Comment on “Activation Gaps and Mass Enhancement of Composite Fermions”

In a recent Letter [1], Park and Jain presented results of extensive Monte Carlo calculations of gaps of polarized fractional quantum Hall states at filling fractions $\nu = 1/3, 2/5, 3/7, 4/9$. In particular, they addressed the effects on these excitation energies of the finite width of the electronic wave function (wf) in the direction perpendicular to the interface. Their findings appeared to demonstrate that finite width corrections alone lead to agreement between experimental and theoretical values for these energy gaps.

This agreement depends crucially on the choice of the width parameter λ in the model interaction

$$V(r) = \frac{e^2}{\epsilon \sqrt{r^2 + \lambda^2}}, \quad (1)$$

that was used to take account of the width of the interface wf in an approximate manner. To fix λ , Park and Jain used results based on calculations of interface wf's by Ortalan *et al.* [2], by fitting the gap at $\nu = 1/3$ for a system of six electrons [2]. This fit led to a value $\lambda = 11.7$ nm for an electronic density $n_S = 2.3 \times 10^{11} \text{ cm}^{-2}$ of the sample used by Du *et al.* [3]. This value for λ leads to the above mentioned agreement between theoretical and experimental gap values (cf. Fig. 3 in [1]).

The main problem with this way of fixing λ is that there are indications that the work of Ortalan *et al.* is not reliable [4]: Analysis of the tabulated values of the Haldane pseudopotentials reveals serious inconsistencies and the calculation of energy gaps is not explained in enough detail to allow a verification of the results. Furthermore, as will be discussed below, the value of λ which fits the gap results does not appear reasonable, although Park and Jain claim that it is.

To see this we employ as interface wf the Fang-Howard variational wf $\Psi(z) \sim \exp(-bz/2)$ [5]. The variational parameter b is given by

$$b = \left(\frac{48\pi m^* e^2 (n_{\text{depl}} + \frac{11}{32} n_S)}{\epsilon \hbar^2} \right)^{1/3}, \quad (2)$$

where m^* is the effective mass in the direction normal to the interface and n_{depl} is the charge density in the depletion layer. Using the values for GaAs, a dielectric constant $\epsilon = 12.7$, $m^* = 0.067m_0$ and neglecting n_{depl} , we obtain $1/b = 4.4$ nm at $n_S = 2.3 \times 10^{11} \text{ cm}^{-2}$. As gaps at filling factors $1/3 \leq \nu \leq 2/3$ are basically determined by the Haldane pseudopotential V_m [2] for angular momentum $m = 1$, the best way to fix λ is by requiring that V_1 is reproduced when calculated with the model interaction (1). Using the Fang-Howard wf with $1/b = 4.4$ nm, we obtain $V_1 = 0.3426e^2/\epsilon\ell_0$ at $\nu = 1/3$ (i.e., at a magnetic field $B = 28.5$ T and a magnetic length

$\ell_0 = 4.8$ nm). This V_1 value is reproduced using the model interaction (1) with $\lambda/\ell_0 = 1.56$, i.e., $\lambda = 7.5$ nm, less than $2/3$ of the value used by Park and Jain [6].

The question of wf width has been explored experimentally by Willett *et al.* [7]. By measuring the energy difference between the first and second subbands, they obtained a more accurate estimate of the subband wf's with the result $1/b = 3.9$ nm. As that sample had a smaller density $n_S = 1.65 \times 10^{11} \text{ cm}^{-2}$, and assuming b scales as $n_S^{1/3}$, we obtain a value $1/b = 3.5$ nm for the sample considered by Park and Jain, which is 20% smaller than the value based on Eq. (2). Again, requiring that V_1 be reproduced by the model interaction at $\nu = 1/3$, we obtain $\lambda/\ell_0 = 1.34$, or $\lambda = 6.4$ nm. On the basis of these results, we feel that the value of $\lambda = 11.7$ nm used by Park and Jain is too large by about a factor of 2.

Clearly, for $\lambda/\ell_0 \approx 1.3$ the gap reduction due to the width of the interface wf is only about half of what Park and Jain obtained. Thus, a significant difference between theoretical and experimental gap values remains. However, effects of disorder and Landau level mixing [8] are expected to further reduce the theoretical gap results.

A last comment regards the use of the model interaction (1). While it is possible to fix λ such that the pseudopotential coefficient V_m is reproduced exactly for $m = 1$, those for $m > 1$ when calculated with the model interaction are generally too large, especially for large λ [2,4]. Consequently, since incompressibility of polarized states in the range $1/3 \leq \nu \leq 2/3$ depends on the ratio V_3/V_1 not becoming too large, a phase transition to compressible states may be predicted erroneously if the model interaction (1) is used (cf. Fig. 3 in [1]). Realistic interactions based on appropriate interface wf's must be used for a reliable study of this question [4].

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