Determination of shear modulus and Poisson's ratio of polymers and foams by the anticlastic plate-bending method

M. Farshad¹, M. W. Wildenberg² and P. Flüeler³

(1) Section Polymers/Composites, EMPA Dübendorf, Titular Professor ETHZ, Switzerland

(2) Student, Haarlem Polytechnic, Haarlem, The Netherlands (on project work at EMPA, Dübendorf)

(3) Section Polymers/Composites, EMPA Dübendorf, Lecturer ETHZ, Switzerland

ABSTRACT

In this paper, results of research towards development of a test method for the determination of shear modulus and Poisson's ratio of polymers and structural foams are presented. Through this work, a test apparatus was developed which employed anticlastic plate bending behaviour; this device was used for calibration tests on steel and aluminium samples and for tests on a number of polymers, including high density Polyethylene (PE-HD), unplasticized Polyvinylchloride (PVC-U), Polymethylmethacrylate (PMMA) and rigid Polyurethane (PUR) foam. Long-term tests and temperature-dependent tests on PVC-U, PE-HD and PMMA samples were also performed. In some cases, results of the tests with the anticlastic bending method were compared with those obtained from tests carried out with the torsion pendulum method. For theoretical simulation, an analytical formula and a number of finite element analyses were performed. Through these simulations, values for the short-term, long-term and temperature-dependent shear modulus and Poisson's ratio of PE-HD, PMMA, PVC-U and PUR foam were obtained.

RÉSUMÉ

On présente les résultats des travaux sur le développement d'une méthode d'essai pour la détermination des coefficients d'élasticité en cisaillement et des coefficients de Poisson des polymères et des mousses structurales. On a conçu un appareil d'essais utilisant le comportement en flexion anticlastique de plaques. Cet appareil a été utilisé pour des essais de calibration des échantillons d'acier et d'aluminium, ainsi que pour des essais sur des polymères, y compris le polyéthylène à haute densité (PE-HD), le chlorure de polyvinyle non plastifié (PVC-U), le polyméthacrylate de méthyle (PMMA) et la mousse rigide en polyuréthane (PUR). Des essais de longue durée et de dépendance de la température ont également été réalisés sur les échantillons de PVC-U, de PE-HD et de PMMA. Dans certains cas, les résultats des essais par la méthode de flexion anticlastique ont été comparés aux résultats obtenus par la méthode de pendule de torsion. Une simulation théorique basée sur une formule analytique et un certain nombre d'analyses au éléments finis a été réalisée. Grâce à ces simulations, les valeurs des coefficients d'élasticité et de Poisson – à court terme, à long terme et dépendantes de la température – ont pu être obtenues pour le PE-HD, le PMMA, le PVC-U et la mousse PUR.

1. INTRODUCTION

Shear modulus and Poisson's ratio of various polymers and rigid foams are needed for the design of structural and composite elements. For these materials, not only the knowledge of the short-term properties, but also information about their long-term, temperature-dependent behaviour is of importance.

In this paper, a well-known testing methodology for determining the short-term and long-term shear moduli and Poisson's ratios of polymers and foams is introduced. This testing system is referred to as the Anticlastic Plate-

Bending (ACPB) method. The ACPB testing methodology is based on the knowledge that a rectangular plate subjected to a torsional couple will undergo an anticlastic bending and will assume a saddle shape [1]. This behaviour is strongly dependent on the in-plane shear properties of the constituent material; hence, by understanding this phenomenon, it would be possible to devise a testing device with which – and with the help of a proper theoretical correlation – the shear modulus in the plane of the plate may be determined. A similar methodology has been applied to find the shear modulus of plywood ([2] and [3]).

Editorial note

M. Farshad, M. W. Wildenberg and P. Flüeler work at the EMPA, Switzerland, a RILEM Titular Member.

In the first part of this paper, a simple theoretical model for the anticlastic bending of rectangular plates is outlined. In addition to this analytical modelling, a Finite Element (FE) simulation is also carried out. Then, an experimental set-up for the determination of short-term and long-term shear moduli is introduced, and the stages in its design and development are briefly described.

Other parts of the paper include the results of the short-term tests on a high-density Polyethylene (PE-HD), an unplasticized Polyvinylchloride (PVC-U), Polymethylmethacrylate (PMMA), and a rigid Polyurethane foam (PUR) along with the results of their experimental and theoretical correlation. Moreover, the results of the long-term and temperature-dependent tests on PVC-U, PE-HD and PMMA are reported. In some cases, independent tests have also been conducted on the same materials with the torsion pendulum and the tensile test methods. The results of these tests are correlated with those of the ACPB tests; through this comparison, correlation factors are obtained.

