Buckling of polymer pipes under internal pressure

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In this work, the buckling phenomenon of slender pressurized pipes under concentrically applied indirect axial force is experimentally and theoretically investigated. The axial force is applied via a frictionless piston. The deformation trend of a polyvinylchloride pipe sample under this compressive axial force is monitored by means of a number of strain gauges. The strain readings are processed by a computer equipped with strain processing software. The buckling modes as well as the buckling load of the system are thus experimentally determined. To describe the stability behaviour of the pipe analytically, several theoretical models are employed and the buckling phenomenon in such a pressurized pipe is analytically ascertained. Numerical values for the critical buckling loads are also obtained. The experimental and theoretical buckling studies are correlated. Through these correlations, the buckling of pressurized pipes under indirect axial pressure is substantiated and an appropriate theoretical model to describe the phenomenon is established.

1. INTRODUCTION

Under certain loading and boundary conditions, the straight form of pressurized slender tubes and pipes could become unstable. The flutter instability of fluid conveying pipes, reported in the literature, is a dynamic mode of instability of pipes containing internal pressurized fluid. In addition to this mode of instability, the pressurized pipes could undergo a static buckling.

The conditions under which the buckling mode of a straight pressurized pipe could occur consist of subjecting a fluid-filled pipe to an axial loading which is exerted on the internal fluid by means of a frictionless piston. This so-called 'ram effect' would keep the pipe under internal pressure without inducing any longitudinal stresses in the pipe wall. For this condition to exist, there should be no friction between the piston and the pipe wall. With an increase in the magnitude of applied axial force, the internal pressure in the pipe would also increase, but the pipe would still remain straight. With further increase in the axial load, a stage is reached at which the straight equilibrium configuration of the pipe would become unstable and, instead, a bent configuration would be stable. This is the buckling of a pressurized pipe.

The buckling of pressurized polymer pipes has, in fact, been observed in the pipes subjected to indirect axial water shock loading [1]. In those experiments, water-filled polyethylene pipes were subjected to axial pressure shock induced by a heavy weight falling on a piston. Consequently, an overall buckling of the polymer pipe followed by bulging of the compression zone had occurred.

In the available literature on elastic instability, theoretical studies have been carried out to determine the critical buckling loads of slender pressurized cylinders. The simplest model is the Euler theory of axially loaded columns. Refinements have also been introduced to take into account the influence of internal

pressure on the buckling behaviour of pipes. Experimental verification of this instability phenomenon and the related theories is, however, not available.

Knowledge of the buckling phenomenon of pressurized pipes is of interest in the polymer pipe industry, where the piping system is sometimes exposed while being subjected to pressure gradients.

In this paper, an experimental and theoretical investigation on the buckling of pressurized polymer pipes under axial ram loading is carried out. The purpose of this study is to investigate the occurrence of buckling in polymer pipes under pressure gradients. To achieve a sound theoretical correlation, various simple buckling models of pressurized pipes under indirect axial force will be developed. An attempt will be made to correlate the experimental data with the theoretical predictions.

2. EXPERIMENTAL PROCEDURE

2.1 Testing apparatus

The apparatus for the buckling testing of pressurized pipes consisted of an Instron testing machine. This machine was calibrated for a maximum axial loading of 10 kN. To monitor and record the trend of straining of the pipe, a computer equipped with software called BEAM was used. This software could process the changing pattern of strains in the pipe with the variation of applied loading. Altogether, five strain gauges were used and their records were processed by this system. The incoming signals were amplified by a digital amplifier.

2.2 Material and pipe properties

The pipe specimen was composed of polyvinylchloride (PVC). The modulus of elasticity of this material, needed

in the theoretical modelling, has been obtained by standard static and/or dynamic (resonance and wave propagation) methods. In previous research in the area of the mechanics of polymeric bodies, the elastic properties of PVC pipes were determined by a non-destructive wave propagation method [2]. Although the elastic properties of polymeric materials show some spatial variations, such spatial changes do not seem to have a primary influence on the global instability of the pipe. The investigation cited resulted in a dynamic value of $E = (3700 \pm 200) \text{ N mm}^{-2}$. For unplasticized PVC material, values of the statically determined elastic modulus falling in the range of 2600 to 3300 N mm^{-2} have been reported in the literature.

