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IFAC PapersOnLine 56-2 (2023) 3751–3756

Designing Fairness in Autonomous Peer-to-peer Energy Trading

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Abstract: Several autonomous energy management and peer-to-peer trading mechanisms for future energy markets have been recently proposed based on optimization and game theory. In this paper, we study the impact of trading prices on the outcome of these market designs for energy-hub networks. We prove that, for a generic choice of trading prices, autonomous peer-to-peer trading is always network-wide beneficial but not necessarily individually beneficial for each hub. Therefore, we leverage hierarchical game theory to formalize the problem of designing locally-beneficial and network-wide fair peer-to-peer trading prices. Then, we propose a scalable and privacy-preserving price-mediation algorithm that provably converges to a profile of such prices. Numerical simulations on a 3-hub network show that the proposed algorithm can indeed incentivize active participation of energy hubs in autonomous peer-to-peer trading schemes.

Keywords: Smart energy grids, Distributed optimization for large-scale systems, Impact of deregulation on power system control

1. INTRODUCTION

Technology improvement for multi-generation and storage systems coupled with an increased penetration of renewable energy sources has led to an unprecedented rise in distributed energy resources, in the form of multi-energy hubs and prosumers. However, this shift also requires markets to evolve from a hierarchical and centralized to a decentralized design that can enable active participation of prosumers. Peer-to-peer (P2P) trading is an emerging feature of new distributed market designs that allows prosumers to directly share energy, that can be used to match demands locally or to reduce the reliance on the grid as well as the overall energy cost. Additionally, P2P trading has also the potential to reduce peak demand and reserve requirements, thus lowering the need for expanding energy infrastructures and energy imports (Tushar et al., 2020).

Several coordination mechanisms have been recently proposed in the literature to facilitate energy management and P2P trading. Existing works formulate the problem either via multi-agent optimization (Sorin et al., 2019; Baroche et al., 2019a; Moret et al., 2020), wherein the autonomous P2P trading is cast as a collective optimization problem and Lagrange relaxation-based methods are used to find the optimal operational set points in a distributed manner, or as noncooperative games (Le Cadre et al., 2020; Cui et al., 2020; Belgioioso et al., 2022a), in which each prosumer has local decoupled objectives and the energy trading is incorporated as coupling reciprocity constraints. Additionally, it can also consider trading with the main grid, and include network operational constraints.

Bilateral trading prices play a key role in defining the outcome of all the aforementioned market models. Naive choices of prices may induce undesired congestions in the power grid or disproportionate costs for the prosumers (Le Cadre et al., 2020). To incentivize active participation in the market model, it is therefore crucial to ensure that individual market participants reap the network-wide benefits of autonomous trading. The development of effective pricing schemes has been already considered in different works. In (Le Cadre et al., 2020), the effect of P2P trading prices is investigated by studying energy preferences and marginal prices (dual variables) connected to the coupling trading reciprocity constraints. In (Fan et al., 2018), a bargaining game is designed to distribute the benefits of P2P trading equally to all hubs, and in (Wang et al., 2020) the transactive prices of each hub are derived based on their internal operation strategy to ensure cost recovery. In (Daryan et al., 2022), a two-stage mechanism is proposed to obtain the optimal payment. Finally, (Sorin et al., 2019) also considers product differentiation and preferences.

In this paper, we leverage hierarchical game theory and distributed optimization to design a novel scalable, locally-benefical, and fair pricing scheme for autonomous peer-to-peer trading in energy hub networks that incentivizes active participation. Our contribution is three-fold:

(i) We formulate the problem of designing fair trading prices as a bilevel game. At the lower level, the "optimal" energy trades and operational setpoints for the energy hubs are formulated as interdependent economic dispatch problems. At the upper level,

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10.1016/j.ifacol.2023.10.1544

the desired P2P trading prices are defined as the minimizers of a fairness metric (the sample variance of the local hub cost reductions) which depends on the optimal setpoints of the lower-level problem.

(ii) We leverage the special structure of the game to decouple the two levels and design an efficient 2-step solution algorithm. In the first step, an ADMM-based algorithm is used for distributively solving the economic dispatch problems. In the second step, a semi-decentralized price mediation algorithm is used to compute fair P2P trading prices in a scalable way.

(iii) We illustrate and validate the proposed autonomous P2P trading and pricing mechanism via numerical simulations on a 3-hub network, using realistic models of energy hubs and demand data.

2. MODELLING THE HUBS

We consider a network of N interconnected energy hubs, labeled by $i \in \mathcal{N} := \{1, \dots, N\}$. As an example, a system of three hubs in illustrated in Fig. 1 and will be used to fix ideas throughout the paper.

