cohesive elements, which are compatible with the surrounding continuum shell elements, with the size of 0.5 mm are inserted in the model to simulate mode I crack growth subjected to fatigue loading. A finite element model, with 45 interface elements with the increased size of 1.5 mm, is also implemented for comparative purposes, see Figure 7b. The latter model has 5,428 S4R and S3 shell steel elements, and 2,261 S4R elements for each CFRP plate. The same fatigue cohesive law parameters, calibrated as in Table 2 with respect to the reference specimen, were used to evaluate the fatigue crack growth curve for the bonded CFRP-strengthened specimen. In order to predict the size and shape of the debonded region at the CFRP-steel interface during fatigue crack propagation, a standard surface-to-surface [29] monotonic contact between CFRP plates and steel is defined which is governed by a proper bilinear cohesive TSL. Note that the coupling of the surface-to-surface contact model with the adopted cyclic CZM increase significantly the computing time. Additional research is needed to optimize the mesh size and speed-up the calculations. The relationships proposed in [31] are used to correlate the thickness and the mechanical properties of the adhesive layer to the main parameters of the surface-to-surface TSL, which are reported in Table 3 with reference to the mode II. Mode I is also taken into account in the simulation but the parameters of the relevant TSL has a marginal influence on debonding since normal tractions at the interface are mainly compressive rather than tensile (peeling) [30].
Table 3. The values of bond-slip parameters for mode II representing the adhesive behavior.

<table>
<thead>
<tr>
<th>Bond shear strength, $\tau_c$ (MPa)</th>
<th>Initial slope, $\tau_c/\delta_c$ (N/mm$^3$)</th>
<th>Fracture energy, $G_f$ (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.4</td>
<td>1170</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Figure 7b shows the simulated fatigue crack growth curve compared to the experimental one in terms of half of crack length (due to the symmetric model) versus number of fatigue cycles. The agreement between experimental and numerical results is rather good. While the crack growth is a discontinuous solution in the simulation, it is expected to be dependent on the size of the CZ element, which determines the increment of crack propagation. This feature is illustrated in Figure 7b, where the difference between the results of the two models, one with the CZ element size of 0.5 mm and one with the size of 1.5 mm, is shown. According to the Figure 7b, the smaller size of CZ element can perform significantly better when the rate of crack propagation is great (the big difference occurs at the end of the curve). However, the coarser mesh is used later in performing the parametric study to decrease the computational effort.
Figure 7. (a) Typical FE mesh used in the analyses of the nonprestressed bonded CFRP-strengthened specimen, and (b) the results of the fatigue crack growth simulation compared to the experimental results from [17].

Figure 8 visualizes the stiffness degradation at the steel-CFRP interface, as a function of the number of applied cycles, at different positions along the bonded region. Stiffness degradation is expressed in terms of an Abaqus built-in scalar damage variable varying from 0.0 (perfect interface) to 1.0 (complete interface failure). From this figure, it is evident how the crack propagation in steel (governed by the cohesive law presented in Section 2.1), due to fatigue loading, induces a progressive debonding along the interface between steel and CFRP (governed by the bilinear cohesive law available in Abaqus), which in turns gradually reduces its effectiveness. In the present approach both cohesive laws are considered simultaneously and the amount of crack propagation in steel and debonded area at the steel-CFRP interface is an output of each single time step of the analysis.
Figure 8. Damage evaluation at given points of the cohesive CFRP-steel interface in the model with coarse CZ element size: (a) position of the selected points, (b) damage evaluation for the points along mode I crack path (X-direction), and (c) damage evaluation for the points along the CFRP plate (Y-direction).
4.3 Fatigue crack growth in prestressed unbonded CFRP-strengthened steel plate

Finally, fatigue crack growth of the prestressed unbonded CFRP-strengthened steel plate is considered. Note that in the experiment, see Figure 9b, the load level was increased twice, by 20% steps, to correspond to a stress range of 105 MPa before any crack propagation could be observed. A typical FE mesh is depicted in Figure 9a. Same as for the bonded model, a total number of 43,664 shell elements with the type of S4R and S3 are used for the steel plate. Each CFRP plate consists of 20,000 S4R elements. The symmetric boundary conditions are also defined on the nodes of the right and bottom edges of the model. The top part of each CFRP plate is tied (a black box at the top of the CFRP plate in Figure 9a indicates the tie) to the steel plate to simulate the mechanical anchors. Furthermore, an initial axial stress equal to 791.40 MPa was applied to the CFRP plate to represent the initial prestressing force.

The same fatigue cohesive law parameters, calibrated in Table 2 with respect to the reference specimen, are used to define 135 four-node linear CZ elements to evaluate the fatigue crack growth curve for the strengthened specimen with the prestressed unbonded system. Figure 9b shows the simulated fatigue crack growth curve compared to the experimental one in terms of half of crack length (due to the symmetric model) versus the number of cyclic loads. A rather good agreement between experimental and numerical results is observed.
5. DISCUSSION AND PARAMETRIC STUDY

In this Section, the achieved numerical results are first validated against the experimental outcomes. Subsequently, a parametric analysis is performed with a reference to a nonprestressed bonded CFRP-strengthened system.

