About APPLE II Operation

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Abstract. The operation of an APPLE II based undulator beamline with all its polarization states (linear horizontal and vertical, circular and elliptical, and continuous variation of the linear vector) requires an effective description allowing an automated calculation of gap and shift parameter as function of energy and operation mode. The extension of the linear polarization range from 0 to 180° requires 4 shiftable magnet arrays [1] [2], permitting use of the APU (adjustable phase undulator) concept. Studies for a pure fixed gap APPLE II for the SLS [3] revealed surprising symmetries between circular and linear polarization modes allowing for simplified operation. A semi-analytical model covering all types of APPLE II and its implementation will be presented.

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APPLE II OPERATION MODEL

To first order, the gap in the APPLE II undulator [4] sets the photon energy, while the phase shift sets the polarization. In more detail, however, the shift is a nonlinear function of the gap. This model links the in situ measurable relation between gap and photon energy (i.e. the fit to the measured data points) with an analytical description of the shift dependency of the K-values. In general, the equations obtained are transcendent and have to be solved numerically.

Gap Dependence

Though made of pure permanent magnets the simplest model $B = a \exp(-\pi g / \lambda_U)$ [5] cannot describe the gap dependence of the shift for a constant ratio B_z / B_x , i.e. the circular mode. A more appropriate description is the model for hybrid undulators $B = a \exp(-bg / \lambda_U + cg^2 / \lambda_U^2)$, with a, b, c constants, g gap and λ_U the period length. Energy and field are related via the resonance condition:

$$E = \frac{C}{1 + K^2/2} \quad (1)$$

with $C = 1.2410^{-6} \cdot 2\gamma^2 / \lambda_U$, $K = 0.934 \cdot B \lambda_U$. In practice, we are interested in the relation between gap and photon energy. For ID control as implemented at SLS, both $E = E(g)$ and $g = g(E)$ are of interest. The functions can be written as:

$$E = \frac{A_0}{1 + A_1 \exp(g(A_2 + A_3 g))}, \quad g = -\frac{A_2}{2A_3} \pm \sqrt{\left(\frac{A_2}{2A_3}\right)^2 + \frac{1}{A_3} \ln\left(\frac{A_0/E - 1}{A_1}\right)}. \quad (2)$$

The fit to the photon energies is needed to involve the red shift of the photon energy due to emittance effects and the angle acceptance of the beamline. The energies have to be fitted only for the cases of horizontal and vertical polarization (LH and LV), which allows calculation of the maxima of the field components K_{z0} and K_{x0} as functions of gap.

Shift Variation

To describe the shift dependence of K_z and K_x at a given gap, the contributions from the 4 magnet arrays (indices $i = 1 \dots 4$) have to be summed up, taking into account their relative phase shifts in the various operation modes.

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Generalizing the approach for the standard APPLE II [6] to 4 shift axes allows phase shifts (diagonal rows) and energy shifts (two rows side by side), ϕ_i and ρ_i respectively:

$$\begin{aligned} K_z(s) &= K_{zi} [\cos(ks + \phi_1 + \rho_1) + \cos(ks + \phi_2 + \rho_2) + \cos(ks + \phi_3 + \rho_3) + \cos(ks + \phi_4 + \rho_4)] \\ K_x(s) &= K_{xi} [\cos(ks + \phi_1 + \rho_1) - \cos(ks + \phi_2 + \rho_2) + \cos(ks + \phi_3 + \rho_3) - \cos(ks + \phi_4 + \rho_4)], \end{aligned}$$

with $ks = 2\pi/\lambda_U s$ and s the longitudinal coordinate. Without limiting generality we restrict to phase shifts using ϕ_1 and ϕ_3 and energy shifts $\rho_1 = \rho_2 = \rho$ and assume equal K_{zi} , respectively equal K_{xi} . For correct calculations it is important to note the different shifts of the maxima:

$$\begin{array}{ll} K_z = s_0 + \frac{\phi}{2} + \frac{\rho}{2} & K_z = s_0 + \frac{\rho}{2} \\ K_x = s_0 + \frac{\lambda_U}{4} + \frac{\phi}{2} + \frac{\rho}{2} & K_x = s_0 + \frac{\rho}{2} \\ \text{in circular mode} & \text{in linear mode} \end{array}$$

The link between the shift variation and the gap dependency is given via the angle and the effective fields ($K_{\text{eff}}^2 = K_x^2 + K_z^2$).

