Density of states for dirty d -wave superconductors: A unified and dual approach for different types of disorder

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A two-parameter field theoretical representation is given of a two-dimensional dirty d -wave superconductor that interpolates between the Gaussian limit of uncorrelated weak disorder and the unitary limit of a dilute concentration of resonant scatterers. It is argued that a duality holds between these two regimes from which follows that a linearly vanishing density of states in the Gaussian limit transforms into a diverging one in the unitary limit arbitrarily close to the Fermi energy.

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Extrinsic defects in high- T_c materials have observable consequences for the local and bulk density of states (DOS) of quasiparticles. This is seen by scanning tunnelling microscopy, nuclear magnetic resonance, and neutron scattering.¹ Popular wisdom distinguishes “weak” extrinsic disorder introduced by defects above or below the copper-oxide planes through oxygen doping, from “strong” extrinsic disorder caused by substitutions of in-plane copper atoms with ions such as zinc.

Given that the symmetry of the superconducting state is predominantly of the d -wave type in high- T_c materials, it is thus not surprising that a considerable theoretical effort has been devoted to the effect of quenched disorder on a d -wave order parameter by reducing the problem to two models. The first one is a model of Anderson localization for a single “nodal” quasiparticle described by a two-dimensional (2D) nonmagnetic random Bogoliubov-de-Gennes (BdG) Hamiltonian.^{2–14} The second one is a Kondo model in which a single magnetic moment couples to a bath of 2D “nodal” quasiparticles.¹⁵ At this level of approximation no attempt is made to describe either the existence of quasi-particle interactions or the self-consistent effect of defects on the superconducting order parameter, although these effects might be important.¹¹ In spite of the simplicity of either model many important theoretical issues remain unresolved to this date.

Even in the simpler case of Anderson localization, the effect of disorder on the quasiparticle DOS of a d -wave superconducting state is poorly understood. The physics of localization is expected to dominate at long distances and small energies in 2D.^{10,13} This fact together with the existence of particle-hole symmetry below T_c makes it essential to account for the level repulsion upon approaching the Fermi energy ε_F . However, the physics of level repulsion and localization are nonperturbative with respect to standard perturbative techniques [expansions in $(k_F l)^{-1} \ll 1$, k_F the Fermi momentum and l the mean-free path in the Born approximation].¹⁶ The existence of four isolated nodes of the superconducting pairing function $\Delta_{\mathbf{k}}$ in the 2D Brillouin zone also invalidates, upon approaching ε_F , expansions in powers of $(k_F l)^{-1} \ll 1$.³ Moreover, there are some theoretical arguments and numerical evidences that have been attributed

to the existence of nodes of $\Delta_{\mathbf{k}}$ suggesting that the dependence on energy of the DOS depends in a dramatic fashion on the microscopic details of the disorder. On the one hand, for a high density of very weak s -wave scatterers [the Gaussian limit (GL)] level repulsion dominates and causes the DOS to vanish linearly upon approaching ε_F .^{10,13} On the other hand, a dilute density of uncorrelated and resonant s -wave scatterers [the unitary limit (UL)] is argued to result in a diverging DOS.^{9,11,12} The purpose of this paper is to present a unified picture of these two limits.

Starting from a BdG Hamiltonian describing a dirty d -wave superconductor with *white-noise correlated disorder* that depends on two parameters, the impurity concentration ρ and the variance g of the disorder, we construct an effective field theory that generates disorder averaged Green functions. We show how this effective action reduces to the nonlinear sigma model (NLSM) describing a random BdG Hamiltonian with time reversal symmetry and spin rotation symmetry (class CI in the classification of Ref. 16) in the limit of $\rho \rightarrow \infty$ with g finite. The effective action is treated by performing an expansion in powers of ρ (virial expansion) in the opposite limit of g fine-tuned to its resonant value $g \rightarrow g_r$ with ρ finite. Carrying out the expansion up to first (second) order in ρ allows to identify this limit with the UL (the expansion parameter). As was the case with the $(k_F l)^{-1}$ expansion, the virial expansion converges only for not too small energies. In order to better understand whether there can or cannot be a crossover from the UL to the GL, we use a variational method and show that the form of the self-consistent equations is controlled by how the chemical potential is introduced in the problem. This suggests that the energy alone cannot connect the two limits if the chemical potential is chosen at the “resonant” condition for the UL. By expanding perturbatively for *large* energies, we identify a duality relation between the GL and UL series that would imply, at *low* energies, that the diverging DOS $1/|\varepsilon| \ln^2 |1/\varepsilon|$ is connected by duality to the vanishing DOS $|\varepsilon|$ in the GL arbitrarily close to ε_F . This nonperturbative argument relies in an essential way on the nodes of $\Delta_{\mathbf{k}}$.

