Emergent magnetic monopoles, disorder, and avalanches in artificial kagome spin ice (invited)

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We study artificial spin ice with isolated elongated nanoscale islands arranged in a kagome lattice and solely interacting via long range dipolar fields. The artificial kagome spin ice displays a phenomenology similar to the microscopic pyrochlore system, where excitations at sub-Kelvin temperatures consist of emergent monopole quasiparticles that are connected via a solenoidal flux line, a classical and observable version of the Dirac string. We show that magnetization reversal in kagome spin ice is fundamentally different from the nucleation and extensive domain growth scenario expected for a generic 2D system. Here, the magnetization reverses in a strictly 1D fashion: After nucleation, a monopole-antimonopole dissociates along a 1D path, leaving a (Dirac) string of islands with reversed magnetization in its wake. Since the 2D artificial spin ice spontaneously decays into a 1D subsystem, magnetization reversal in kagome spin ice provides an example of dimensional reduction via frustration. © 2012 American Institute of Physics. [doi:10.1063/1.3670441]

I. INTRODUCTION

Dipolar interactions are ubiquitous, even in the most ideal nanolithographic systems. Dipolar interactions at the microscopic level have also been attracting interest in the context of so-called pyrochlore spin ice,^{1,2} where the rare-earth spins exhibit highly noncollinear easy-axis anisotropies of Ising-type, which point toward the center of an elementary tetrahedron cf. Fig. 1. In conjunction with the dipolar interaction, this noncollinearity leads to a high degree of frustration. As a consequence, the dipolar interaction is “screened” at the smallest possible length scale, effectively enforcing the “2-in/2-out” Bernal-Fowler ice rule, where two of the four spins point towards the center of an elementary tetrahedron while the two remaining ones point away from it. This leads to a large degeneracy of the ground state and, hence, to a residual entropy, even at the lowest temperatures measured.³ The name “spin ice” derives from the analogy of the Ising spin orientations with the proton displacement in water ice⁴ (I_h -phase), where two of the four protons are shifted from the symmetric midpoints toward the tetrahedral center while the other two protons are pushed away from the center.

It has then been predicted theoretically⁶ that the particular geometry of pyrochlore systems may give rise to excitations in the form of flux tubes (“strings”) of overturned spins. The ice rule is only violated at the end points of these tubes, which play the role of magnetic monopole quasipar-

ticles, while the (tensionless) flux tube plays the role of an observable version of the Dirac string.^{7,8} In passing, we note that, in a condensed matter setting, such Dirac-like strings are indeed observable, even if they carry an integer multiple of the flux quantum, and one could hence argue that strings are “invisible” to the surrounding electrons. The most prominent example of such observable “strings” are flux lines in a superconductor.^{9–11} In pyrochlores, signatures of excitations resembling such (unquantized) flux tubes or “Dirac strings” have been observed in neutron scattering experiments,^{12–14} which were also carried out in an applied magnetic field.¹³

Two-dimensional square lattice nanolithographic systems were first intensely investigated by P. Schiffer and co-workers as analogues of 3D pyrochlore spin ice.¹⁵ Initially restricted to square lattices, these so-called “artificial” spin ice systems were demonstrated to exhibit states that obey the ice rules. Such systems have the advantage that, unlike pyrochlore systems, the anisotropy barriers are of the order of 10^4 K and, hence, the magnetization is “frozen” and can be observed by quasi-static probes, such as magnetic force microscopy¹⁵ (MFM). Nonetheless, it should be noted that judiciously applied oscillating fields may play a role reminiscent of temperature.¹⁶ Unfortunately, the square lattice version¹⁷ of 2D artificial spin ice suffers from the disadvantage that the interactions of the four islands centered around one vertex are inequivalent^{18–21} in contrast to other geometries, such as artificial kagome ice^{5,22–26} (cf. Fig. 1), where all islands surrounding a vertex are equivalent.