2.3 Sample preparation and instrumentation

The geometrical features of the tested pipe are shown in Fig. 1; these consisted of:

- Outer diameter, $D = 66 \pm 3 \text{ mm}$
- Pipe wall thickness, $T = 4.1 \pm 0.1 \text{ mm}$
- Pipe length, $L = 1210 \text{ mm}$
- Distance between two end hinges, $H = 1481 \text{ mm}$

Attempts were made to prepare and test a relatively slender pipe, so that the buckling failure would be distinguished from material failure and would occur at a relatively early stage of loading.

Fig. 1 (a, b) Schematic representations of a simply supported pressurized pipe under indirect concentric axial force, showing the location of strain gauges, pipe dimensions, lower cap, and the upper piston.

Of prime importance in the present investigation was the testing of the hypothesis that the buckling of pressurized pipes under indirect axial loading could occur. It was, therefore, quite essential that the axial force was applied to the contained fluid and not to the pipe wall. As the contained fluid in this set of experiments, a water medium was chosen.

To simulate the condition of a pressurized pipe loaded by indirect axial pressure, special end fittings were designed and constructed. As the boundary conditions, simply supported ends were conceived. So, at the lower end, a frictionless hinge capable of allowing planar rotations was devised. The condition at the upper end, i.e. at the end of load application, was not, however, so simple. The upper cap should fulfil all following requirements: (i) to act as a planar hinge, (ii) to be water-tight; and (iii) to transfer the axial load directly to the contained fluid without causing any stresses in the upper end.

To record the deformation of the pipe, a strain measuring system composed of five strain gauges was employed. Two of these gauges were longitudinally installed on the outer surface of the pipe at the end joints. Two other longitudinally installed strain gauges were located diagonally at the mid-length of the pipe segment. The fifth strain gauge was mounted along the hoop direction at the mid-length section.

With the longitudinally mounted end gauges the existence (or absence) of axial stress in the pipe wall at the end hinged section could be ascertained. Two longitudinal gauges at the mid-length would yield information about the flexural buckling of an initially straight pipe. Finally, the hoop strain gauge would give the hoop stress and, therefore, the internal pressure in the pipe. Fig. 1a shows the pipe with the mounted strain gauges. Fig. 2 depicts the instrumented pipe in the Instron testing machine. In this figure, the end caps and the mounted strain gauges are also shown. The instrumented pipe was linked to a computer as described above.

2.4 Testing procedure

The pipe buckling experiments were performed on segments of slender water-filled PVC pipes subjected to an axial force which acted on a relatively frictionless piston. Through a proper ram design, an attempt was made to eliminate possible friction between the end piston and the pipe wall. With such a design the axial load would be applied, by means of the end piston, directly to the internal fluid and not to the pipe wall.

The instrumented pipe was filled with tap water at room temperature. The required volume of water was calculated and measured while pouring into the pipe. The end piston was slowly placed on the water-filled pipe.

The instrumented pipe was subjected to pseudo-statically increasing axial force. Experiments were performed with two distinct rates of load increase. The faster load increase had a rate of 800 mm s^{-1} , while the slower load had a rate of 200 mm s^{-1} . The axial load was increased until overall lateral buckling of the pipe

Fig. 2 Photograph of the instrumented pipe mounted on the testing machine.

Fig. 3 Photograph of the buckled pipe.

occurred. In both cases of applied axial loading, buckling of the pipe occurred under the same value of the applied force. Fig. 1b shows a schematic diagram of the buckled shape of the water-filled pipe. In the course of the experiments, the strain gauge readings were continuously monitored and recorded as described above.

The buckling test was performed beyond the buckling stage and ran into the post-buckling region. To check the elastic behaviour of the pipe, the buckled sample was unloaded at the same rate as for loading. The unloading curve coincided with the loading graph, thus indicating that virtually no inelastic deformation had occurred.