To model a dirty 2D d -wave superconductor, we use the tight-binding Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{V}$,

$$\mathcal{H}_0 = - \sum_{\langle ij \rangle} \left(t_{ij} \sum_{\sigma=\uparrow, \downarrow} c_{i\sigma}^\dagger c_{j\sigma} + \Delta_{ij} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger + \text{H.c.} \right),$$

$$\mathcal{V} = \sum_i \sum_{\sigma=\uparrow, \downarrow} V_i c_{i\sigma}^\dagger c_{i\sigma}, \quad (1)$$

that describes electrons hopping on a square lattice made of $\mathcal{N}$ sites with lattice spacing a . The real valued hopping matrix t_{ij} contains in its diagonal the chemical potential μ_0 . Its off diagonal elements describe nearest-neighbor hopping (t), next-nearest hopping (t') etc., and yield the normal state dispersion $\varepsilon_{\mathbf{k}}$ in reciprocal space. The order parameter $\Delta_{ij} = (\Delta_0/2)(\delta_{i,j\pm\hat{x}} - \delta_{i,j\pm\hat{y}})$ enforces the $d_{x^2-y^2}$ symmetry. The disorder potential V_i describes N uncorrelated s -wave scatterers with the individual potential strength V_0 . Moreover, N is randomly distributed according to a Poisson distribution. Hence, moments of this disorder potential are given by $\overline{V_i^p V_j^q} = \rho V_0^{p+q} \delta_{ij}$, $\rho = N/\mathcal{N}$, $p, q \in \mathbb{N}$, where the overline denotes disorder averaging (the subscript c refers to the cumulant). We choose the parametrization of the disorder (ρ, g) where $g = \rho V_0^2$. We can then identify two distinct limits: (i) The GL $\rho \rightarrow \infty$, g finite. (ii) The UL $g \rightarrow g_r$, ρ finite. The resonant disorder strength $g_r = \infty$ when the normal state dispersion is particle-hole symmetric, whereas it is a finite-energy scale that depends on μ_0 , t' , etc., otherwise. To simplify notation we will assume a particle-hole symmetric normal state dispersion from now on: $g_r = \infty$.

Effective field theories in Anderson localization are constructed to compute the disorder average over products of the Green functions $G(z) = (iz - \mathcal{H})^{-1}$ at different Matsubara energies z . For the problem at hand, the (bosonic) replicated canonical partition function

$$\Xi_{2N,n} \propto \frac{1}{\mathcal{N}^N} \sum_{i_1=1}^{\mathcal{N}} \cdots \sum_{i_N=1}^{\mathcal{N}} \int \mathcal{D}[\boldsymbol{\phi}] e^{-1/2 \boldsymbol{\phi}^T (iz - \mathcal{H}) \boldsymbol{\phi}}$$

$$= \int \mathcal{D}[\boldsymbol{\phi}] e^{-1/2 \boldsymbol{\phi}^T (iz - \mathcal{H}_0) \boldsymbol{\phi}} [\mathcal{A}(\rho, g)]^N,$$

$$\mathcal{A}(\rho, g) = \frac{1}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} e^{-1/2 \sqrt{g/\rho} \sum_{a=1}^n \sum_{\tau, \tau'=1}^n \phi_{ia\tau}(-)^{\tau} \delta_{\tau, \tau'} \phi_{ia\tau'}}, \quad (2)$$

yields, upon insertion of a source term, the disorder average of the single-particle Green function $G(z)$ whereby disorder averaging is defined by weighing equally all the spatial configurations that N impurities can take on the lattice. Here, $\mathcal{H} = \mathcal{H}_0 + \mathcal{V}$ is a $4\mathcal{N}n \times 4\mathcal{N}n$ matrix representation of Eq. (1) (in the Nambu representation with the Pauli matrices $\gamma_{1,2,3}$ to represent the particle-hole grading) and $\boldsymbol{\phi}$ is a replicated bosonic real field with $4\mathcal{N}n$ components. In order to make a connection between the GL and the UL we need to relax the fact that N is fixed in Eq. (2), i.e., we define

