Determination of the Spectroscopic Quadrupole Moments of $^{75}_{33}\text{As}$ and $^{63}_{29}\text{Cu}$

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The HFS-splitting of the $2s-2p$ and $3d-2p$ transitions of muonic arsenic and $3d-2p$ transitions of copper have been measured with high resolution Ge-semiconductor detectors to determine the nuclear quadrupole moments. A coincidence measurement was performed to separate the $2s-2p$ transition from the background. The nuclear spectroscopic quadrupole moments were found to be:

$$Q_{\text{spec}} = 0.314(6)\text{b} \quad \text{for } ^{75}_{33}\text{As}$$

$$Q_{\text{spec}} = 0.220(15)\text{b} \quad \text{for } ^{63}_{29}\text{Cu}.$$

In addition, generalized Barrett moments were determined in a model-independent analysis of muonic transition energies.

I. Introduction

Ever since the first experiments in muonic-atom physics there has been a continuous effort to extract information about nuclear sizes and shapes from the measured transition energies [1]. For the measurement of monopole charge radii, recent work has led to a method of analysis which, in a nearly 'model-independent' way, has yielded generalized moments for the nuclear size [2, 3]. For deformed nuclei an equivalent method of analysis exists since the work of the LAMPF group [4]. The situation for these deformed nuclei, however, is much more complicated because of the strong quadrupole mixing of excited muonic states. The results of this kind of evaluation yield not only quadrupole moments for the ground state but also for excited states. Such an approach meets the requirements for deformed nuclei in the rare earth region and beyond. For most other nuclei of lower Z , however, no measurable nuclear excitation during the muonic cascade can

occur. Moreover, for $Z \lesssim 40$ the orbit size of even the $2p$ level is large compared to the nuclear dimensions but also small compared to the size of the electron shell so that the muonic atom can be treated as a 'hydrogen-like' system. This fact had been used by Leisi and coworkers [5] in order to determine spectroscopic quadrupole moments from $4-3$, $5-3$, $5-4$ transitions in the rare earth region. The quadrupole splitting ΔE_2^{HFS} of these hydrogen-like states can be described by a product $W(I, j, F) \times A_2$. Where $W(I, j, F)$ is only a function of the quantum numbers of the states [14]

$$W(I, j, F) = \frac{6[D(D+1) - 4/3 I(I+1)j(j+1)]}{2I(2I-1)2j(2j-1)}$$

with $D = F(F+1) - I(I+1) - j(j+1)$ and I = Nuclear spin; F = Total angular momentum of the state; j = Total angular momentum of the muon.

The quadrupole constant A_2 is a product of the nuclear spectroscopic quadrupole moment Q_{spec} and the field gradient the muon produces at the nuclear

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site as in the case of electronic atoms [6]

$$A_2 = e^2(2j-1)/(8\{j+1\})Q_{\text{spec}}\langle\Phi_0|(1/r^3)|\Phi_0\rangle$$

with Φ_0 : Muonic wavefunction for pointlike nucleus.

For electronic atoms A_2 can be determined with high precision because of the higher developed techniques in this field [7] but the quadrupole moments extracted from these data have large inherent systematic errors because of the badly known field gradient in multi-electronic systems. The resulting systematic errors are typically of the order of 20 %. In muonic atoms the difficulty is twofold. Even in the case of muonic As the $2p_{3/2}$ -quadrupole splitting deviates by about 5 % from the calculated splitting taking Dirac wave-functions for a point-like nucleus. Because of that reason all our calculations for hyperfine splittings have been performed with the program RUP [10]. In our case, a diagonalisation of the Hamiltonian of the quadrupole interaction in the model space, consisting of the $2p_{3/2}$, $2p_{1/2}$ and nuclear ground states, has been performed. Dynamical excitation of the nucleus was shown to play no sensitive role. Moreover, second order vacuum polarisation was included in the calculations. The second difficulty in muonic atoms, however, is of experimental origin due to the relatively bad resolution of the X-ray detectors, when compared with the accuracy obtained in experiments with electronic atoms.