To fulfil its electrical and thermal demand, each hub is equipped with different energy conversion and storage devices. Additionally, cooling demand may also be present which is not considered in this work. The heating devices can include gas boilers (GB), heat pumps (HP), as well as thermal energy storage (TS) such as borehole field, water tanks, etc. Electricity can be locally produced via photovoltaic (PV), Combined Heat and Power (CHP), and micro-CHP (μCHP), devices and stored using batteries (ES). Each hub is connected to the electricity and gas grid, and can trade electrical energy with the other hubs via peer-to-peer (or bilateral) trading through the electricity grid. Electricity can be directly drawn from the electricity

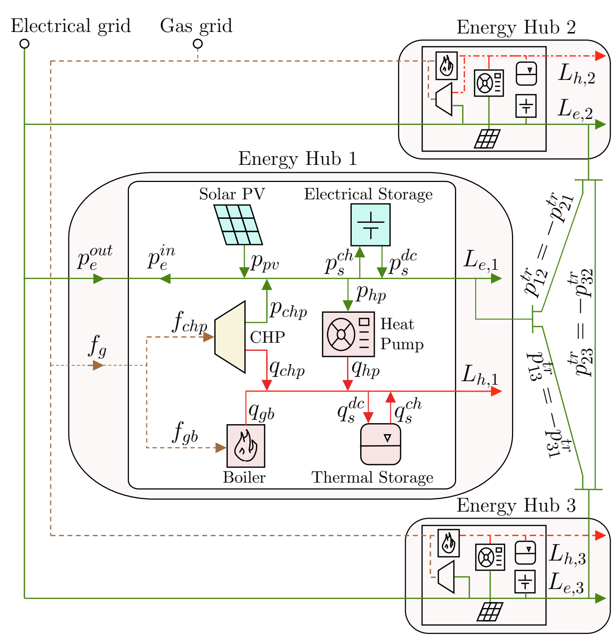

Fig. 1. A network of three interconnected energy hubs. Each hub can import energy from the electricity (green) and gas (brown) grids, and can feed-in electricity to the electricity grid; additionally, each hub can also trade electrical energy with the other hubs.

grid and additionally excess electrical energy produced in the hub can also be fed to the grid. The hubs are connected to the heating and electricity demand via a downstream distribution network. In this study, we assume that a perfect forecast is available for this demand as well as for the temperature and the solar radiation. In the next sections, we provide an overview of the models used for the components in the energy hubs.

Energy Converters: The CHP simultaneously generates heat, $q_{\mathrm{chp,i}}$, and power $p_{\mathrm{chp,i}}$ using natural gas f_{chp} based on its feasible operation region (Navarro et al., 2018). HP extracts heat, $q_{\mathrm{hp,i}}$, from the ground using electricity, $p_{\mathrm{hp,i}}$, and similarly, GB uses natural gas, $f_{\mathrm{gb,i}}$, to generate heat, $q_{\mathrm{gb,i}}$. The electrical energy output of the PV at any time is $p_{\mathrm{pv,i}}$. The output of all energy converters are also limited by capacity constraints. A detailed model of all the components and their corresponding equations are provided in an extended version of this work (Behrunani et al., 2023).

Energy Storage: The dynamics of the electrical storage (ES) is modelled by the discrete-time LTI system:

$$p_{\mathrm{s,i}}(h) = \gamma_{\mathrm{e,i}} \cdot p_{\mathrm{s,i}}(h-1) + \eta_{\mathrm{e,i}} \cdot p_{\mathrm{s,i}}^{\mathrm{ch}}(h) - \left(\frac{1}{\eta_{\mathrm{e,i}}}\right) \cdot p_{\mathrm{s,i}}^{\mathrm{dc}}(h), \quad (1)$$

$$p_{\mathrm{j,i}}^{\min} \leq p_{\mathrm{j,i}} \leq p_{\mathrm{j,i}}^{\max} \quad \mathrm{j} = \{\mathrm{s, dc, ch}\}. \quad (2)$$

where $h \in \mathcal{Z}_{\geq 0}$ is the time index, p_{s} is the battery state of charge, $p_{\mathrm{s,n}}^{\mathrm{dc}}$ and $p_{\mathrm{s}}^{\mathrm{ch}}$ are the energy discharged and charged into the battery, and γ_{e} and η_{e} are the standby and cycle efficiency. Additionally, (2) ensures that the storage levels also satisfies the battery capacity limits.

The heat storage (TS) dynamics are modelled analogously with the corresponding state of charge, q_{s}, the heat discharged and charged into the thermal storage, $q_{\mathrm{s}}^{\mathrm{dc}}$ and $q_{\mathrm{s}}^{\mathrm{ch}}$, and standby and cycle efficiency, γ_{h} and η_{h}, respectively.