II. EFFECTIVE CHARGES IN KAGOME SPIN ICE

We consider nanoscale magnetic islands arranged in a kagome lattice, as sketched in the bottom layer in Fig. 1. Photoemission electron spectroscopy²⁷ (PEEM) reveals²⁸

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FIG. 1. (Color online) Relation between 2D kagome and 3D pyrochlore lattice. The pyrochlore lattice contains [111] lattice planes, which form a kagome lattice structure (Ref. 5). The islands of the artificial kagome spin ice investigated here are shown in white, with their uniform magnetization indicated by arrows.

that the nanoscale islands are in a single-domain state, with the magnetization pointing along the island axis. Thus, the islands may be effectively regarded as Ising spins, and, since the islands are only coupled via dipolar interactions, they are described by the energy

$$E_{\text{dip}} = \frac{1}{2} \sum_{i,j} \frac{1}{r_{ij}^3} [\mathbf{m}_i \cdot \mathbf{m}_j - 3(\mathbf{m}_i \cdot \hat{\mathbf{r}}_{ij})(\mathbf{m}_j \cdot \hat{\mathbf{r}}_{ij})], \quad (1)$$

where i denotes a site of the kagome lattice and $\mathbf{m}_i = \tau_i m_0 \mathbf{e}_i$. Here, $\tau_i = \pm 1$ denotes the Ising degree of freedom of the spin along the local anisotropy direction $\mathbf{e}_i$ of site i and m_0 is the magnetic moment of an island. The anisotropy directions $\mathbf{e}_i$ are pointing along the bonds of the honeycomb lattice of Fig. 2 and are chosen here such that they all have a positive x -component.

According to an idea put forward by Castelnovo, Moessner, and Sondhi,⁶ these point dipoles may be imagined to be stretched into dumbbells which carry magnetic charges of opposite sign and magnitude $q = m_0/\ell$, where ℓ is the length of the dumbbell. We are interested in the limiting case, where ℓ approaches the nearest neighbor distance a_h of the honeycomb lattice, cf. Fig. 2. Thus, the N dipoles $\mathbf{m}_i$ are replaced by $2N$ charges $q_{i\delta}$ situated at $\mathbf{r}_{i\delta} \equiv \mathbf{r}_i + \delta(\ell/2)\mathbf{e}_i$ (with $\delta = \pm$) and, thus, located arbitrarily close to the sites of the honeycomb lattice (for $\ell \rightarrow a_h$). The dipolar energy in Eq. (1) may then be approximated by the Coulomb interaction of these charges,

$$E_{\text{dip}} \simeq \frac{1}{2} \sum_{m,n} \mathcal{V}(r_{mn}), \quad (2)$$

where

$$\mathcal{V}(r_{mn}) = \begin{cases} \frac{q_m q_n}{r_{mn}}, & r_{mn} > a_h/2, \\ v_0 q_m q_n, & r_{mn} < a_h/2 \end{cases}, \quad (3)$$

and, for simplicity, we used the compact notation $n = (i, \delta)$. Since there are 3 charges near a honeycomb site, it is convenient to rewrite the above sum by rearranging $n = (\alpha, \nu)$, with α characterizing a honeycomb site, and the index $\nu = 1, \dots, 3$ runs over those three charges that meet at the honeycomb site $\mathbf{r}_\alpha$. Introducing the total vertex charge $Q_\alpha = \sum_\nu q_{\alpha\nu}$, the Coulomb interaction then takes the following form:

FIG. 2. (Color online) Geometry of the nanoisland array on a kagome lattice. (a) The centers of the islands form a kagome lattice with nearest neighbor distance a_k . Note that the kagome lattice can be represented as a hexagonal lattice spanned by the indicated vectors with lattice constant a and consisting of the 3 sublattices A, B, C. The island tips meet at vertices, which form a honeycomb lattice with nearest neighbor distance a_h . Note that $a = 2a_k$ and $a_k = (\sqrt{3}/2)a_h$. (b) Island geometry near one of the honeycomb vertices. The islands of width $2r$ have semicircular ends of radius r . The island tips have a distance d from the honeycomb sites. (c) Within the charge model, the point dipoles are thought to be stretched into dumbbells of length ℓ , carrying positive and negative charge of magnitude $q = m_0/\ell$. Shown is a head-to-tail configuration (I) and a head-to-head configuration (II) connecting the vertices 1, 2, and 3.

$$E_{\text{dip}} \simeq \frac{1}{2} \sum'_{\alpha,\beta} \frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} + \frac{v_0}{2} \sum_\alpha Q_\alpha^2, \quad (4)$$