$$\mathcal{Z}_{\alpha,n} = \sum_{N=0}^{\infty} \Xi_{2N,n} \frac{\alpha^N}{N!} e^{-\alpha} = \int \mathcal{D}[\boldsymbol{\phi}] e^{-S_0 - S_1 - \rho \mathcal{N}},$$

$$S_0 = \frac{1}{2} \boldsymbol{\phi}^T (iz - \mathcal{H}_0) \boldsymbol{\phi}, \quad (3)$$

$$S_1 = -\rho \sum_{i=1}^{\mathcal{N}} \exp \left[-\frac{1}{2} \sqrt{\frac{g}{\rho}} \sum_{a=1}^n \sum_{\tau=1}^2 \phi_{i\tau a}(-)^{\tau} \delta_{\tau\tau'} \phi_{i\tau' a} \right].$$

This Poisson grand canonical partition function is normalized to one in the replica limit $n \rightarrow 0$. Denoting averaging with $\mathcal{Z}_{\alpha,n}$ by $\langle (\cdots) \rangle$, we deduce that the average number of impurities is $\langle N \rangle = \alpha$ and that the standard deviation in the number of particles scales like $\sqrt{\alpha}$. From now on we will identify the impurity concentration ρ with $\alpha/\mathcal{N}$. The interacting field theory described by Eq. (3) has already been introduced by Mezard, Parisi, and Zee as an effective field theory for random Euclidean matrices¹⁷ (REM) (see also Refs. 18–20). Our derivation shows that the problem of Anderson localization with Poisson distributed impurities is a special case of REM.

The GL for a white noise distribution of impurities should be recovered as $\rho \rightarrow \infty$. It is then natural to expand the exponential in S_1 . The first nonvanishing contribution to the action in the grand canonical partition function is of order g ; $\mathcal{Z}_{\alpha,n}^g = \int \mathcal{D}[\boldsymbol{\phi}] \exp(-S_0^g - S_1^g)$,

$$S_0^g = \frac{1}{2} \boldsymbol{\phi}^T (iz - \mathcal{H}_0^g) \boldsymbol{\phi}, \quad S_1^g = -\frac{g}{8} (\boldsymbol{\phi}^T \gamma_3 \boldsymbol{\phi})^2, \quad (4)$$

aside from a shift in the chemical potential $\delta\mu = \sqrt{\rho g}$. Equation (4) describes the effective action needed to calculate the average Green function of the random BdG Hamiltonian $\mathcal{H}_0^g + \mathbf{v}$. The superscript “ g ” indicates that the chemical potential $\mu_0^g = \mu_0 - \sqrt{\rho g}$ in $\mathcal{H}_0^g$. The random potential $\mathbf{v}$ is white noise correlated and Gaussian distributed with vanishing mean and variance $g/4$. The usual justification for neglecting terms of order $\rho(g/\rho)^{3/2} \ll 1$ is that they are irrelevant at the fixed point to which $S_0^g + S_1^g$ flows. This fixed point is itself described by a NLSM on the manifold $\text{Sp}(2n)$ in the limit of weak disorder, i.e., when the mean free path in the Born approximation $\sim 2t/g$ is much larger than the lattice spacing a and for energies close to the band center $\mu_0^g = 0$.^{10,13} At this fixed point the DOS $\nu(\varepsilon)$ is only sensitive to the existence of nodes for not too small energies.^{21,22} The physics of weak localization comes into play below the energy scale $\varepsilon_1^g \sim (B/A) \exp(-\pi B^2/2g)$ at which a pseudogap opens up. Below an even smaller energy scale $\varepsilon_2^g \sim \varepsilon_1^g [g/(A^2 B^2)] \exp(-A^2)$ random matrix theory (RMT) predicts that the DOS becomes linear and insensitive to the nodes of $\Delta_{\mathbf{k}}$ (the prediction of RMT can break down for correlated disorder in which case sensitivity of the DOS to the nodes of $\Delta_{\mathbf{k}}$ is recovered^{3,6,13,14}). Here, $B \propto \sqrt{2t\Delta_0}$ is the bandwidth and $A \propto \sqrt{2t/\Delta_0}$ is the Dirac cone anisotropy. For notational simplicity $2t = \Delta_0 = 1$ from now on.