2. ANTICLASTIC BENDING OF PLATES

2.1. Anticlastic bending behaviour

An anticlastic plate-bending (ACPB) of a thin elastic rectangular plate is a mode of deformation in which the plate assumes a saddle shape containing two opposing principal curvatures. To produce an anticlastic deformation, one may apply two equal but opposite uniformly-distributed bending moments $M_x = -M_y$ along the two pairs of intersecting edges (Fig. 1a). An ACPB behaviour can also be produced by applying a uniformly-distributed torsional couple along the edges. This loading is statically equivalent to two pairs of corner forces which are applied at two diagonal corners (Fig. 1b). The value of the corner forces equals twice the distributed twisting couple [4]. The result of both of the above-mentioned loading types is a general twist in the xy-direction; twist-

ing of the plate causes a dominantly shear field which may be advantageously related to the torsional rigidity or the shear modulus of the constituent material.

2.2. Anticlastic plate-bending formula

From classical plate theory, the corner deflection of a square plate loaded by concentrated forces applied at its four corners can be derived as:

$$\Delta = \frac{Fa^2}{4D(1-\nu)} \quad (1)$$

where F is the force applied at each corner (total force W applied to the plate is $W = 2F$), a is the side dimension of the plate, and D is the bending stiffness of the plate which is defined by:

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (2)$$

where h is the plate thickness, E is the Young's modulus, and ν is the Poisson's ratio. Hence, the expression for the Poisson's ratio is:

$$\nu = \frac{Eh^3\Delta}{3Fa^2} \quad (3)$$

For an isotropic material $G = E/2(1 + \nu)$; therefore, the shear modulus would be:

$$G = \frac{3Fa^2}{2\Delta h^3} \quad (4)$$

For anisotropic materials, this method may be used to estimate the torsional rigidity of anisotropic plate samples [5].

Formula (4) is based on classical plate bending theory; it only considers bending effects decoupled from the in-plane stresses and neglects shear deformation. Correction due to shear deformation may be implemented [6]. In the present work, however, due to the choice of a thickness-to-width ratio of about 1/25, the shear deformation is expected to be insignificant.

2.3. Finite element simulation

For numerical simulation of experiments and for illustration of the shear stress field in the plate and corner effects, a Finite Element (FE) analysis was performed. The FE analysis was carried out on a square PVC-U plate for which a linear elastic and isotropic behaviour was assumed. For the finite element analysis, the following data were employed:

$a = b = 100$ mm, $h = 5$ mm, $E = 2900$ N/mm², $\nu = 0.33$

The loading consisted of two concentrated forces each equal to 30 N applied normally to the plate surface at diagonal corners. The other two diagonal corners were partially restrained against rotation and translation. In Section 5, the results of the Finite Element analysis will be correlated with the experimental data.

Fig. 1 - (a) Applied opposite moments on a rectangular plate to induce anticlastic bending; (b) applied corner forces.

Fig. 2 - (a) A sketch of the ACPB device; (b) photograph of the ACPB device.

3. ACPB TESTING SYSTEM

To achieve an optimal ACPB testing system, various designs were conceived and constructed. One of these testing rigs consisted of a portable device which could be placed in the thermal conditioning chamber and could also be used for the long-term (creep) tests. Another device could be used together with a standard tension-compression testing machine. Fig. 2a shows schematically the principle of this ACPB testing device; Fig. 2b displays a photograph of the constructed system. In

Fig. 3 - Load deflection curves for three metallic samples.

developing an optimal system, the effects of boundary conditions, sample size and the manner of load application were all experimentally investigated.

4. CALIBRATION TESTS

To assess the functionality of the testing system, a number of calibration tests were performed. These calibration tests included short-term ACPB tests on polymers and foams, long-term tests, temperature-dependent tests, and comparison tests with the torsion pendulum testing method. Calibration tests were also carried out with the installed testing machine as well as with the portable testing device. Calibration samples consisted of steel, brass, and aluminium with known shear moduli; these moduli were determined by tensile tests. A number of pre-tests were also performed on the PVC-U samples. The calibration tests did lead to improvements in the design of the testing system and to construction of a device whose system errors were substantially reduced.