where the prime denotes omission of terms with equal indices, and we recall that this result was obtained under the condition $\ell \rightarrow a_h$. As illustrated in Fig. 3, there are always 3 charges at one honeycomb site, and, therefore, the allowed values for the total vertex charge are $Q_\alpha/q = -3, -1, 1, 3$. Leaving aside, for the moment, an estimate for $v_0 > 0$, Eq. (4) tells us that the dipolar interaction in Eq. (1) can be understood as the Coulomb interaction of discrete (magnetic) charges at honeycomb sites with four possible values. The second term in Eq. (4) describes a “charging energy,” where $1/v_0$ plays the role of a capacity. For large v_0 (quantitative estimates given below), this term plays a role similar to that of a Hubbard repulsion, and we expect the low energy states to have $Q_\alpha/q = \pm 1$ at each site. The energy of such a configuration may further be minimized by taking advantage of the bipartite nature of the honeycomb lattice, which allows us to place the positive and the negative charges on the two sublattices, respectively, thus minimizing the long range Coulomb interaction in Eq. (4). In the limit $\ell \rightarrow a_h$, we thus expect the ground state to be an ordered state with an alternating NaCl-type charge ordering. This intuitive argument is indeed confirmed by Monte Carlo simulations.^{25,29} Note that such a charge ordered state still has a large degeneracy with respect to the underlying (pseudo-) spin states and exhibits a finite entropy S of approximately 0.108 per island.²⁸ These results are modified if one considers the case where the charge dumbbells are shorter than the honeycomb links, i.e., $\ell < a_h$. As discussed in Refs. 25 and 29, the higher order terms of the multipole expansion of the charge distribution around one honeycomb site lead to the appearance of a fully ordered ground state, albeit at temperatures that are at least one order of magnitude smaller than the transition temperatures into the charge-ordered spin ice phase for $(\ell - a_h)/a_h < 0.3$.

Thus, this phase will most likely be masked by disorder effects that exist in realistic samples, as discussed below. The “onsite repulsion” v_0 can be determined in two independent ways. Either v_0 may be determined by requiring the total interaction energy of the charges on two adjacent dumbbells to equal that of two adjacent dipoles, as shown in Fig. 2(c). This leads to $v_0 = 3.565/a_k$. Or one may take into account the finite extension of the islands, as shown in Fig. 2(b), with $d = r = 60$ nm, as determined from scanning electron microscope (SEM) images,²⁸ one arrives at $v_0 = 3.5/a_k$, where, for the systems investigated in Ref. 28, we have $a_k = 500$ nm.

III. HYSTERESIS IN KAGOME SPIN ICE

Rather than investigating thermodynamic equilibrium states, we now focus on the hysteresis after initial saturation of the system in a field along the negative x -axis (cf. Fig. 2). This prepares the system in one particular realization of the NaCl-type charge ordered state, which is a low energy state of the system^{25,29} cf. Fig. 3(a). Upon applying an external field to the right, a 1D path of islands with reversed magnetization will form with charge defects $\Delta Q = \pm 2q$ relative to the charge background shown in Fig. 3(a). At the beginning of the hysteresis, these charge defects are freely mobile and play the role of emergent monopoles. This becomes most evident if one introduces a “smeared” charge density by convoluting the discrete charge distribution with a Gaussian of

FIG. 3. (Color online) Charge-ordered saturated state and formation of emergent monopoles at the ends of Dirac string. (a) Initial state, which is saturated to the left and which is one particular realization of a NaCl-type charge-ordered state, where the two sublattices have opposite charge $Q = \pm q$. (b) State which exhibits a “Dirac string” of reversed islands (shown in green). Note that the charge order of the background is maintained on the string, but monopole-type charge defects emerge at the endpoints of the string, which have charge $\Delta Q = \pm 2q$ relative to the charge background shown in (a) and which are indicated by a diffuse disk.

width $2a_h$, as illustrated in Ref. 28. The ordered alternating background charges $Q = \pm q$ then lead to charge density that is exponentially small, and it is only significant in the vicinity of the monopole charge defects sketched in Fig. 3(b).

Figure 4 shows examples of the magnetization state observed via synchrotron-based PEEM²⁸ together with the ascending branch of the hysteresis. As shown in the inset of the first panel, bright islands indicate a magnetization pointing to the left, while dark islands indicate the islands that have flipped the magnetization in a reversed field along the positive x -axis. Therefore, the dark, one-dimensional string-like objects in Fig. 4 correspond to the Dirac string. Notice the strikingly one-dimensional nature of these strings, which lends support to the scenario outlined in Fig. 3(b). The corresponding hysteresis of the magnetization, as shown in black in Fig. 4(b), reveals a characteristic plateau in the field range $1 \lesssim H/H_c \lesssim 2$. This plateau is a manifestation of a stripe-like phase that may already be discerned in the panel for $H = 0.99H_c$. The red curve in Fig. 4(b) shows the corresponding density of charge defects, which is shifted toward considerably higher fields. The charge density thus remains low until the magnetization plateau is reached. This is a manifestation of the basic mechanism underlying magnetization reversal in this frustrated spin system: After nucleation of a monopole-antimonopole pair via the flipping of one island, adjacent islands will flip as well, thus forming a one-dimensional, Dirac-type string of reversed islands. During this “extension” of the Dirac string, the already nucleated

