Direct Experimental Verification of Neutron Acceleration by the Material Optical Potential of Solid $^2\text{H}_2$

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We have measured the acceleration of neutrons by the material optical potential of solid $^2\text{H}_2$. Using a gravitational spectrometer, we find a minimal kinetic energy $E_c = (99 \pm 7)$ neV of neutrons from a superthermal ultracold neutron (UCN) source with solid $^2\text{H}_2$ as an UCN converter. The result is in excellent agreement with theoretical predictions, $E_c = 106$ neV.

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Measuring the very low energy end of the neutron spectrum from a solid $^2\text{H}_2$ crystal in an ultracold neutron (UCN) source gives a direct estimate for the material optical potential, see, e.g., Ref. [1], i.e., the averaged or coherent strong interaction potential, which arises in the theoretical treatment of the interaction of very low energy neutrons with materials in terms of refraction, e.g., Ref. [2]. Such a treatment is a specific example of the general problem of the multiple scattering of waves, which covers topics such as neutron diffusion, sound in turbid media, fundamental particles in media and electromagnetic waves by turbid media (Thomson, Rayleigh, Mie scattering). As these topics share a common theoretical basis they also serve as mutual homologues. Some comments on the general problem are given to assist putting the work being reported here into this general physics context.

The rigorous general solution of the multiple scattering of waves using wave mechanical arguments was made by Lax [3] and reference therein, who generalized the result of Foldy [4]

$$(\tilde{\chi})^2 - (\tilde{k})^2 = 4\pi G(\tilde{r}); \quad G(\tilde{r}) = \int_V g(s, \omega) \rho(\tilde{r}, s) ds, \quad (1)$$

obtained from consideration of the scattering of sound by bubbles in water. Here $\tilde{\chi}$ and $\tilde{k}$ are the momenta inside and outside the system, ω is the wave frequency, $G(\tilde{r})$ is an effective potential obtained from an integral of the microscopic scattering function, $g(s, \omega)$ with the density distribution $\rho(\tilde{r}, s)$ over the scattering parameter, s , and the subscript V is to indicate that an average over some suitable volume is involved.

The common basis is that the properties of the individual scattering processes come from solutions of the wave equation. Lax considers the scattering can be split between (i) a *coherent* part that leads to the macroscopic phenomena of refraction and specular reflection and (ii) an *incoherent* part that represents the effect of the scattering ensemble by creating the difference between the average and the root mean square of the outgoing wave function.

This leads to

$$(\tilde{\chi})^2 - (\tilde{k})^2 = 4\pi\rho Y f(a \leftarrow a). \quad (2)$$

Here, $f(a \leftarrow a)$ is the forward elastic scattering amplitude and Y is a correction factor equal to the ratio of the effective and coherent fields formed from averaging the scattering over some N -particle density distribution function (for random systems Y is normally very close to unity). The result can be applied to systems with anisotropic scattering, inelastic scattering, scattering of quantized waves, creation and absorption of particles, motion of the scattering centers, randomly and partially or completely ordered scattering centers.

Specializing to very low energy neutrons and using the Born approximation, Lax shows that the forward scattering amplitude for a single scattering is given by

$$\lim_{r \rightarrow \infty} \int_r V(\tilde{r} - \tilde{r}_i) - \mu \tilde{\sigma} \cdot \tilde{B}(\tilde{r} - \tilde{r}_i) d\tilde{r}, \quad (3)$$

where $V(\tilde{r} - \tilde{r}_i)$ is the material optical potential, μ the magnetic moment, $\tilde{\sigma}$ the spin vector and $\tilde{B}$ the internal magnetic field, and leads to the familiar result, e.g., Ref. [5], which contains references to the earlier work of Halpern and co-workers, and Ekstein:

$$(\tilde{\chi})^2 - (\tilde{k})^2 = -\left(\frac{2m}{\hbar^2}\right) [V_{\text{av}} \pm \mu |B_{\text{av}}|]. \quad (4)$$

Expressions for the potential V_{av} were given by Foldy [4], and Goldberger and Seitz [6].