$$\tan \alpha = K_z/K_x, \quad E = \frac{C}{1 + K_{\text{eff}}^2/2}. \quad (3)$$

APPLE II with 2 Shift Axes

The standard APPLE II offers only the phase shifts with ϕ_1 and ϕ_3 and no energy shift. In the **circular case** with a symmetric motion the phase shift is set to $\phi_1 = \phi_3 = \phi$:

$$\begin{aligned} K_z(s) &= K_{zi} (2 \cos(ks + \phi) + 2 \cos ks) \\ K_x(s) &= K_{xi} (2 \cos(ks + \phi) - 2 \cos ks) \end{aligned}$$

Using the addition theorems $\cos x + \cos y = 2 \cos(x+y)/2 \cos(x-y)/2$ and $\cos x - \cos y = -2 \sin(x+y)/2 \sin(x-y)/2$, we get for the maxima

$$K_z = K_{z0} \cos \frac{\phi}{2}, \quad K_x = -K_{x0} \sin \frac{\phi}{2} \quad (4)$$

In the **linear case**, with antisymmetric motion $\phi_2 = -\phi_1$ and $|\phi_2| = \phi_1 = \phi$:

$$\begin{aligned} K_z(s) &= K_{zi} (\cos(ks + \phi) + \cos(ks - \phi) + 2 \cos(ks)) \\ K_x(s) &= K_{xi} (\cos(ks + \phi) + \cos(ks - \phi) - 2 \cos(ks)) \end{aligned}$$

Using the addition theorems twice, the maxima show a quadratic dependence:

$$K_z = K_{z0} \cos^2 \frac{\phi}{2}, \quad K_x = -K_{x0} \sin^2 \frac{\phi}{2} \quad (5)$$

For circular polarized light the K_x is equal to K_z ($\tan \alpha = 1$), respectively $K_x = 0.8(0.6) \times K_z$ for elliptical light from the 3rd (5th) harmonics).

$$\begin{array}{ll} \text{circular} & \text{linear} \\ \phi = 2 \arctan \left(R_h \frac{K_{z0}}{K_{x0}} \right), \quad R_h = 1, 0.8, 0.6 & \phi = 2 \arctan \sqrt{\frac{1}{\tan(\alpha)} \frac{K_{z0}}{K_{x0}}} \end{array} \quad (6)$$

$$\begin{array}{ll} E = \frac{C}{1 + 0.5 \left(K_{z0}^2 \cos^2 \frac{\phi}{2} + K_{x0}^2 \sin^2 \frac{\phi}{2} \right)} & E = \frac{C}{1 + 0.5 \left(K_{z0}^2 \cos^4 \frac{\phi}{2} + K_{x0}^2 \sin^4 \frac{\phi}{2} \right)} \end{array} \quad (7)$$

Implementing the phase shifts ϕ in the energy equations 7 allows numeric calculation of the gap using equations 1 and 2, followed by calculation of the phase shifts.

APPLE II with 4 Shift Axes

In addition to allowing 180° operation [2] by using shift axes 2 and 4 instead of 1 and 3, the 4 shift axes allow improved operation conditions. The energy shift ρ can replace the gap g to vary the energies, as first proposed by R. Carr [7] [8] with the adjustable phase undulator (APU). Again, summing up the contributions, in the **circular case** ($\phi_3 = \phi_1 = \phi$, $\rho_1 = \rho_2 = \rho$)

$$\begin{aligned} K_z(s) &= K_{zi} [\cos(ks + \phi + \rho) + \cos(ks + \rho) + \cos(ks + \phi) + \cos ks] \\ K_x(s) &= K_{xi} [\cos(ks + \phi + \rho) - \cos(ks + \rho) + \cos(ks + \phi) - \cos ks] \end{aligned}$$

the maxima can be expressed as:¹

$$\begin{aligned} K_z &= K_{z0} \cos \frac{\phi}{2} \cos \frac{\rho}{2} \\ K_x &= -K_{x0} \sin \frac{\phi}{2} \cos \frac{\rho}{2} \end{aligned}$$