The natural expansion for small impurity concentrations is the virial expansion, i.e., one in powers of ρ of $\exp(-S_1)$ in Eq. (3). We have calculated the increment $\delta\nu^{(p)}(\varepsilon)$ in the DOS due to $p=1$ and $p=2$ impurities, respectively. As it

should be we reproduce the known increment $\delta\nu^{(1)}(\varepsilon)$ induced by one impurity off and at resonance ($g = g_r$) to first order in ρ .^{23,4,5,8} Off resonance the DOS crosses over to a linearly vanishing DOS upon approaching ε_F . *This is not so at resonance.* We infer from the calculation of the averaged single particle Green function $\langle G(z) \rangle$ up to second order in ρ that the virial expansion is an expansion in the small parameter $\alpha^u \equiv \pi\rho/[\varepsilon^2 \ln(z^2)]$. Hence, the virial expansion is not uniform in the Matsubara energy z . This nonuniformity of the virial expansion is the counterpart to the fact that in the GL the expansion in $1/k_F l$ of $\langle G(z) \rangle$ is really an expansion in the small parameter $\alpha^g \equiv g \ln(z^2)/\pi$.³ [The simplest guess, as suggested by the first terms in the virial expansion, for a small expansion parameter that interpolates between these two limits is $\alpha^{\text{eff}} \equiv \alpha^u \alpha^g / (\alpha^u + \alpha^g)$.] The nonuniformity of the perturbative expansions in $\alpha^{g,u}$ reflects the importance of quantum interferences already at the level of the single-particle Green function. Both perturbative breakdowns below the thresholds $\varepsilon_1^{g,u}$ ($\varepsilon_1^u \sim \sqrt{\rho/|\ln \rho|}$) are signatures of the existence of isolated nodes of $\Delta_{\mathbf{k}}$ in the 2D Brillouin zone that require nonperturbative approaches to access the DOS arbitrarily close to ε_F . We have not been able to find a counterpart to the $\text{Sp}(2n)$ NLSM that describes the GL and is amenable to a calculation of the DOS arbitrarily close to ε_F . On the other hand, comparison of the expansions in α^g and α^u up to second order suggests that the GL and UL are related by $g/\pi \leftrightarrow \pi\rho$ and $\ln(z^2) \leftrightarrow 1/z^2 \ln(z^2)$. If this relationship holds to all orders and extends to the nonperturbative regime as well it would allow access to $\lim_{\varepsilon \rightarrow \varepsilon_F} \nu(\varepsilon)$ in the UL from knowledge of $\lim_{\varepsilon \rightarrow \varepsilon_F} \nu(\varepsilon)$ in the GL.

For practical purposes $\varepsilon_1^{g,u}$ can be considered small and the crossover of the DOS between the GL and UL is needed when $\varepsilon > \max \varepsilon_1^{g,u}$. This can be achieved by treating the interacting theory (3) within a variational approximation. We thus replace the kernel of $S_0 + S_1 + \rho\mathcal{N}$ in Eq. (3) by the noninteracting, translationally invariant, and replica diagonal kernel $iz' - \mathcal{H}^{\text{sc}}$, $z' \equiv z + \bar{z}$. In reciprocal space $\mathcal{H}_{\mathbf{k}}^{\text{mf}} = \varepsilon'_{\mathbf{k}} \gamma_3 + \Delta'_{\mathbf{k}} \gamma_1$, $\varepsilon_{\mathbf{k}} \equiv \varepsilon_{\mathbf{k}} - \mu_0 + \bar{\varepsilon}$, and $\Delta'_{\mathbf{k}} \equiv \Delta_{\mathbf{k}} + \bar{\Delta}$. The self-consistent parameters $\bar{z}$, $\bar{\varepsilon}$, and $\bar{\Delta}$, i.e., the disorder induced self-consistent shifts in the self-energy, chemical potential, and gap, respectively, are obtained by solving the self-consistent equations

$$\Omega(z') = \rho \frac{\bar{z}}{\bar{z}^2 + \bar{\varepsilon}^2 + \bar{\Delta}^2}, \quad (5a)$$

$$\mathcal{E}(z') = \rho \frac{\bar{\varepsilon}}{\bar{z}^2 + \bar{\varepsilon}^2 + \bar{\Delta}^2} - \sqrt{\frac{\rho}{g}}, \quad (5b)$$

$$\Delta(z') = \rho \frac{\bar{\Delta}}{\bar{z}^2 + \bar{\varepsilon}^2 + \bar{\Delta}^2}, \quad (5c)$$

$$(\Omega, \mathcal{E}, \Delta)(z') = \frac{1}{\mathcal{N}} \sum_{\mathbf{k}} \frac{(z', \varepsilon'_{\mathbf{k}}, \Delta'_{\mathbf{k}})}{(z')^2 + (\varepsilon'_{\mathbf{k}})^2 + (\Delta'_{\mathbf{k}})^2}. \quad (5d)$$