The aim of this work is to go a step further by choosing ‘novel techniques’ with semiconductor detectors in order to bridge the gap between the up to now accessible region ($Z \approx 40-50$) and the region of $Z \approx 10-15$ where crystal spectrometers work best [8].

II. The Experiment

The aim of the As-experiment was to detect the weak $2s-2p$ transitions with coincidence techniques, described elsewhere [9]. These transitions are rather sensitive to nuclear parameters and have a simpler structure than the $3d-2p$ transitions. Furthermore the absolute energy resolution of the detectors is better in the energy region of the $2s-2p$ transitions ($\approx 30-60$ keV) than in the energy region of the $2p-1s$ transitions (≈ 1.5 MeV), both being most sensitive to nuclear parameters.

The experiment was performed at the high-intensity $\mu\text{E}1$ beam at SIN. The stopping of the muons was defined by a conventional scintillator telescope. The X-ray spectra were measured with an intrinsic Ge-detector (IGP87) with an area of 80 mm^2 and a

Table 1. Experimental counting rates at the measurement of arsenic

Detector coincidence	Counting rate
μ -stop	$\approx 10^6$
IGP $\cap$ μ -stop	$\approx 10^3$
CsF $\cap$ μ -stop	$\approx 10^4$
CsF $\cap$ IGP $\cap$ μ -stop	≈ 50

thickness of 7 mm. It had a resolution of about 380 eV at 55 keV and about 1.15 keV at 430 keV.

The used isotopically enriched (99 %) ^{63}Cu target of 10 g was packed in a cylindrical box with an inner diameter of 7.5 cm. The ^{75}As target (40 g) was also cylindrical with a diameter of 5 cm. The targets were placed under $2 \times 45^\circ$ with respect to the beam. In order to detect the $2s-2p$ transitions in arsenic, a coincidence measurement was performed between a CsF-crystal, which identified the high energetic $2p-1s$ transitions, and the Ge-detector. A CsF-detector, although of poor energy resolution (25 % at 1.6 MeV) compared to a more conventional NaJ(Tl)-detector, was chosen because of its ability to stand high counting rates (up to several 10^6 counts/s). The coincidence requirement allows detection only of events coming from the target material, suppressing muonic N, C, O lines from the scintillation counters and the wrapping around the target and the target box. Typical counting rates during the measurement of arsenic are shown in Table 1.

The calibration of the $2s-2p$ transition of the As-spectrum was performed in the following way:

Firstly, the singles spectrum was calibrated with the energies of the N, C, O lines. The energies of arsenic transitions were determined using this calibration, which then in turn served to calibrate the coincidence spectrum.

The calibration of the spectrum in the energy region of the $3d-2p$ transition was performed in a direct way by feeding through events of radioactive sources in the prompt coincidence window with a width of 9 ns.

III. The Analysis

Energies and relative intensities of the muonic X-rays were computed with the programs RUP and MUON [10]. The finite size effect and QED-corrections as well as the relativistic nuclear recoil and the electron screening have been taken into account [11, 12]. The program MUON computes these corrections with a numerical precision of about 1 eV. The $2p_{3/2}$ level is split by the quadrupole interaction