Network Modelling: The network and internal connections describe the input-output equations of different energy carriers. For hub i, the energy balance constraint of electricity and heat read:

$$L_{\mathrm{e,i}} = p_{\mathrm{chp,i}} + p_{\mathrm{pv,i}} - p_{\mathrm{hp,i}} + \left(p_{\mathrm{e,i}}^{\mathrm{out}} - p_{\mathrm{e,i}}^{\mathrm{in}}\right) + \left(p_{\mathrm{s,i}}^{\mathrm{dc}} - p_{\mathrm{s,i}}^{\mathrm{ch}}\right) + \sum_{j \in \mathcal{N} \setminus \{i\}} p_{ij}^{\mathrm{tr}}, \quad (3)$$

$$L_{\mathrm{h,i}} = q_{\mathrm{chp,i}} + q_{\mathrm{gb,i}} + q_{\mathrm{hp,i}} + \left(q_{\mathrm{s,i}}^{\mathrm{dc}} - q_{\mathrm{s,i}}^{\mathrm{ch}}\right), \quad (4)$$

where $L_{\mathrm{e,i}}$ and $L_{\mathrm{h,i}}$ are the electrical and thermal demand of the energy hub i respectively, and $p_{\mathrm{e,i}}^{\mathrm{out}}$ and $p_{\mathrm{e,i}}^{\mathrm{in}}$ are the energy imported and fed into electricity grid respectively; The energy hubs can also trade electrical energy amongst each other. The power traded between hub i and hub j is p_{ij}^{tr}. The value is positive if the energy is imported by the hub i and negative otherwise. While a district heating network may also be present, it is not included here. In the absence of the grid, we assume demand can be met exactly at all times by conversion or storage.

Trading agreement is enforced via the so-called reciprocity constraints (Baroche et al., 2019b)

$$p_{ij}^{\mathrm{tr}} + p_{ji}^{\mathrm{tr}} = 0, \quad \forall i, j \in \mathcal{N}, \; i \neq j, \quad (5a)$$

$$p_{ij}^{\mathrm{tr}} \leq \kappa_{ij}, \quad \forall i, j \in \mathcal{N}, \; i \neq j, \quad (5b)$$

The reciprocity constraints (5a) ensures that the energy exported from hub i to hub j is the same as the energy imported by hub j from hub i; additionally, the constraints (5b) limit the trade between hubs where κ_{ij} is the maximum that can be traded between the hubs i and j. Specific trading network topologies can be defined by restricting some of the trading limits (5b) to zero.

3. AUTONOMOUS P2P TRADING

3.1 Economic Dispatch as a Noncooperative Game

For each hub, the economic dispatch problem consists of choosing the local operational set points p_i, over a horizon $\mathcal{H} := \{1, \ldots, H\}$, to minimize its energy cost. The cost of each hub $i \in \mathcal{N}$ is the sum over all costs of its available assets (including the energy exchanged with the electricity grid), $\ell \in \mathcal{A}_i = \{\text{chp, gb, gshp, pv, grid}\}$, and the bilateral trades with the other hubs in the network.

We model the cost of each asset $\ell \in \mathcal{A}_i$ as a strongly convex function $f_i^\ell(p_i^\ell)$, where $p_i^\ell \in \mathbb{R}^H$ is the vector of setpoints of asset ℓ over the horizon $\mathcal{H}$. Typical choices in the literature are quadratic and linear functions (Baroche et al., 2019a; Moret et al., 2020; Le Cadre et al., 2020). The total cost of bilateral trades with hub j is given by

$$c_{(i,j)}^\top p_{ij}^{\text{tr}} + \gamma \|p_{ij}^{\text{tr}}\|_2^2, \tag{6}$$

where $p_{ij}^{\text{tr}} \in \mathbb{R}^H$ collects the trades with hub j over $\mathcal{H}$, $c_{(i,j)} \in \mathbb{R}^H$ defines the prices of the bilateral trades with hub j, while γ is a marginal trading tariff imposed by the grid operator to use the network for bilateral trades.

Overall, the economic dispatch problem of each hub i can be compactly written as the following convex program:

$$\min_{p_i} \quad \overbrace{\sum_{\ell \in \mathcal{A}_i} f_i^\ell\left(p_i^\ell\right) + \sum_{j \in \mathcal{N}} \left(c_{(i,j)}^\top p_{ij}^{\text{tr}} + \gamma \|p_{ij}^{\text{tr}}\|_2^2\right)}^{=: J_i(p_i, c_i)} \tag{7a}$$

$$\text{s.t.} \quad p_i \in \mathscr{P}_i \tag{7b}$$

$$p_{ij}^{\text{tr}} + p_{ji}^{\text{tr}} = 0, \quad \forall j \in \mathcal{N} \setminus \{i\}, \tag{7c}$$

where p_i collects all the decision variables of hub i (local set points p_i^ℓ, and import/exports from/to other hubs p_{ij}^{tr}), and $\mathscr{P}_i := \{p_i \mid (2) - (5\text{b}) \text{ hold}\}$ all its operational constraints; finally, the cost function $J_i(p_i, c_i)$ combines the costs of all the local assets and the bilateral trades, and its parametric dependency on the bilateral trading prices $c_i = (c_{(i,1)}, \ldots, c_{(i,N)}) \in \mathbb{R}^{(N-1)H}$ is made explicit.