Fig. 3 shows an overview of the results of calibration tests on steel, aluminium and brass plates; these curves were used to calculate the shear moduli of the tested samples.

Table 1 presents the outcome of the calculation leading to the value of the shear modulus for aluminium and PVC-U; for this calculation, formula (4) was employed.

Table 1 - Shear moduli obtained with the ACPB device		
Materials and samples	Shear modulus [N/mm ²]	
	Reference values	ACPB
Aluminium	26,161	29,877
PVC (h = 2 mm)	1,380	1,630
PVC (h = 4 mm)	1,310	1,457
PVC (h = 5 mm)	1,300	1,345

Fig. 4 – Variation of correlation factor with plate thickness for aluminium samples.

This table includes values of the reference shear moduli for these materials which may be found in the classical literature [7].

To compare the test results obtained from the ACPB rig with those obtained from other methods, a series of tests were performed with the torsion pendulum method [8]. In addition, a number of tensile tests on strips of the same materials were also carried out; these samples were instrumented with a longitudinal and a lateral strain gauge. Thus, through tension testing the value of Poisson's ratio was estimated [9].

From calibration tests, a correlation factor k was obtained. This correlation factor k is defined as the ratio of the reference shear modulus G_R to the shear modulus value G_A obtained with the ACPB test. The correlation factor is derived from the assumptions inherent in the ACPB theory (relation 4), the boundary and size effects, and the difference between the ACPB method and those related to the reference values.

$$k = G_R/G_A \quad (9)$$

The correlation factor k is, in general, a function of the plate thickness: Fig. 4 shows the variation of the k -factor with plate thickness for aluminium samples. However, with reference to Fig. 4, a linear thickness dependency for the k -factor may be assumed. Through linear curve-fitting, the following formula is derived:

$$k = 0.0307 h - 0.768 \quad (10)$$

Table 2 – Correlation factor calculated from the comparison values and the ACPB tests

Material	Thickness, h [mm]	Shear modulus [$10^3 \times \text{N/mm}^2$]		Correlation factor, k
		Comparison values	Anticlastic plate bending	
Steel	1.009	82.0	105	0.78
Aluminium	2.021	26.2	30.3	0.86
Brass	2.029	40.0	47.8	0.84
Aluminium	2.022	26.2	30.5	0.86
Aluminium	2.993	26.0	29.8	0.87
Aluminium	4.953	26.6	28.9	0.92

Fig. 5 – Load deflection curves for PVC, PE and PMMA.

If the correlation factor deduced from Fig. 4 is implemented in the classical relation for the corner-loaded plates, the correlated relation would read as follows:

$$G_{AC} = k \frac{3Fa^2}{2\Delta h^3} \quad (11)$$

Table 2 presents the results of calibration tests which have led to deriving correlation factors.

5. SHORT-TERM ACPB TESTS

5.1. Polymers

Plate samples from PVC, PE-HD, and PMMA had dimensions of 110 mm $\times$ 110 mm $\times$ 5 mm. The correlation factor for these materials ranged from 0.921 to 0.924. Fig. 5 shows the load deflection diagrams for the three polymers. The bold points in this figure represent the result of the finite element simulation for PVC (see Section 3). As noted previously, the result of the finite element analysis matches the experimental results. Due to linear behaviour, it would be expected that the finite element simulation of the other materials would also match the experimental values.

Table 3 presents the comparative values of the shear moduli of PVC, PE-HD and PMMA obtained from the anticlastic plate bending tests along with those found from the torsion pendulum tests.

Table 3 – Comparison of results from the ACPB tests with the torsion pendulum tests

Material	Shear modulus [N/mm^2]	
	Torsion pendulum test	ACPB test
PVC-U	1301	1240
PE-HD	616	396
PMMA	1575	1367

5.2. PUR foam

Due to the cellular structure of foams, the preparation of small samples of foams and their tensile and torsion testing may involve some difficulty. Testing of foams with the ACPB testing method, on the other hand, proved to be simple and very efficient. Fig. 6 shows the average load-deflection curves for the PUR samples with various thicknesses.

The correlation factor for all PUR samples was found to be equal to 0.922. The shear modulus obtained with the ACPB method was compared with that of the torsion pendulum test. Table 4 summarizes the results of the two tests.