Here $\nu_{\text{sc}}(\varepsilon) = \lim_{\eta \rightarrow 0} \lim_{i\eta \rightarrow \varepsilon + i\eta} \text{Im}[i \text{sgn}(\eta) \Omega(z')]$ is the self-consistent DOS. It is sufficient to consider Eqs. (5a) and (5b) alone since $\Delta_{k_x, k_y} = -\Delta_{k_y, k_x}$ allows $\bar{\Delta} = 0$.

That details of disorder matter in 2D d -wave superconductors (see Ref. 11) already follows from the variational approach. Indeed, Eqs. (5a) and (5b) are not so much sensitive to the energy z as they are to the chemical potential μ_0 when it comes to describing the crossover from the UL to the GL. Equations (5a) and (5b) reduce to the two well-known self-consistent equations

$$\rho \rightarrow \infty, \quad g < \infty, \quad \mu_0 = \sqrt{\rho g} \Rightarrow \Omega(z') = \frac{\bar{z}}{g}, \quad \bar{\varepsilon} = \mu_0, \quad (6a)$$

$$g \rightarrow \infty, \quad \rho < \infty, \quad \mu_0 = 0 \Rightarrow \Omega(z') = \frac{\rho}{z}, \quad \bar{\varepsilon} = 0, \quad (6b)$$

in both the GL (Refs. 21 and 22) and the UL.^{24,25} It is thus imperative to scale μ_0 with $\sqrt{\rho g}$ for $\bar{\varepsilon}$ to be tuned away from the UL resonance $|\bar{\varepsilon}| \ll |\bar{z}|$.

The self-consistent Eq. (6a) does not capture the interplay between the physics of localization and that of level repulsion below the energy scale ε_1^g .^{3,10} The same should be true of Eq. (6b) below the energy scale ε_1^u at which the virial expansion breaks down. We are going to argue that the physics of localization and level repulsion results in a diverging DOS upon approaching ε_F in the UL. Our argument relies first on identifying a duality relating the perturbative expansions of the self-energy in the GL and UL valid at *large* energies relative to the disorder strength. We then postulate that the same duality extends to *small* energies relative to the disorder strength. Let z be a large Matsubara energy, i.e., $g(\rho)$ is sufficiently small so that level repulsion can be ignored and the physics of localization is that of the standard orthogonal symmetry class. In this regime, we can safely remove the self-consistency condition in

$$\Omega(z + \bar{z}) = \sum_{p=0}^{\infty} \begin{cases} \frac{g^p}{p!} [\Omega(z')]^{+p} (\partial_z^p \Omega)(z), & \text{GL,} \\ \frac{\rho^p}{p!} [\Omega(z')]^{-p} (\partial_z^p \Omega)(z), & \text{UL,} \end{cases} \quad (7a)$$

by substituting $\Omega(z + g\Omega(z + \dots))$ [$\Omega(z + \rho/\Omega(z + \dots))$] with $\Omega(z) + (\partial_z \Omega)(z) \Omega(z) g + \mathcal{O}(g^2) [\Omega(z) + (\partial_z \ln \Omega)(z) \rho + \mathcal{O}(\rho^2)]$. In this way, the self-energy reduces to the perturbative expansion

$$\Omega(z + \bar{z}) \approx \frac{z \ln(z^{-2})}{\pi} + \frac{2z [\ln(z^2) + 2]}{\pi} \begin{cases} \frac{\ln(z^2)}{2} \frac{g}{\pi} \\ \frac{\pi \rho}{2z^2 \ln(z^2)} \end{cases} - \frac{2z}{\pi} \sum_{p=2}^{\infty} \begin{cases} \beta_p^g [\ln(z^2)]^{+p} \left(\frac{g}{\pi}\right)^p, & \text{GL,} \\ \beta_p^u [z^2 \ln(z^2)]^{-p} (\pi \rho)^p, & \text{UL.} \end{cases} \quad (7b)$$