FIG. 4. (Color online) (a) XMCD images of the kagome array at different field values. The dark contrast indicates magnetization pointing to the right, while the bright contrast indicates magnetization toward the left, as indicated in the inset. Reprinted with permission from E. Mengotti, L. J. Heyderman, A. F. Rodriguez, F. Nolting, R. V. Hügli, and H.-B. Braun, Nature Phys. **7**, 68 (2011). © 2011. Nature. (b) Hysteresis and the total density of charge defects and the density of mobile monopole-type defects (Ref. 28). Configurations shown in (a) are indicated by full disks.

FIG. 5. (Color online) A Dirac string avalanche as an example of dimensional reduction due to frustration. The (Dirac) string, consisting of islands with reversed magnetization and, hence, dark contrast, is shown together with the Ising pseudospins that capture the magnetization orientation. The reversal of the island magnetization along a 1D path rather than a 2D patch is an example of dimensional reduction due to frustration, and the generation of monopoles of opposite charge is closely related to the nucleation of solitons of opposite charge in 1D systems (Refs. 30 and 32).

monopole and antimonopole are increasingly separated from each other, thus leading to a sharp increase of the magnetic moment while keeping the monopole density small. Also shown is the fraction of truly mobile monopoles, which have not yet been trapped at a neighboring string.

IV. DIMENSIONAL REDUCTION DUE TO FRUSTRATION

It is remarkable that magnetization reversal in this dipolar system proceeds in an essentially 1D fashion, reminiscent of the nucleation of soliton-antisoliton pairs in nanowires.³⁰ However, here an intrinsically 2D system spontaneously decays into 1D subsystems, a phenomenon also known as “dimensional reduction due to frustration.”³¹ We illustrate this phenomenon in Fig. 5, which shows a configuration similar to Fig. 3(b), which exhibits a segment of overturned moments, which is delimited by defects in the ordered charge background, the monopoles. Projecting the pseudospin direction τ_i along the 1D path defined by the direction of the Dirac string, as shown in Fig. 5 below the sample plane, we recog-

nize the analogy between the monopoles and the solitons or domain walls in 1D ferromagnetic³⁰ or antiferromagnetic chains³² with strong easy-axis or Ising anisotropy.

V. AVALANCHES AND DISORDER

The experimental observations shown in Fig. 4 are closely linked to the existence of disorder. In a realistic experimental system, the anisotropy and, hence, the switching fields of the individual islands differ due to slight differences in the microstructure. Typically, the switching fields will be Gaussian, distributed around a mean switching field H_0 with standard deviation $H_0\sigma$ with typically $\sigma \approx 0.1 - 0.15$. The initial reversal of an island and, hence, the nucleation of the monopole-antimonopole pair occurs at an island of low anisotropy and, hence, low individual switching field. After an individual island has switched, the local fields on the neighboring islands become favorable for switching, and, therefore, the magnetization switches in an avalanche-type fashion until it comes to rest at an island with large anisotropy. As the phenomenon of dimensional reduction suggests that this is essentially a 1D process, the probability $P(s)$ for s islands to switch sequentially is simply proportional to p^s , where p is the disorder dependent (conditional) probability for an individual island to switch at a given field, provided that its neighbor has already switched. However, this implies that the probability of encountering an “avalanche” of length s decays exponentially,

$$P(s) \propto e^{-\alpha s}, \quad (5)$$

where $\alpha = -\ln p$. Note that this relation will hold even for arbitrarily small values of disorder σ , as long as the avalanche process is one-dimensional. Indeed, such exponential decay is observed for avalanches in purely 1D systems,³³ while, in 2D systems, there is a power law dependence for sufficiently small disorder,^{34–36} in agreement with measurements on thin films.³⁷

While it is extremely challenging to control the disorder during sample fabrication, we obtained insight into the

FIG. 6. (Color online) Simulated samples at different fields (columns) and for different values σ of the intrinsic disorder. Top row: $\sigma = 0.025$, middle row: $\sigma = 0.13$, and bottom row: $\sigma = 0.2$. The value of $\sigma = 0.13$ gives the best agreement with the experimentally observed configurations and hysteresis as shown in Fig. 4. The numbers indicate the external field in units of the (disorder dependent) coercivity H_c .