3. EXPERIMENTAL RESULTS

In Fig. 3, a photograph of the buckled pipe is presented. This photograph was taken at a post-buckling stage of the pipe behaviour. Fig. 4 shows one of the recorded force-deformation diagrams. In this graph, variation of applied axial force with the measured axial strain at the locations identified by strain gauge number is presented. As we observe, the load-deformation curve has a peak point beyond which a marked drop in the force required to maintain the equilibrium of the 'buckled pipe' is observed. The axial force corresponding to this peak point denotes the buckling load of the pressurized pipe.

Fig. 4 Variation of measured longitudinal strains with applied axial force.

The value of this critical buckling load is read from Fig. 4 to be 8.4 kN.

In Fig. 4 all longitudinal strains have similar trends, while the trend of hoop strain variation is markedly distinct from those of other strains. We also note that beyond a certain load value, the hoop strain readings are no longer linear but have a constant value. This is obviously an artefact, and can be attributed to malfunction of the hoop strain gauge in this range of loading.

In processing the experimental data, it would be useful to calculate the value of internal water pressure at the buckling stage. The internal pressure can be obtained by the following simple formula:

$$p = \frac{F}{\pi r^2}$$

where F is the axial force exerted on the fluid by means of the piston. The radius of the piston was assumed to be the same as the inner radius of the pipe. The common value was denoted by r . The experimentally determined magnitude of internal pressure at the buckling stage, obtained by this formula, is 3.2 N mm^{-2} .

In Fig. 5, variations of the axial force with the lateral deflection at the mid-length of the pipe is plotted. The bending deflection of the pipe at its mid-length is calculated from two longitudinal strain gauges located diagonally at the mid-length section. To calculate the central deflection, Δ , an appropriate relation has been derived in the following fashion. If ε_1 and ε_2 denote the longitudinal strains at the mid-length of the pipe, then the bending strain at the mid-length of the pipe is

$$\varepsilon_b = (\varepsilon_1 - \varepsilon_2)/2 \quad (1)$$

Fig. 5 Variation of lateral experimentally obtained bending deflection at mid-length of the pipe with applied axial force.

The bending moment would then be

$$M = \frac{EI}{R} \left(\frac{\varepsilon_2 - \varepsilon_1}{2} \right) \quad (2)$$

Hence the central deflection is

$$\Delta = \frac{M}{F} = \left[\frac{EI}{R} \left(\frac{\varepsilon_2 - \varepsilon_1}{2} \right) \right] / F \quad (3)$$

Variation of the central deflection, Δ , with the axial force, F , is depicted in Fig. 5. The axial load–bending deflection curve shows a trend typical of the buckling of most unidimensional bodies. Again, the peak load of 8.4 kN may be interpreted as the buckling load of the pipe.

4. THEORETICAL MODELLING

An internally pressurized pipe can also buckle under certain loading conditions. If the axial load is applied via a piston directly to the contained fluid (ram action) then, with increase of induced internal pressure, the straight configuration of the tube may become unstable. We shall, therefore, refer to the internal pressure at the onset of instability as the ‘buckling pressure’. In the following, a simplified stability analysis of this phenomenon is presented. In this study, the second-order effects arising from the pressure action on the walls of the tube will be neglected.

A pressurized pipe under indirect axial loading, i.e. an axial loading applied to the internal fluid, acts as a concentrically loaded strut. This is so while at any section of the pipe a resultant pressure force does exist. In addition to the axial pressure resultant, the pipe wall is under lateral internal pressure loading. Therefore, to investigate the buckling phenomenon of pressurized pipes, the influence of both the axial and lateral internal pressure loading should be taken into consideration. However, as a simple model, only the axial force may be considered and thus the well-known Euler theory of slender columns may be employed for a preliminary buckling investigation. However, some features have been observed in the experimental data which should be theoretically explained. For this purpose, a simple model of the so-called ‘limit equilibrium’ will be introduced.