Fig. 1. Spectrum of the $2s_{1/2} - 2p_{3/2}$ transition in As. The relative intensities of the HFS-components are indicated by lines. The total angular momentum F of the HFS levels, to which the transitions occurred, is indicated

into 4 HFS levels with spin $F=0, 1, 2, 3$ [6]. The $2s_{1/2}$ level shows no quadrupole splitting, because all wave functions with a spherically symmetric charge distribution, such as the $s_{1/2}$ and $p_{1/2}$ levels possess no electric field gradient. They only show a splitting caused by the magnetic dipole interaction ($M1$ HFS), which, for muonic atoms, is smaller by a factor 207 than the quadrupole interaction, since the magnetic moment is inversely proportional to the mass. The interesting part of the spectrum is shown in Fig. 1. Here the six HFS-transitions are visible. The transition $10-6$ also lies in this energy region, which always was kept fixed in position during the evaluation. The energy difference between the two lines to the $F=2$ and to the $F=1$ corresponds to the magnetic splitting of the $2s_{1/2}$ level. The energy resolution of the detector did not allow an evaluation of the $M1$ HFS-splitting with a relative accuracy of better than 65 %. Hence this was not treated further.

In the spectrum of the $3d-2p$ transitions (Fig. 2) more HFS lines appear also because the $3d_{3/2} - 2p_{3/2}$ transition lies in the energy region of the $3d_{5/2} - 2p_{3/2}$ transition. There are 18 HFS lines al-

together, which in Fig. 2 are averaged for ease of interpretation.

In the fitting program EVAL [13] the energy differences of the HFS lines as a function of the quadrupole moment were given as input for a best fit procedure.

The quadrupole moments from the $2s_{1/2} - 2p_{3/2}$ and $3d_{5/2} - 2p_{3/2}$ transitions for arsenic were determined to be:

$$Q_{\text{spec}} = 0.307(13)b \quad \text{from } 2s_{1/2} - 2p_{3/2}$$

$$Q_{\text{spec}} = 0.315(6)b \quad \text{from } 3d_{5/2} - 2p_{3/2}.$$

The weighted average is:

$$Q_{\text{spec}} = 0.314(6)b.$$

For the $A=63$ -isotope of copper no coincidence measurement could be performed. The evaluation proceeded along the same lines as in arsenic. The following result for the quadrupole moment evaluated from the $3d_{5/2} - 2p_{3/2}$ transition was obtained

$$Q_{\text{spec}} = 0.220(15)b.$$

Fig. 2. Spectrum of the $3d-2p$ transitions in As. The relative intensities of the HFS-components, calculated by the program RURP, are averaged and indicated by lines

IV. Nuclear Monopole Charge Distributions

From our measurements we deduced nuclear monopole charge distributions using the generalized Barrett moments [3] defined as:

$$\langle r^k e^{-\alpha r} \rangle = 4\pi \int_0^{R_k} \rho(r) r^k e^{-\alpha r} r^2 dr.$$

This means that the transition potential is fitted with the following function:

$$V_{\mu}^{i \rightarrow f} = B r^k e^{-\alpha r}$$

α is a specific constant for each element and nearly a linear function of Z and R_k is the radius for a uniformly charged sphere which gives the same Barrett moments. The results are shown in Table 2. $C_z = dR_k/dE$ is the sensitivity of the equivalent radius R_k to a change in the muonic transition energy.

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Table 2. Results of the nuclear charge distribution analysis

Element	Transition	k	R_k (fm)	C_z (fm/keV)
$^{75}_{33}\text{As}$	$2s_{1/2} - 2p_{3/2}$	1.969(20)	5.225(2)	+0.069
	$2s_{1/2} - 2p_{1/2}$	1.956 (8)	5.241(2)	+0.063
	$3d_{3/2} - 2p_{3/2}$	4.106 (5)	5.69(17)	-1.561
	$3d_{3/2} - 2p_{1/2}$	3.382 (1)	5.31(11)	-1.012
	$3d_{5/2} - 2p_{3/2}$	4.098(11)	5.54(18)	-1.663
$^{63}_{29}\text{Cu}$	$3d_{3/2} - 2p_{3/2}$	4.119 (4)	5.29(13)	-3.821
	$3d_{3/2} - 2p_{1/2}$	3.318(20)	4.883(9)	-2.395
	$3d_{5/2} - 2p_{3/2}$	4.116 (4)	5.16(14)	-4.080

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