5.1 Validation of the numerical results

Numerical fatigue crack growth curves in the three cases mentioned earlier are presented in Figure 10. A unique set of the CZM parameters (see Table 2) calibrated with respect to the reference specimen results in a reasonable agreement for both strengthened specimens using the nonprestressed bonded and prestressed unbonded CFRP plates.
Figure 10. Comparison between the experimental fatigue crack growth curve from [17] and numerical simulation.

According to the crack growth curves presented in Figure 10, a rather good agreement can be observed between the numerical and experimental results in all three cases. Moreover, it can be seen that for both of the CFRP-strengthened specimens (nonprestressed bonded and prestressed unbonded), the decrease in the crack propagation rate, due to strengthening, is clearly captured by the proposed CZM, endowed with the same CZM parameters calibrated with respect to the reference specimen. Moreover, the complete crack arrest that was experimentally observed in the prestressed unbonded specimen at lower fatigue load levels (see [17] for further details) is also well-captured by using the proposed numerical simulation.

5.2 Parametric analyses on nonprestressed bonded CFRP-strengthened system

A set of parametric analyses are performed in order to further investigate the effect of the parameters governing the behaviour of the steel-CFRP interface as well as the stiffness of the strengthening system on the
effectiveness of the strengthening technique, i.e., the fatigue lifetime extension. Parametric analyses are performed with respect to the bond-slip (BS) parameters, namely $\tau_c$ (shear strength), $\delta_c$ (critical displacement) and $G_f$ (fracture energy). Table 4 reports the sets of the BS parameters adopted in the parametric analyses. The reference set for the parametric study of mode I fatigue crack growth in the strengthened steel plates corresponds to the BS1. In Table 4, the BS2 and BS3 are intended to study the effect of the bond shear strength, the BS4 and BS5 are considered to reveal the effect of the fracture energy, and, finally, BS6 and BS7 are included to investigate the effect of the critical displacement. The last column in Table 4 represents the initial slope $\tau_c / \delta_c$ of the bond-slip.

Table 4. The values of the steel-CFRP bond-slip parameters adopted in the parametric analyses.

<table>
<thead>
<tr>
<th>Group</th>
<th>Bond shear strength, $\tau_c$ (MPa)</th>
<th>Fracture energy, $G_f$ (N/mm)</th>
<th>Critical displacement, $\delta_c$ (mm)</th>
<th>Initial slope, $\tau_c / \delta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS1</td>
<td>23.4</td>
<td>1.29</td>
<td>0.02</td>
<td>1170</td>
</tr>
<tr>
<td>BS2</td>
<td>11.7</td>
<td>1.29</td>
<td>0.02</td>
<td>585</td>
</tr>
<tr>
<td>BS3</td>
<td>35.1</td>
<td>1.29</td>
<td>0.02</td>
<td>1755</td>
</tr>
<tr>
<td>BS4</td>
<td>23.4</td>
<td>0.64</td>
<td>0.02</td>
<td>1170</td>
</tr>
<tr>
<td>BS5</td>
<td>23.4</td>
<td>1.93</td>
<td>0.02</td>
<td>1170</td>
</tr>
<tr>
<td>BS6</td>
<td>23.4</td>
<td>1.29</td>
<td>0.01</td>
<td>2340</td>
</tr>
<tr>
<td>BS7</td>
<td>23.4</td>
<td>1.29</td>
<td>0.03</td>
<td>780</td>
</tr>
</tbody>
</table>

The results of the parametric analyses on the effect of the steel-CFRP interface properties are reported in Figure 11, where the response of the different models are compared under the same applied load history. This figure clearly indicates that $G_f$ has a marginal effect on the fatigue lifetime. In fact, a 50% increment or decrement of the fracture energy (group BS4 and BS5) has almost no effect in practice on the fatigue crack growth curve. A deep investigation on the damage distribution at the steel-CFRP interface reveals that the interface was significantly damaged close to the crack tip, but the damage level is less than one. On one hand, this means that the strengthening plate is not yet fully debonded but the interface stiffness is anyway significantly reduced influencing the effectiveness of the strengthening system. On the other hand, this also means that the fracture
energy is not fully released. This could also explain the marginal influence of the fracture energy in the parametric analyses.

Moreover, it is also seen from Figure 11 that $\tau_c$ has a strong effect on the fatigue lifetime. In fact, a 50% increment or decrement of the bond shear strength produces (group BS2 and BS3) completely different fatigue crack growth curves. In particular, the greater the bond shear strength, the longer the fatigue lifetime. The same holds with the reference to the initial slope of the TSL. A larger initial slope (group BS6) provides a longer fatigue lifetime. These findings are in line with the results of the parametric analyses reported in [30].