4.1 Euler buckling models

The buckling theory of axially loaded round columns is well-known. According to classical stability theory, a slender compression member may buckle if the axial load reaches a certain value known as the critical buckling load. For a tubular compression member with bending rigidity EI and length L , the critical buckling load is

$$F_{cr} = \pi^2 EI / (kL)^2 \quad (4)$$

In this formula, E is the elastic modulus, I is the second moment of cross-sectional area, L is the column length and k is the effective length factor which depends on the end conditions.

In the present set of experiments, the axial force is applied to the pipe via a piston which, upon downward movement, creates a pressure increase in the pipe as well as an axial force imparted by the piston on the inner fluid. Fig. 1 shows a schematic diagram of the loading applied to the pipe, which is now looked on as a column under axial loading. The axial force F acting on the fluid is equal to

$$F = (\pi r^2)p \quad (5)$$

where r is the inner radius of the pipe and p is the internal pressure.

By equating Equations 4 and 5 at the critical stage of pipe buckling, and substituting for I the value $I = \pi t R^3$, related to a thin-walled tube of average radius R , we obtain the following expression for critical internal pressure:

$$p_{cr} = \frac{\pi^2 E t R^3}{r^2 (kH)^2} \quad (6)$$

This relation is known and is cited in the literature (e.g. [3]). It is noted that the boundary conditions are such that axial displacement of the tube is, to some extent, constrained. Therefore, in addition to indirect impact via the fluid, some pressure is also directly exerted on the pipe wall. This would accelerate the buckling of the pipe. Hence the above formula is expected to give an 'upper bound' to the critical buckling load.

For the present system, the following measured data are used: $T = 4.1 \pm 1$ mm, $R = 33 \pm 1.5$ mm, $E = 3700 \pm 200$ N mm⁻², $H = 1481$ mm. Moreover, for the chosen hinged end conditions, the effective length factor $k = 1$ may be used. Using these data in Equations 5 and 6, the following theoretical values for the critical axial load and critical internal pressure are obtained: $F_{cr} = 7695$ N and $p_{cr} = 2.9$ N mm⁻². The critical axial stress in the pipe wall is easily calculated by simple statics.

In adopting a classical Euler model, attention was paid to the experimental set-up of the pipe. As we observe from Fig. 1a, there exists a distance between the end hinge and the piston, i.e. the point of load application. In order to refine the Euler model to take this situation into account, we consider Fig. 6 in which the end hinge is distinguished from the location of the piston. The arm between the piston and the end hinge is assumed rigid. Furthermore, the rotation of this rigid arm is assumed to be equal to the rotation of the upper end of the pipe.

Now let us assume the following load parameter:

$$\lambda = \left(\frac{F}{EI} \right)^{1/2} \quad (7)$$

The refined boundary conditions at the upper end of the pipe are

$$\begin{aligned} x = 0: \quad w(x) &= 0 \\ x = L: \quad w(x) &= h \tan \alpha = h \frac{d[w(x)]}{dx} \end{aligned} \quad (8)$$

Fig. 6 Pipe with linkage connection at the upper end.

Applying these conditions to solve the differential equation of the column, we obtain the characteristic relation

$$\tan(\lambda L) = h\lambda \quad (9)$$

For the sample pipe geometry, the following data are assumed:

$$h = 120 \text{ mm}$$

$$L = 1400 \text{ mm}$$

With these data, the characteristic Equation 9 yields $p_{cr}^* = 0.9 p_{cr}$, where p_{cr}^* is the buckling pressure corresponding to the present pipe model having an end linkage. As we see, a more refined Euler model yields a lower buckling load (pressure) as compared with the pipe model with fixed hinges.

4.2 A non-linear limit equilibrium model

The experimental data, as presented in Figs 4 and 5, show that the applied axial force has a descending character beyond the load plateau. This cannot be explained by classical Euler buckling theory; it should be traced to the behaviour of so-called 'imperfection-sensitive' structures, including shells. To model this phenomenon, we consider the simple one degree-of-freedom system of Fig. 7. In this figure, a rigid bar is connected to a linear end-spring and is acted upon by an end force. An initial imperfection, θ_0 , has also been incorporated in this model.