Note that the economic dispatch problems (7) are coupled via the reciprocity constraints (7c) that enforce agreement on the bilateral trades. Therefore, the collection of N parametric interdependent optimization problems (7) constitutes a game with coupling constraints (Facchinei and Kanzow, 2010). A relevant solution concept for the game (7) is that of generalized Nash equilibrium (GNE). Namely, a feasible action profile $p^* = (p_1^*, \ldots, p_N^*)$ for which no agent can reduce their cost by unilaterally deviating (Belgioioso et al., 2022b, Def. 1). Here, we focus a special subclass of GNEs known as variational GNEs (v-GNEs) and characterized by the solution set, $S(c)$, of the variational inequality $\text{VI}(F(\cdot, c), \mathscr{P})$. Namely, the problem of finding a vector $p^* \in \mathscr{P}$ such that

$$F(p^*, c)^\top (p - p^*) \geq 0, \quad \forall p \in \mathscr{P}, \tag{8}$$

where $\mathscr{P} := \{p \mid (7\text{b}) - (7\text{c}) \text{ hold for all } i \in \mathcal{N}\}$, and F is the so-called pseudo-gradient mapping, defined as

$$F(p, c) = \text{col}(\nabla_{p_1} J_1(p_1, c_1), \ldots, \nabla_{p_N} J_N(p_N, c_N))$$

and parametrized by the bilateral trading prices c. This subclass of GNEs enjoys the property of "economic fairness", namely, the shadow price (dual variable) due to the presence of the coupling reciprocity constraints is the same for each hub (Facchinei and Kanzow, 2010).

Interestingly, since the cost functions J_i are decoupled, the variational GNEs of (7) correspond to the minimizers of the social cost problem

$$\left\{ \min_p \sum_{i \in \mathcal{N}} J_i(p_i, c_i) \quad \text{s.t.} \quad p \in \mathscr{P} \right\} =: W(c). \tag{9}$$

This equivalence directly follows by comparing the KKT conditions of $\text{VI}(F(\cdot, c), \mathscr{P})$ with that of the social cost problem (9), and was noted in a number of different works (Le Cadre et al., 2020; Moret et al., 2020). In the remainder, we exploit this connection in a number of ways.

First, we prove that the trading game (7) has a unique variational GNE which does not depend on the specific choice of the bilateral trading prices.

Lemma 1. Let $S(\cdot)$ be the price-to-variational GNE mapping, i.e., $S(c) = \{p^* \mid F(p^*, c)^\top (p - p^*) \geq 0, \ \forall p \in \mathscr{P}\}$. Then, $S(c) = \{p^*\}$ for all price profiles $c \in \mathbb{R}^{N(N-1)H}$, for some unique profile $p^* \in \mathscr{P}$ independent of c.

Proof: A formal proof of the equivalence between $S(c)$ and the solution set to the social cost problem (9), $S^{\text{sc}}(c)$, can be found in (Le Cadre et al., 2020). Here, we show that $S^{\text{sc}}(c)$ has a unique element independent of c. First, note that the objective functions $J_i(p_i, c_i)$ are decoupled (namely, depend only on local decision variables) and are strongly convex, for all c_i. Hence, (9) has a unique solution $p^*(c)$, that is, $S^{\text{sc}}(c) = \{p^*(c)\}$. Next, we show that $p^*(c)$ is the same for all c. Note that, each pair of twin trading terms in $J_i(p_i^*(c_i), c_i)$ and $J_j(p_j^*(c_j), c_j)$, satisfy $c_{(i,j)}^\top p_{ij}^{*,\text{tr}} = -c_{(i,j)}^\top p_{ji}^{*,\text{tr}}$, due to the reciprocity constraints (7c). Hence, these terms cancel out when summed up in the objective function (9), for any choice of $c_{(i,j)}$. It follows that the minimizers of (9) do not depend on c. □

To find a variational GNE of (7), a number of different methods are available that exploit the optimization reformulation in (9), e.g. (Baroche et al., 2019a). For example, a fully-distributed algorithm can be obtained by relying on the consensus version of the alternating direction method of multipliers. The resulting algorithm and its detailed derivation are provided in the extended version of this work (Behrunani et al., 2023).

3.2 Undesired Effects of Autonomous P2P Trading

In this section, we show that this autonomous peer-to-peer trading model provably leads to a social cost decrease, but not necessarily to a local cost decrease for each hub.