![Figure 11. The results of the parametric analyses on the bond-slip (BS) parameters of the steel-CFRP interface.](image)

Finally, the effect of the stiffness of the strengthening strips is investigated. The bond-slip parameters are equal to that of BS1 case available in Table 4. The parametric analyses are performed with respect to $t_f$ (CFRP thickness) and $E_f$ (Young’s modulus). Table 5 presents the sets of the selected parameters and the corresponding CFRP stiffness in the last column for each case considered (designated as ST).
Table 5. CFRP thickness and Young’s modulus adopted in the parametric analyses.

<table>
<thead>
<tr>
<th>Group</th>
<th>Thickness, $t_f$ (mm)</th>
<th>Young’s modulus, $E_f$ (GPa)</th>
<th>Stiffness, $E_f \cdot t_f$ (kN/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST1</td>
<td>1.5</td>
<td>435</td>
<td>652.5</td>
</tr>
<tr>
<td>ST2</td>
<td>0.75</td>
<td>435</td>
<td>326.25</td>
</tr>
<tr>
<td>ST3</td>
<td>1.5</td>
<td>205</td>
<td>307.5</td>
</tr>
<tr>
<td>ST4</td>
<td>2.25</td>
<td>435</td>
<td>978.75</td>
</tr>
<tr>
<td>ST5</td>
<td>0.75</td>
<td>205</td>
<td>153.75</td>
</tr>
<tr>
<td>ST6</td>
<td>2.25</td>
<td>205</td>
<td>461.25</td>
</tr>
</tbody>
</table>

The results of the parametric analyses on the effect of CFRP stiffness is reported in Figure 12. As expected, and as already demonstrated in the previous studies in literature, (see e.g., [33]), Figure 12 shows that the CFRP stiffness plays a very important role in the lifetime extension. A stiffer strengthening system produces a significant increment of the fatigue lifetime. No crack propagation was observed for a very stiff CFRP plate (ST4, not shown for this reason in Figure 12). A very stiff strengthening system, in fact, reduces both traction and separation in the CZ elements.

Figure 12. Results of the parametric analyses on the CFRP stiffness.
6. CONCLUSIONS AND FUTURE WORK

Performing fatigue tests (especially in high cycle regime) is time-consuming, costly and often requires the implementation of very complex test setups. Therefore, a more widespread use of predictive numerical models is demanded to investigate the effect of various design parameters on the performance of the system used for the strengthening of metallic members under high cycle fatigue loading. In this paper, a numerical procedure to simulate the fatigue driven crack propagation is presented, which allows the modelling of both crack retardation and arrest. This model is implemented through a UEL subroutine in Abaqus FEA commercial software and adopted to simulate mode I crack propagation in precracked steel plates, subjected to high cycle fatigue loading. A unique set of the CZM parameters is calibrated in relation to the crack growth curve experimentally observed in the case of an unstrengthened reference specimen. The same CZM parameters are then adopted to simulate the results of two precracked specimens strengthened by using nonprestressed bonded and prestressed unbonded CFRP plates. In the simulation of the nonprestressed bonded specimen, a bilinear cohesive TSL is also defined to simulate the interface between the CFRP plate and the steel substrate.

Based on the results achieved the following main conclusions can be drawn:

- After calibration of the cyclic cohesive law for fatigue driven crack propagation, with respect to the reference specimen, the cohesive model can be successfully adopted to investigate other configurations, such as steel specimens strengthened with CFRP patches.
- A very good agreement between the numerical and experimental results is observed for the CFRP-strengthened specimens, which permits to validate the cohesive zone model adopted.
- The proposed numerical approach can simulate both the bond behaviour between the strengthening patch and steel substrate, and fatigue crack growth during one analysis. A fatigue CZM is adopted to model the fatigue crack propagation in the steel substrate and implemented in Abaqus FEA through the definition of an ad hoc UEL. Additionally, the surface to surface contact model available in Abaqus FEA is used to model the steel-CFRP debonding. Finally, the crack retardation and potential crack arrest in the steel bulk material are captured by the adopted properly calibrated fatigue CZM.
- Parametric studies on the bond-slip law as representative of the adhesive behaviour reveals the dominant effect of higher value of TSL’s initial slope in the extension of fatigue lifetime; either by increasing the bond shear strength (BS3) or by decreasing the critical displacement (BS6).
the results show that the duration of fatigue lifetime has a direct relationship to the stiffness of strengthening system.

Finally, it must be mentioned that further studies are needed to better calibrate the cohesive law expressing the interaction between the CFRP and the steel plate in the case of bonded strengthening system. The promising results obtained for the simulation of mode I fatigue crack propagation also encourage the extension of the present model to capture the fatigue behaviour under mixed mode loading conditions.

ACKNOWLEDGEMENTS

The research grant provided by Fondazione Fratelli Confalonieri is gratefully acknowledged by the first author. The financial support provided by Politecnico di Milano is also acknowledged.

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