By writing down the equilibrium equations in a deformed state, we derive the following non-linear force-deformation relation:

$$F^* = \frac{F}{kL} = \frac{x}{x + x_0} [1 - (x + x_0)^2]^{1/2} \quad (10)$$

Fig. 7 A simple model of symmetrical equilibrium limit buckling.

Fig. 8 Non-linear force-deformation curve of the simplified pipe model of Fig. 7.

where

$$x = \frac{\Delta}{L} \quad x_0 = \frac{\Delta_0}{L} \quad (11)$$

Plots of this force-deformation relation are presented in Fig. 8. This figure shows the variation of axial force with the dimensionless end displacement, $x = \Delta/L$, for some values of the imperfection parameter $x_0 = \Delta_0/L$. The decreasing trend of axial force with end displacement is quite evident from this figure. Moreover, a decrease in the critical load of the imperfect system, as compared with the perfect system, is observable in these graphs. This behaviour is an indication of the imperfection sensitivity of the system. These curves resemble the experimental graphs presented in Figs 4 and 5.

5. CORRELATIONS AND CONCLUSIONS

In this work, the plateau of the experimentally obtained load-displacement plots has been interpreted as being indicative of buckling of the pipe. A critical buckling axial force of 8.4 kN, was deduced on this basis. It is to be expected, however, that the bifurcation of equilibrium would have started before such a loading plateau. So the above-mentioned number could be interpreted as being an upper bound to the true buckling load of the pipe.

The classical Euler buckling model, employed herein, predicts a buckling load of about 7.1 kN. Comparison of this predictive model with the experimental data shows a weak but promising correlation. In the post-buckling region, however, a marked difference between the Euler model and the behaviour of the test specimen is observed. The difference lies firstly in the linearity of the Euler model as compared with non-linear post buckling behaviour, and secondly in the combined water-pipe system interaction which has not been taken into account in the Euler model.

So far as the effect of internal pressure on the buckling behaviour is concerned, a refined Euler model can be developed. It is hypothesized that the internal pressure would have a destabilizing effect and would lower the buckling load of the system.

The non-linear behaviour of the pipe has been modelled by a simple mechanical model. The experimental curves depict 'softening' of the system beyond a certain load value. This drop in the load-deformation curve is modelled by an imperfection-sensitive mechanical system. This model gives a quantitative explanation of the internally pressurized pipe behaviour.

It would be interesting to compare the experimental as well as theoretical values of critical buckling pressure, determined in this investigation, with the *bursting pressure* of PVC pipes subjected to internal pressure. For this purpose, a tensile strength of the PVC pipe material would be needed. The tensile strength of rigid PVC has ranges from 35 to 63 N mm⁻² [4]. Taking the minimum value of 35 N mm⁻², the corresponding bursting pressure is calculated to be 4.6 N mm⁻². As we see, the critical buckling pressure is well below the bursting pressure of this pipe. In fact, in the course of experiments, no bursting failure occurred. The pipe failed due to its buckling instability and not as a result of bursting failure. This conclusion has certain practical applications in making designers aware of the fact that the design of pressurized pipes should not always be carried out on strength but also on stability considerations.

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RESUME**Flambage de tuyaux en polymères sous pression interne**

On examine ici sur le plan théorique et expérimental le phénomène de flambage de tuyaux minces pressurisés sous charge axiale indirecte appliquée concentriquement par l'intermédiaire d'un piston sans friction. On a contrôlé au moyen d'extensomètres la direction de la déformation d'un échantillon de tuyau en polyvinylchloride (PVC) sous charge de compression axiale. Les mesures relevées ont été traitées par un ordinateur équipé d'un logiciel de traitement des déformations.

On a pu ainsi déterminer expérimentalement le mode et la charge de flambage du système. On a utilisé plusieurs modèles théoriques pour décrire de façon analytique la stabilité du tuyau et on a identifié de la même manière le phénomène du flambage dans ce tuyau pressurisé. On a obtenu également des valeurs numériques pour la charge de flambage critique. On a établi une corrélation entre les flambages théorique et expérimental, ce qui a permis de déterminer le flambage de tuyaux pressurisés sous compression axiale indirecte, et d'établir un modèle théorique approprié pour décrire le phénomène.