The economic dispatch problem without bilateral trading corresponds to (7) with $p_{ij}^{\text{tr}} = 0$ for all hubs. Clearly, the corresponding social cost, $W^{\text{nt}}(c)$, will be greater than $W(c)$ in (9), since the feasible set of the non-trading scenario $\mathscr{P}^{\text{nt}} = \mathscr{P} \cap \{p \mid p_{ij}^{\text{tr}} = 0, \ \forall j \in \mathcal{N} \setminus \{i\}, \forall i \in \mathcal{N}\}$ is a subset of the feasible set with bilateral trades $\mathscr{P}$. Hence, allowing bilateral trades cannot increase the social cost, regardless of the trading prices.

The decrease in social cost is not necessarily reflected in the individual costs of all agents. In other words, there may exist profiles of bilateral trading prices for which certain hubs are worse off than if they were not participating in the autonomous trading mechanism. We illustrate this phenomenon via a numerical example for a 3-hub network in Section 5. A natural question is whether there exists a price profile for which all the hubs benefit by participating in the autonomous bilateral trading mechanism. The following theorem gives an affirmative answer to this question.

Theorem 1. Let p^* be the unique variational GNE of (7). There exists a price profile $c^* \in \mathbb{R}^{N(N-1)H}$ such that

$$J_i(p_i^*, c_i^*) \leq J_i^{\text{nt}}, \quad \forall i \in \mathcal{N}, \tag{10}$$

where J_i^{nt} are the local costs of the non-trading scenario.

Proof: The detailed proof can be found in an extended version of this work (Behrunani et al., 2023).

In the following section, we develop a scalable mechanism to identify bilateral trading prices that are not only locally beneficial for each hub but also fair.

4. DESIGNING FAIRNESS VIA BILEVEL GAMES

To ensure that the equilibrium of the autonomous peer-to-peer trading game (7) is "fair", we set the prices of the bilateral trades by solving the following bilevel game

$$\min_{c,\,p} \quad \varphi(p, c) \tag{11a}$$

$$\text{s.t.} \quad c \in \mathcal{C}, \tag{11b}$$

$$p \in S(c), \tag{11c}$$

where $\mathcal{C}$ is a set of feasible trading prices that can be used to model price regulations, such as capping, and S as in Lemma 1. The lower level (11c) imposes the operational setpoints and the bilateral trades p to be in the variational GNE set $S(c)$ of the price-parametrized game (7). At the higher level, the optimal trading prices are chosen to minimize a certain *fairness metric* φ which depends also on the operational setpoints of the lower level.

Here, we define the fairness metric as the sample variance of the normalized cost reductions d_i achieved by enabling peer-to-peer trading between hubs, namely

$$\varphi(p, c) = \sum_{i \in \mathcal{N}} \left(d_i(c_i, p_i) - \frac{1}{N} \sum_{j \in \mathcal{N}} d_j(c_j, p_j) \right)^2, \tag{12}$$

where the normalized cost reduction d_i is defined as

$$d_i\left(c_i, p_i\right) = \left(J_i^{\text{nt}} - J_i\left(p_i, c_i\right)\right)/J_i^{\text{nt}}, \quad \forall i \in \mathcal{N}. \tag{13}$$

The non-trading cost J_i^{nt} can be locally computed by each hub by solving their optimal economic dispatch problem in which trading is disabled. This metric ensures that the social wealth generated by enabling peer-to-peer trading is "fairly" distributed across the hubs.

Next, we show that a solution to (11) can be efficiently obtained by solving sequentially the game (11c) and the optimization (11a). By Lemma 1, the v-GNEs set $S(c)$ is a singleton, independent on the bilateral trading prices c. Hence, the bilevel game (11) boils down to

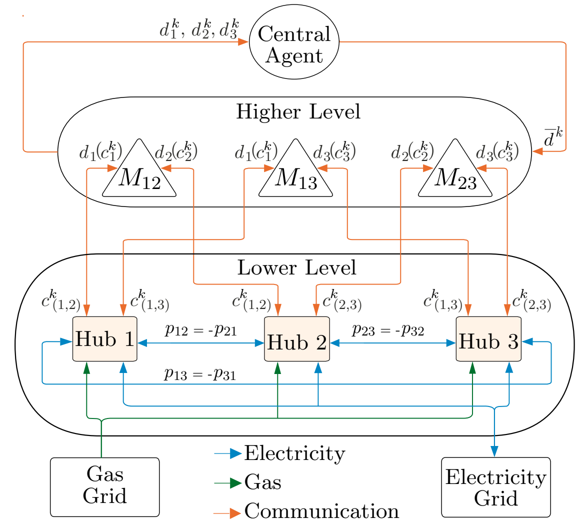

Fig. 2. Flowchart of the price mediation scheme in Alg. 2.

$$\min_{c} \quad \varphi\left(p^*, c\right) \quad \text{s.t.} \quad c \in \mathcal{C}, \tag{14}$$

where p^* is the unique price-independent v-GNE of (7). Hence, a solution to (11) can be found in two steps:

1. Compute the unique v-GNE p^* of (7), for any fixed c;
2. Compute a solution to (14), where p^* is fixed.

Solving (14) centrally requires global knowledge over the optimal operational setpoints $p^* = (p_1^*, \ldots, p_N^*)$, the cost functions J_i, and the non-trading cost, J_i^{nt}, which is unrealistic. Additionally, the dimensionality of (14) increases quadratically with the number of hubs, thus making it rapidly intractable for large-scale hub networks. Motivated by these challenges, in the next section, we design a scalable and privacy-preserving algorithms to solve (14).