disorder of these avalanches by performing Monte Carlo simulations. The energy was based on gives the model in Eq. (1) supplemented with Gaussian-distributed switching fields and an external field of varying magnitude. The dipolar interaction was implemented exactly via Ewald's summation over an infinite array of periodically repeated patches containing up to 10 000 islands. The results for configurations at different fields and increases from top to bottom. For low disorder $\sigma = 0.025$, which is experimentally inaccessible, one recognizes a very sharp hysteresis with a sharp increase at the coercivity. At the same time, the monopole density is very small, and this is directly related to the fact that, after nucleation, a monopole-antimonopole pair gets immediately separated and eventually winds several times around the toroidal sample, as enforced by the periodic boundary conditions. Hence, the magnetization increases dramatically without a concomitant increase in monopole density. Notice the distinct plateau in the hysteresis, which is a direct consequence of the nearly perfectly striped phase, where the magnetization in the two sublattices A, B is switched, as shown in the panel on the far right.

It is then evident that, in order to be able to observe emergent monopoles, one relies on a moderate amount of disorder, whose effect, however, is smaller than the typical dipolar coupling between the islands. Configurations are shown in the second row in Fig. 6 for a value of $\sigma = 0.13$, which provides the best fit to the experiment, as shown in Fig. 4. In the bottom row, the disorder has the high value of $\sigma = 0.2$, which leads to short avalanches and, hence, short Dirac strings. Correspondingly, the hysteresis is increasingly smoothed out and the plateau develops into a broad shoulder for $\sigma = 0.2$.

Finally, we investigate the avalanche statistics $P(s)$ as a function of disorder, as shown in Fig. 7. Clearly, the $P(s)$ decays exponentially, even for the smallest values of disorder, and hence is clearly distinct from the critical behavior observed by Sethna and co-workers³⁴ for the random field Ising model³⁸ for moderately low disorder. We conclude that the phenomenon of dimensional reduction due frustration persists to extremely small values of disorder.

FIG. 7. (Color online) The probability $P(s)$ of an avalanche of length s occurring decays exponentially with its length s , even for very small values of the disorder σ . This demonstrates the 1D nature of the avalanches in 2D artificial kagome ice (Ref. 28). Reprinted with permission from E. Mengotti, L. J. Heyderman, A. F. Rodriguez, F. Nolting, R. V. Hügli, and H.-B. Braun, *Nature Phys.* 7, 68 (2011). © 2011. Nature.