The important consequence is that the interaction of very low energy neutrons with material can be changed into a barrier penetration or reflection problem, described by solution of the Schrödinger equation using the potential given by

$$V_{\text{av}} = \frac{2\pi N_0 b \hbar^2}{m} \left\{ \pm \sqrt{1 - \frac{k^2 \sigma_a^2}{4\pi \sigma_s}} + i \frac{k \sigma_a}{\sqrt{4\pi \sigma_s}} \right\} \pm \mu |B_{\text{av}}|, \quad (5)$$

where N_0 is the scattering center density, b the coherent scattering length (the thermal scattering cross-section

$\sigma_s = 4\pi b^2$) and σ_a the cross section for loss (absorption plus energy up-scattering), again at thermal energies.

Strictly and generally, the use of refractive index formulations applies only to systems of *infinite* extent, where it is valid to ignore the incident wave and boundary effects. However, it will be a good approximation [3] as long as the system under consideration has dimensions that are large compared with all three of (i) the extinction length (Laplace, Ewald-Oseen extinction theorem, see [7], in the case of light and, perhaps, the penetration length for the wave function [6] in the case of neutrons), (ii) the interparticle separation, and (iii) the wavelength of the particle [3]. Each condition is necessary but not sufficient. In such cases, the use of the continuity of the wave function and its gradient at the system interfaces are valid criteria for any solution. Because of the large variety of scattering problems that share this common theoretical treatment (neutron diffusion, etc., see above), there is extensive experimental evidence that application to finite systems does give reasonable agreement.

The material optical potential has been measured for most materials using reflectometry and thin film material transmission and is required for computing the center-of-mass velocity associated with absorption (or other $1/\nu$) cross-section values obtained from transmission data measured with very low energy neutrons, see, e.g., [8–10]. A further consequence of the material optical potential is that the energy of neutrons produced within a sample will be boosted as they cross the material boundary, leading to the external spectrum being shifted in energy; such a measurement will give a model independent value of the material optical potential but requires (i) a material that produces significant quantities of very low energy neutrons and (ii) measurement of the lower energy limit of the external neutron spectrum. This is possible, and has been carried out, by measuring for the first time the spectrum of UCNs produced in a solid $^2\text{H}_2$ moderator.

The experiment was performed at the TRIGA Mark II reactor of the Mainz University [11,12] which can be operated at a steady state power of 100 kW and in a pulsed mode with a maximal power of 250 MW and an energy release of about 10 MWs [13]. At this reactor, a new UCN source with a solid $^2\text{H}_2$ converter operating at 5 K was recently installed [14]. This material is widely used for the design of new and very intense UCN sources [15–18]. Detailed reports on the source principle and first measurements of the source parameters and performance will be published elsewhere [19].

In order to determine the UCN energy spectrum from the superthermal solid $^2\text{H}_2$ converter, we used a gravitational spectrometer [20], see Fig. 1. The TRIGA reactor was operated in the pulsed mode with a reactivity insertion of $1.5\rho/\beta$ in the pulse (in the language of reactor users termed 1.5 \$ pulse), corresponding to a released energy of about 6.7 MWs. Here, $\rho = \delta k/k$ is the reactivity with

the effective multiplication factor k being the number of fissions in one generation divided by the number of fissions in the previous generation. The quantity β represents the number of delayed neutrons. For the TRIGA fuel elements with aluminum cladding and 8% by weight $U\text{-ZrH}$ the value of β is 0.0073 [13].

Ultracold neutrons leave the 5 K solid $^2\text{H}_2$ converter, pass through a 0.1 mm thin aluminum foil with material optical potential of 54 neV [21] installed for safety reasons. The neutrons are guided through the biological shielding of the reactor strictly horizontally in electropolished stainless steel tubes of 66 mm inner diameter and about 4 m length. Outside the reactor core shielding, the neutrons enter a U -shaped chicane built from the same material. The “ U ” uses gravity to determine the minimum energy of thoroughgoing UCNs; its height and hence the minimum UCN energy may be adjusted by rotation around the neutron guide axis. A vacuum shutter was installed in front of the U , cf. Fig. 1, to prevent an inrush of air into the vacuum system of the cold source during the rotation process. Unfortunately, about 40% of the UCNs were absorbed in this gadget. Passing neutrons are registered in a gas counter containing 18 hPa ^3He and 12 hPa CO_2 in about 1070 hPa Ar. The detector with its 0.1 mm aluminum entrance window was mounted 60 cm below the beam axis. In this way, gravitation accelerates UCNs such that all of them can penetrate the detector entrance window with a material optical potential of 54 neV.