4.1 A Semi-decentralized Price Mediation Protocol

We design a semi-decentralized projected-gradient algorithm which preserves privacy and achieve scalability with respect to the number of hubs. To distribute the computation, a mediator, $M_{(i,j)}$, is introduced between each pair of hubs i and j, whose objective is to determine a fair trading price, $c_{(i,j)}$. The mediator updates the price according to

$$c_{(i,j)}^{k+1} = \Pi_{\mathcal{C}}\left(c_{(i,j)}^{k} - \beta \frac{\partial \varphi(p^*, c^k)}{\partial c_{(i,j)}} \right), \tag{15}$$

where $\Pi_{\mathcal{C}}$ is the projection onto a feasible set of prices $\mathcal{C}$. The gradient of $\varphi(p, c)$ with respect to a price $c_{(i,j)}$ is

$$\frac{\partial \varphi(p, c)}{\partial c_{(i,j)}} = \frac{2}{N} \left(\frac{p_{ij}^{\text{tr}}}{J_i^{\text{nt}}} \left(d_i(c_i, p_i) - \bar{d}\left(c, p\right) \right) + \frac{p_{ji}^{\text{tr}}}{J_j^{nt}} \left(d_j\left(c_j, p_j\right) - \bar{d}\left(c, p\right) \right) \right)$$

where $\bar{d}(c, p) = \frac{1}{N} \sum_{i=1}^{N} d_i\left(c_i, p_i\right)$ is the average cost reduction. A central coordinator gathers the $\bar{d}_i$ and broadcasts $\bar{d}$ to all the mediators resulting in a semi-decentralized structure (Belgioioso and Grammatico, 2023).

The resulting algorithm is summarized in Algorithm 2 and its information structure is illustrated in Figure 2. At every iteration, each mediator M_{ij} receives the normalized cost reductions, d_i and d_j, from the hubs it manages, and the

ALGORITHM 2: Price Mediation Mechanism

Initialization ($k = 0$): $c^0_{(i,j)} = 0$, for all trades (i, j).
Iterate until convergence:

For all mediators M_{ij}:

1. Receives $d_\ell(c^k_\ell)$ from each hub $\ell \in \{i, j\}$,
2. Receives $\bar{d}^k$ from the coordinator,
3. Update price $c^{k+1}_{(i,j)}$ as in (15),

Coordinator:

1. Gather $\bar{d}^{k+1} = \sum_{i \in \mathcal{N}} d^k_i (c_k)$,
2. Broadcast $\bar{d}^{k+1}$ to all mediators,

$k \leftarrow k + 1$

average network cost reduction $\bar{d}$ from the coordinator. Then, it updates the price $c_{(i,j)}$ with a projected-gradient step (15). This process continues until the prices of all trades reach convergence. Since the objective function $\varphi(p, c)$ is convex and L-smooth [1], for some $L > 0$, uniformly in p, taking the step size $\beta \in (0, 2/L)$, in the price update (15), guarantees convergence of Algorithm 2.

5. NUMERICAL RESULTS

We illustrate the proposed price mediation mechanism on an example 3-hub network. The configuration, parameters and capacities for the three hubs used are available in the extended version (Behrunani et al., 2023). The demand profiles of hubs are based on buildings in the village Rolle, Switzerland, and the hubs were designed using the energy hub modelling tool (Muriel et al., 2020). Large-scale simulations with more hubs is left as a topic of future work. In general, the cost functions for electricity and gas are linear and the v-GNE is not unique. In this study, an additional regularization term that minimizes the total energy imported is added to the cost to find a unique solution. Alternatively, selection algorithms can be used instead of regularization to handle the non-uniqueness of the v-GNE (Ananduta and Grammatico, 2022). The price of input energy carriers (electricity and gas) are and for utilizing the electricity grid are summarized in Table 1. We solve the optimization for a horizon $H = 24$ h with a sampling resolution of 1 h. The electricity demand and PV production for the three hubs over a span of 24 hours are shown in Figures 3(a) and (b), respectively, thereby illustrating the relative sizes and capacities of the hubs.