6. TEMPERATURE-DEPENDENT TESTS

Shear moduli and Poisson's ratios of polymers are temperature-dependent: it is known that an increase in temperature will generally result in a decrease in the shear modulus of polymers. To explore the potential of the ACPB testing method for temperature-dependent tests, a number of tests at different temperatures were performed. The materials tested included PE-HD, PVC-U, and PMMA. Samples of these materials were taken from the same plates as those for the short-term test samples.

Fig. 6 – Average load-deflection curves for PUR foam samples with various thicknesses.

Table 4 – Shear moduli derived from ACPB tests and torsion pendulum tests on PUR foam

Sample	Thickness, h [mm]	Shear modulus [N/mm ²]	
		Torsion pendulum test	Anticlastic plate bending test
PUR S1	4.91	28.6	34.77
PUR S2	5.98	28.6	33.51
PUR S3	8.05	28.6	33.91
PUR S4	9.70	28.6	37.11
PUR S5	11.83	28.6	35.56

Fig. 7 – Shear modulus as a function of temperature for PE, PMMA and PVC-U.

Fig. 8 – Corner deflection of the anticlastically-bent PE-HD plate as a function of time under a maintained load of 12.67 Newtons.

Various test samples were thermally conditioned to 23, 40, 60 and 80 degrees; then, they were placed in the ACPB device and their deflections under the applied load were measured. Fig. 7 represents the results of these tests. Curves were fitted to the experimental data; Fig. 7 shows the descending trend of all these curves.

7. ACPB CREEP TEST OF POLYMERS

To explore the potential of the constructed ACPB testing device for long-term tests, a series of long-term anticlastic bending tests on PE-HD were performed. During these tests, a constant load was maintained on the samples and the resulting deformation was measured at various time intervals. Fig. 8 shows the result of ACPB creep tests; it represents the corner deflection of the plate as a function of time.

Fig. 9 shows the long-term shear modulus of the PE-HD material which was determined from the creep tests through a correlation with ACPB theory.

Fig. 9 – Time-dependent shear modulus of PE-HD obtained with the ACPB test.

8. FURTHER TESTS

To assess the influence of loading, boundary conditions and geometrical parameters, a number of other tests were performed. Furthermore, in order to study the deformation due to indentation and the in-plane (membrane) effects, some subsidiary tests were also carried out. Moreover, to investigate local contact deformation due to corner forces, samples from aluminium, steel and brass were subjected to indentation tests. It would be expected that, due to local plastification, the measured deflection in a real anticlastic test would be larger than the deflection in the ideal anticlastic bending. At the loading levels used for ACPB tests, however, the local plastification proved to be insignificant.

To evaluate the cable/membrane effects, which might lead to non-linear behaviour, a group of three-point bending tests on strips were also performed. In the three-point bending tests, the supports of the actual device were chosen and the loading was applied through a steel cylinder. At the lower level of loading, the load-deflection curves of these strips showed a linear behaviour; no pronounced non-linearity was observed.

9. DISCUSSION OF THE ACPB METHOD

The main goal of the present work has been to introduce the well-known ACPB testing methodology in the area of polymers and foams, and to extend it to the creep testing of these materials as well as to the determination of their temperature-dependent shear moduli.

The ACPB test method proves to be particularly useful for the testing of materials whose sample preparation and handling presents some difficulties. Foams, in particular foams with an open cell structure, and some polymers are examples of such materials. Other features of the ACPB test device are its simple construction and portability; these features enable the construction of many of these devices at low cost for the long-term (time scale of several years) testing of polymers and foams, along with the possibility of performing temperature-dependent tests in small conditioning chambers.

A comparison of the results from the ACPB tests with those from the torsion pendulum tests has shown a certain degree of discrepancy; this may be attributed to differences in the sample preparation procedure, size effects, testing speed, as well as the theories used for the processing of data. In order to fully investigate the influence in these parameters, additional comparative tests would be required.

The ACPB theory should be better refined to take into account the second-order effects arising from the boundary conditions and from the sample size. Furthermore, it should be noted that formula (4) yields an average value for the G modulus of the samples; this value may indeed characterize the shear modulus of the material itself. For anisotropic materials, such as laminated and sandwich plates, however, this method can only give the overall torsional rigidity of the plate.

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ISSN 0025-5412

JULY 1997

VOL. 30
N° 200

MATERIALS AND STRUCTURES

MATÉRIAUX ET CONSTRUCTIONS