The data were acquired during two consecutive days. Over night, the 5 K $^2\text{H}_2$ converter had to be warmed up and cooled down the next morning before the second block of data taking was started. This might have caused a non-statistical uncertainty, which was taken into account as described below. Furthermore, it cannot be excluded that certain source parameters may have changed with time leading to a drift in UCN intensity. In order to minimize the influence of such drifts, the sequence of data taking at different heights was determined randomly. Figure 2 shows

FIG. 1. Sketch of the gravitational spectrometer; 1: vacuum shutter; 2: 10 cm borated polyethylene; 3: UCN detector with 0.1 mm aluminum entrance window. The detector was shielded against background in a box of 10 cm borated polyethylene (not shown).

FIG. 2. Data taken with the gravitational spectrometer. The UCN transmission rates per megajoule of the reactor power after background subtraction are plotted against the vertical height of the gravitational spectrometer. The two lines represent the fit to the data, see text.

the registered neutrons normalized to the reactor power. Each data point consists of counting neutrons after two consecutive reactor pulses with a time interval between 3 and 10 min. In order to determine the detector background during a reactor pulse, two measurements were performed, one before freezing of the $^2\text{H}_2$ and a second one with the vacuum shutter closed. The two methods showed identical background rates of 20 counts per megajoule (MWs).

The reactor pulses had an energy of typically 6.7 MWs. A 2% uncertainty of the reactor yield in the pulsed mode at constant released energy was estimated and added in quadrature to the statistics uncertainty. To the left of the data shown in Fig. 2, one can see a flat distribution over increasing height of the spectrometer. Above about 100 cm, corresponding to ~ 100 neV neutron kinetic energy, the spectrum decreases. This can be compared to the material optical potential of solid $^2\text{H}_2$ at 5 K calculated to be about 106 neV [21]. For the analysis, we have introduced a simple model: UCNs gain energy through acceleration in the coherent strong interaction surface potential when they leave the solid $^2\text{H}_2$ converter. Consequently, the minimal energy of UCN from the new source at the Mainz TRIGA reactor corresponds to the material optical potential of solid $^2\text{H}_2$. Above this threshold, the energy spectrum is expected to rise linearly (i.e., quadratically with velocity, v^2) [22]. This can be seen in Fig. 2 from the linear intensity decrease of dN/dE with increasing height. The minimal energy of UCNs from the source was determined by fitting a constant A to the flat distribution and a straight line, $y =$

$Bx + C$, to the beginning of the slope at height values above 100 cm; see Fig. 2. From the crossing of these two functions, we deduce the minimal UCN energy, i.e., the material optical potential of solid $^2\text{H}_2$ at 5 K. The material optical potential is proportional to the density of the material. The density gradient in solid $^2\text{H}_2$ at that temperature is about $0.00006 \text{ g/cm}^3/\text{deg}$. Uncertainties from the determination of the temperature are thus negligible.

The fitted parameters are $A = 1002 \pm 12$, $B = -10.1 \pm 0.6$ and $C = 2002 \pm 86$. The χ^2 per degree-of-freedom of the two fits were 1.0 for determining A and 0.93 for the straight line. With these values, we find $E_c = (99 \pm 7) \text{ neV}$, in agreement with expectations. The bend in the spectrum around 200 cm originates from the material optical potential of stainless steel (190 neV) [23]: UCNs with higher energies are no longer totally reflected under all angles and the U -chicane serves as an energy filter. The small number of events above 190 neV is attributed to scattered neutrons passing this filter.

In conclusion, we have measured for the first time the acceleration of neutrons by the material optical potential of solid $^2\text{H}_2$. Using a gravitational spectrometer, we find a minimal kinetic energy $E_c = (99 \pm 7) \text{ neV}$ of neutrons from a superthermal UCN source with solid $^2\text{H}_2$ as an UCN converter. The result is in excellent agreement with theoretical predictions, $E_c = 106 \text{ neV}$.

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