5.1 The impact of autonomous peer-to-peer trading

First, we compare the performance of the system without and with autonomous peer-to-peer energy trading. All the bilateral trades and the operational setpoints of the converters are calculated by solving (7) using the distributed ADMM algorithm in (Behrunani et al., 2023). The results are illustrated in Figure 4(a). In the absence of PV production, power is traded from the larger Hub 1 to Hub 2 and Hub 3, owing to its larger production capacity and CHP to

[1] The convexity of $\varphi(\cdot, p)$ can be proven by showing that its Hessian is positive semidefinite. A formal proof is omitted here due to space limitation; smoothness follows since $\phi(\cdot, p)$ is quadratic.

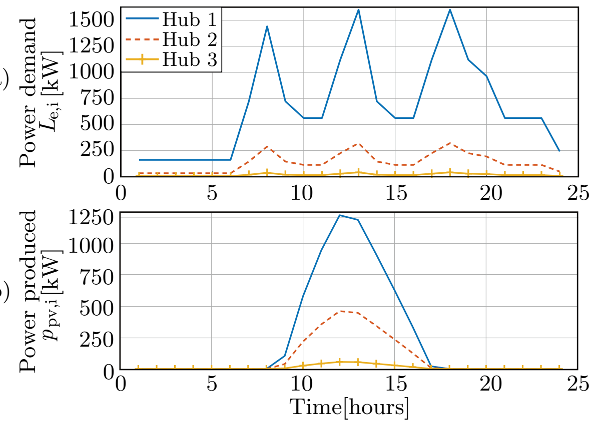

Fig. 3. Electric power demand, $L_{e,i}$, and the PV production, $p_{pv,i}$, for the three hubs($i = 1, 2, 3$) over 24 hours.

Tariff	Price(CHF/kW)
Electricity output	0.22
Electricity feed-in	0.12
Gas	0.115

Table 1. Tariffs for electricity and gas utility.

produce electricity. In the absence of trading, Hub 2 and Hub 3 import electricity from the grid at a higher price than the price of producing the energy using CHP. Part of the power traded from Hub 1 to Hub 3 is transferred to Hub 2 to minimize the quadratic grid tariff levied to the hubs. When PV output is high, the trades drop to 0 as the output is sufficient to fulfill the local demand and the cost is equivalent to that in the non-trading case.

5.2 The impact of the bilateral trading prices

We verify that the optimal power traded between the hubs and the optimal setpoints, p^*, are independent of the trading price (Lemma 1) by solving (7) with different trading prices. Figure 5 shows the cost reduction (13) achieved by each hub for three different trading prices (uniform across all peer-to-peer trades), namely, $c = 0.1, 0.18, 0.2$ CHF/kWh. The figure also shows the reduction in the social cost compared to when no trading occurs, and the benefit of autonomous trading to the hub network is evident by the 2.5% reduction of social cost, independently of the trading price. For $c = 0.1$ CHF/kWh, since the trading price is low (even lower than the feed-in tariff), Hub 2 and Hub 3 benefit by trading as they import cheap energy from Hub 1. However, this results in an increase of the cost for Hub 1, as the trading price does not cover the additional production costs of the power traded to the other hubs. For $c = 0.18$ CHF/kWh, although each of the hubs benefits from trading, the cost reduction varies drastically between the hubs. Finally, for $c = 0.2$ CHF/kWh, Hub 1 and Hub 2 continue to benefit whereas the smaller Hub 3 loses since the higher trading price for import and the trading tariff increase the net price to more than the grid price.

Finally, the trading prices are calculated using the price mediation mechanism in Algorithm 2. The resulting cost reduction for the three hubs using the optimal price profile is shown in Figure 5. Interestingly, the trading price is different for each trade and also varies at different times

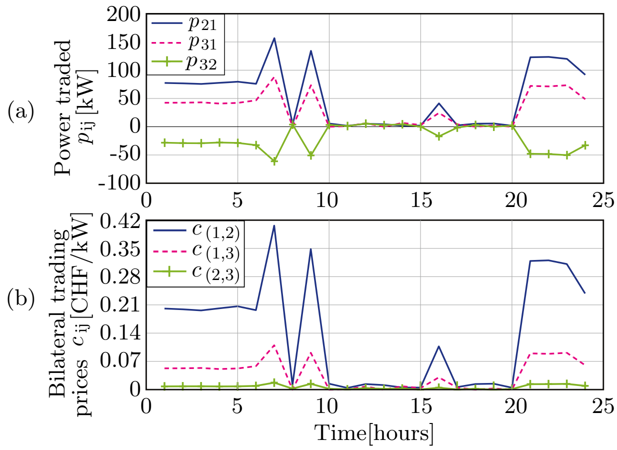

Fig. 4. (a) Bilateral power traded and (b) trading price for the bilateral trades in the 3-hub network.