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- ¹M. J. Harris, S. T. Bramwell, D. F. McMorro, T. Zeiske, and K. W. Godfrey, *Phys. Rev. Lett.* **79**, 2554 (1997).
- ²S. T. Bramwell and M. Gingras, *Science* **294**, 1495 (2001).
- ³A. P. Ramirez, A. Hayashi, R. J. Cava, R. Siddhant, and B. S. Shastry, *Nature* **399**, 333 (1999).
- ⁴L. Pauling, *J. Am. Chem. Soc.* **57**, 2680 (1935).
- ⁵Y. Tabata, H. Kadowaki, K. Matsuhira, Z. Hiroi, N. Aso, E. Ressouche, and B. Fak, *Phys. Rev. Lett.* **97**, 257205 (2006).
- ⁶C. Castelnovo, R. Moessner, and S. L. Sondhi, *Nature* **451**, 42 (2008).
- ⁷P. A. M. Dirac, *Proc. R. Soc. London, Ser. A* **133**, 60 (1931).
- ⁸L. D. C. Jaubert and P. C. W. Holdsworth, *Nature Phys.* **5**, 258 (2009).
- ⁹D. R. Nelson, *Defects and Geometry in Condensed Matter Physics* (Cambridge University Press, Cambridge, England, 2002).
- ¹⁰M. Laver and E. M. Forgan, *Nature Commun.* **1**, 1 (2010).
- ¹¹A. D. Bianchi, M. Kenzelmann, L. DeBeer-Schmitt, J. S. White, E. M. Forgan, J. Mesot, M. Zolliker, J. Kohlbacher, R. Movshovich, E. D. Bauer, J. L. Sarrao, Z. Fisk, C. Petrovich, and M. R. Eskildsen, *Science* **319**, 177 (2008).
- ¹²T. Fennell, P. P. Deen, A. Wildes, K. Schmalzl, D. Prabhakaran, A. T. Boothroyd, R. J. Aldus, D. F. McMorro, and S. T. Bramwell, *Science* **326**, 415 (2009).
- ¹³D. J. P. Morris, D. A. Tennant, and S. A. Grigera, *Science* **326**, 411 (2009).
- ¹⁴H. Kadowaki, N. Doi, Y. Aoki, Y. Tabata, T. J. Sato, J. W. Lynn, K. Matsuhira, and Z. Hiroi, *J. Phys. Soc. Jpn* **78**, 103706 (2009).
- ¹⁵R. F. Wang, C. Nisoli, R. S. Freitas, J. Li, W. McConville, B. J. Cooley, M. S. Lund, N. Samarth, C. Leighton, V. H. Crespi, and P. Schiffer, *Nature* **439**, 303 (2006).
- ¹⁶C. Nisoli, J. Li, X. Ke, D. Garand, P. Schiffer, and V. Crespi, *Phys. Rev. Lett.* **105**, 047205 (2010).
- ¹⁷J. P. Morgan, A. Stein, S. Langridge, and C. H. Marrows, *Nature Phys.* **7**, 75 (2011).
- ¹⁸G. Möller and R. Moessner, *Phys. Rev. Lett.* **96**, 237202 (2006).
- ¹⁹Z. Budrikis, P. Politi, and R. L. Stamps, *Phys. Rev. Lett.* **105**, 017201 (2010).
- ²⁰L. A. Mol, R. L. Silva, R. C. Silva, A. R. Pereira, W. A. Moura-Melo, and B. V. Costa, *J. Appl. Phys.* **106**, 063913 (2009).
- ²¹A. Remhof, A. Schumann, A. Westphalen, H. Zabel, E. Y. Vedmedenko, T. Last, and U. Kunze, *Phys. Rev. B* **77**, 134409 (2008).
- ²²A. S. Wills, R. Ballou, and C. Lacroix, *Phys. Rev. B* **66**, 144407 (2002).
- ²³Y. Qi, T. Brintlinger, and J. Cumings, *Phys. Rev. B* **77**, 094418 (2008).
- ²⁴E. Mengotti, L. J. Heyderman, A. F. Rodriguez, A. Bisig, L. L. Guyader, F. Nolting, and H. B. Braun, *Phys. Rev. B* **78**, 144402 (2008).
- ²⁵G. Möller and R. Moessner, *Phys. Rev. B* **80**, 140409 (2009).
- ²⁶S. Ladak, D. E. Read, G. K. Perkins, L. F. Cohen, and W. R. Branford, *Nature Phys.* **6**, 359 (2010).
- ²⁷J. Stöhr, Y. Wu, B. D. Hermseier, M. G. Samant, G. R. Harp, S. Koranda, D. Duham, and B. P. Tonner, *Science* **259**, 658 (1993).
- ²⁸E. Mengotti, L. J. Heyderman, A. F. Rodriguez, F. Nolting, R. V. Hügli, and H.-B. Braun, *Nature Phys.* **7**, 68 (2011).
- ²⁹G. W. Chern, P. Mellado, and O. Tchernyshyov, *Phys. Rev. Lett.* **106**, 207202 (2009).
- ³⁰H. B. Braun, *Phys. Rev. Lett.* **71**, 3557 (1993).
- ³¹L. Balents, *Nature* **464**, 199 (2010).
- ³²H. B. Braun, J. Kulda, B. Roessli, D. Visser, K. W. Kramer, and H. U. Güdel, *Nature Phys.* **1**, 159 (2005).
- ³³O. A. Chubykalo, J. M. Gonzales, and J. Gonzales, *Physica D* **113**, 382 (1998).
- ³⁴J. P. Sethna, K. A. Dahmen, and O. Perkovic, in "The Science of Hysteresis, Vol. II", edited by G. Bertotti and I. Mayergoyz, Elsevier Academic Press, Oxford (2006), Cond-mat, arXiv:0406320 v1 (2004).
- ³⁵J. P. Sethna, K. A. Dahmen, S. Kartha, J. A. Krumhansl, B. W. Roberts, and J. D. Shore, *Phys. Rev. Lett.* **70**, 3347 (1993).
- ³⁶J. P. Sethna, K. A. Dahmen, and C. R. Meyers, *Nature* **410**, 242 (2001).
- ³⁷M.-Y. Im, P. Fischer, D.-H. Kim, and S.-C. Shin, *Appl. Phys. Lett.* **95**, 182504 (2009).
- ³⁸K. Christensen and N. R. Molloney, *Complexity and Criticality* (Imperial College Press, London, 2005).