Here, the combinatoric coefficient $\beta_p^{g,u}$ depends on the order in g, ρ to which the self-consistency has been truncated. For example, the substitution $\Omega(z') = \Omega(z)$ gives $\beta_p^g = 1/p(p-1) = \beta_p^u$ and accounts only for the pure rainbow diagrams in the expansion of the Dyson equation for the self-energy in the GL. Truncation to higher order in g accounts for diagrams that are not purely of the rainbow type but in which impurity lines never cross. Neglecting diagrams with crossing of impurity lines is only justified for sufficiently weak disorder at a fixed energy.³ Correspondingly, the condition on the disorder strength for the physics of localization and level repulsion to be unimportant is

$$-\alpha^g \equiv \frac{g}{\pi} \ln(1/z^2) < 1 \Leftrightarrow |z| > \varepsilon_1^g, \quad \text{GL}, \quad (8a)$$

$$-\alpha^u \equiv \frac{\pi\rho}{z^2 \ln(1/z^2)} < 1 \Leftrightarrow |z| > \varepsilon_1^u, \quad \text{UL}. \quad (8b)$$

Note that the expansions in the GL and UL are related by $g/\pi \leftrightarrow \pi\rho$ and $\ln(z^2) \leftrightarrow 1/z^2 \ln(z^2)$. By undressing the propagator in Eq. (7b), two expansions of the same scaling form $\Omega(z') - \Omega(z) = 2(z/\pi) \{ \frac{1}{2} [\ln(z^2) + 2] \alpha^{g,u} - f(\alpha^{g,u}) \}$ have been derived for *large* energies in the GL and UL, respectively. We now assume that the self-energy can also be written as some scaling function $\tilde{f}$ that can be expanded in powers of $\alpha^{g,u}$ outside of the radius of convergences (8), i.e., at *low* energies. The scaling function $\tilde{f}$ should be specified by the symmetry class CI, i.e., the physics of level repulsion induced by the particle-hole symmetry of a time-reversal and spin-rotation symmetric superconductor. This scaling function is not completely unknown since RMT predicts that $\tilde{f}(\alpha^g) \propto \alpha^g$ for $|\alpha^g| \gg 1$. [$(-2z/\pi)(g \ln z^2/\pi)$ is the combination that, upon analytical continuation, leads to $\nu^g(\varepsilon) \propto |\varepsilon|$ arbitrarily close to $\varepsilon=0$.] By the duality assumption $\tilde{f}(\alpha^u) \propto \alpha^u$ must also hold outside of the radius of convergences

(8b) [leading to the combination $(-2z/\pi)(\pi\rho/z^2 \ln z^2)$]. Upon analytical continuation, duality then predicts a diverging DOS $\nu^u(\varepsilon) \propto 1/|\varepsilon| \ln^2|\varepsilon|$ arbitrarily close to the Fermi energy ε_F in the UL. The robustness of this conjecture requires a careful study of the role played by diagrams with crossing impurity lines (to which belong the Goldstone modes in NLSM terminology) in the UL that is beyond the scope of this paper.

In this paper we have constructed an effective field theory to calculate the average DOS of a dirty d -wave superconductor in the presence of Poissonian disorder. There are two bare couplings V_0 and ρ that characterize the disorder. For very weak single impurity potential strength V_0 but very large impurity concentration ρ this effective theory coincides with the Gaussian model of disorder, and reduces to a NLSM studied in Refs. 10 and 13. In the (unitary) limit in which ρ is very small and V_0 is tuned to the resonance value V_{0r} the DOS diverges upon approaching the Fermi energy term by term in an expansion in ρ . Within a variational approach, we identify a duality relation between the series for the Gaussian and the (resonant) unitary limits that relates the linearly vanishing DOS in the Gaussian limit to the diverging one $1/|\varepsilon| \ln^2|\varepsilon|$ in the unitary limit. In this way the apparent dichotomy between a DOS controlled solely by RMT in the Gaussian limit and one controlled solely by the nodes of the gap is elegantly resolved. However, within the same variational approximation we conclude that the DOS vanishes linearly at ε_F for generic bare values of V_0 and ρ , i.e., as soon as $V_0 \neq V_{0r}$. The emerging picture is that the Gaussian limit^{10,13} is a stable fixed point governing the scaling of the DOS at low energies, while the unitary limit is an unstable fixed point, dual to the Gaussian fixed point, and accessible only by exact tuning to the resonance.

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