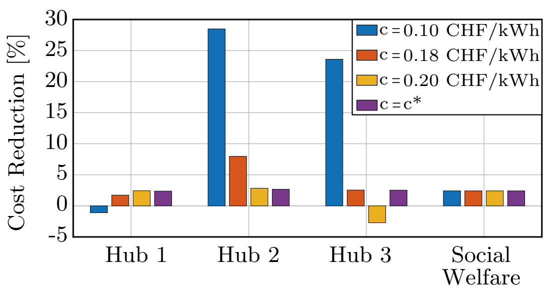

Fig. 5. Cost reduction (13) for three different trading prices and the optimized prices c^* illustrated in Figure 4(b).

within the control horizon (shown in Figure 4(b)). The normalized cost reduction achieved by each of the hubs is nearly equal and matches the social cost reduction achieved by the network.

6. CONCLUSION

Energy trading prices play a major role in incentivizing participation in autonomous peer-to-peer trading mechanisms. We proposed a privacy-preserving and scalable price-mediation algorithm that provably finds price profiles that are not only locally-beneficial for each hub but also network-wide fair. Numerical simulation on a 3-hub network supported this theoretical result.

REFERENCES

Ananduta, W. and Grammatico, S. (2022). Equilibrium seeking and optimal selection algorithms in peer-to-peer energy markets. *Games*, 13(5), 66.

Baroche, T., Moret, F., and Pinson, P. (2019a). Prosumer markets: A unified formulation. In *2019 IEEE Milan PowerTech*, 1–6. IEEE.

Baroche, T., Pinson, P., Latimier, R.L.G., and Ahmed, H.B. (2019b). Exogenous cost allocation in peer-to-peer electricity markets. *IEEE Transactions on Power Systems*, 34(4), 2553–2564.

Behrunani, V., Irvine, A., Belgioioso, G., Heer, P., Lygeros, J., and Dörfler, F. (2023). Designing fairness in autonomous peer-to-peer energy trading. *arXiv preprint arXiv:2302.04771*.

Belgioioso, G., Ananduta, W., Grammatico, S., and Ocampo-Martinez, C. (2022a). Operationally-safe peer-to-peer energy trading in distribution grids: A game-theoretic market-clearing mechanism. *IEEE Transactions on Smart Grid*, 13(4), 2897–2907.

Belgioioso, G. and Grammatico, S. (2023). Semi-decentralized generalized Nash equilibrium seeking in monotone aggregative games. *IEEE Transactions on Automatic Control*, 68(1), 140–155.

Belgioioso, G., Yi, P., Grammatico, S., and Pavel, L. (2022b). Distributed generalized Nash equilibrium seeking: An operator-theoretic perspective. *IEEE Control Systems Magazine*, 42(4), 87–102.

Cui, S., Wang, Y.W., Shi, Y., and Xiao, J.W. (2020). A new and fair peer-to-peer energy sharing framework for energy buildings. *IEEE Transactions on Smart Grid*, 11(5), 3817–3826.

Daryan, A.G., Sheikhi, A., and Zadeh, A.A. (2022). Peer-to-peer energy sharing among smart energy hubs in an integrated heat-electricity network. *Electric Power Systems Research*, 206, 107726.

Facchinei, F. and Kanzow, C. (2010). Generalized Nash equilibrium problems. *Ann. Oper. Res.*, 175(1), 177–211.

Fan, S., Li, Z., Wang, J., Piao, L., and Ai, Q. (2018). Cooperative economic scheduling for multiple energy hubs: A bargaining game theoretic perspective. *IEEE Access*, 6, 27777 – 27789.

Le Cadre, H., Jacquot, P., Wan, C., and Alasseur, C. (2020). Peer-to-peer electricity market analysis: From variational to generalized Nash equilibrium. *European Journal of Operational Research*, 282(2), 753–771.

Moret, F., Tosatto, A., Baroche, T., and Pinson, P. (2020). Loss allocation in joint transmission and distribution peer-to-peer markets. *IEEE Transactions on Power Systems*, 36(3), 1833–1842.

Muriel, B., Perera, A.T.D., Hanmin, C., Bollinger, A., and Orehounig, K. (2020). Improving energy sustainability of suburban areas by using distributed energy systems: A case study. *Proceedings*, 58(1).

Navarro, J.P.J., Kavvadias, K.C., Quoilin, S., and Zucker, A. (2018). The joint effect of centralised cogeneration plants and thermal storage on the efficiency and cost of the power system. *Energy*, 149, 535–549.

Sorin, E., Bobo, L., and Pinson, P. (2019). Consensus-based approach to peer-to-peer electricity markets with product differentiation. *IEEE Transactions on Power Systems*, 34(2), 994–1004.

Tushar, W., Saha, T.K., Yuen, C., Smith, D., and Poor, H.V. (2020). Peer-to-peer trading in electricity networks: An overview. *IEEE Transactions on Smart Grid*, 11(4), 3185–3200.

Wang, X., Liu, Y., Liu, C., and Liu, J. (2020). Coordinating energy management for multiple energy hubs: From a transactive perspective. *International Journal of Electrical Power & Energy Systems*, 121, 106060.