The ratio of K_z/K_x and hence the degree of polarization keeps constant for all energies or ρ -settings[8], the ratio K_z/K_x varies only with a gap change. Again, using angle and energy relations, (eqn 3), now allows ϕ and ρ to be calculated:

$$\phi = 2 \arctan R_h \frac{K_{z0}}{K_{x0}}, \quad R_h = 1(0.8, 0.6) \quad (8)$$

$$\begin{aligned} E &= \frac{C}{1 + \frac{K_{\text{eff}}^2}{2}} = \frac{C}{1 + 0.5 \cos^2 \frac{\rho}{2} \left[K_{z0}^2 \cos^2 \frac{\phi}{2} + K_{x0}^2 \sin^2 \frac{\phi}{2} \right]} \\ \rho &= 2 \arccos \sqrt{2 \left(\frac{C}{E} - 1 \right) \frac{1}{K_{z0}^2 \cos^2 \frac{\phi}{2} + K_{x0}^2 \sin^2 \frac{\phi}{2}}} \end{aligned} \quad (9)$$

The phase shift is indeed independent of ρ , which allows a closed analytical description with equations 8 and 9 in the circular (elliptical) mode. In the **linear case** with $\phi_3 = -\phi_1$ the magnetic fields sum up to:

$$\begin{aligned} K_z(s) &= K_{zi} [\cos(ks + \phi + \rho) + \cos(ks + \rho) + \cos(ks - \phi) + \cos ks] \\ K_x(s) &= K_{xi} [\cos(ks + \phi + \rho) - \cos(ks + \rho) + \cos(ks - \phi) - \cos ks] \end{aligned}$$

$$\begin{aligned} K_z(s) &= K_{z0} \cos \frac{\phi}{2} \cos \frac{2ks + \rho}{2} \cos \frac{\phi + \rho}{2} \\ K_x(s) &= K_{x0} \sin \frac{\phi}{2} \cos \frac{2ks + \rho}{2} \sin \frac{\phi + \rho}{2} \end{aligned}$$

The angle and energy condition are:

$$\begin{aligned} \tan \alpha &= \frac{K_{z0}}{K_{x0}} \cot \frac{\phi}{2} \cot \frac{\phi + \rho}{2} \\ \rho &= 2 \cdot \arctan \left(\frac{K_{z0}}{K_{x0}} \cot \frac{\phi}{2} \cot \alpha \right) - \phi \end{aligned} \quad (10)$$

$$E = \frac{C}{1 + 0.5 (K_{z0}^2 \cos^2 \frac{\phi}{2} \cos^2 \frac{\phi + \rho}{2} + K_{x0}^2 \sin^2 \frac{\phi}{2} \sin^2 \frac{\phi + \rho}{2})}. \quad (11)$$

Substitution of ρ in equation 11 allows the numerical calculation of $\phi = \phi(E, \alpha)$. These relations have been calculated for use in the fixed gap APPLE II undulator UE44, which will be installed in fall 2006 at SLS [3]. Calculations based on the algorithm above have been cross-checked with the RADIA model. The calculations agree to within 1% in energy and 0.5° in angle.

¹ More detailed calculations can be found in http://sfs.web.psi.ch/view.php/lsy/group_id/group_id_home.html

Symmetry Phase

During calculations for the fixed gap undulator, a deep symmetry in the APPLE II emerged. At a fixed gap APPLE II, in circular mode at $\phi = \arctan(K_{z0}/K_{x0})$ the energy shift ρ causes only a change of the photon energy while the degree of polarization remains constant, as already described in [8] (note that this is not the case for the gap change in the standard APPLE II). At exactly the setting for the phase shift for the circular condition, but now in the asymmetric mode, the photon energy variation with the energy shift ρ disappears for the entire range, and the angle variation becomes for this symmetry phase ϕ_s a linear function of ρ . The energy shift is no longer regarded as a mechanism for altering the energy. For calculating the condition of vanishing ρ dependency, it has to be shown that the $K_{\text{eff}} = K_z^2 + K_x^2$ is constant for a certain ϕ . The maxima shifts with half of the energy shift ($ks \rightarrow ks - \rho/2$), so that we get from equation 10:

$$E \propto K_{z0}^2 \left[\cos \frac{\phi}{2} \cos \frac{\phi + \rho}{2} \cos ks \right]^2 + K_{x0}^2 \left[\sin \frac{\phi}{2} \sin \frac{\phi + \rho}{2} \cos ks \right]^2 \equiv \text{const.}$$

Taking into account only the maxima and calculating the condition for the derivative being zero, results in

$$\begin{aligned} \frac{\partial K_{\text{eff}}(\phi, \rho)}{\partial \rho} &= 0 = -K_{z0}^2 \cos^2 \frac{\phi}{2} \cos \frac{\phi + \rho}{2} \sin \frac{\phi + \rho}{2} + K_{x0}^2 \sin^2 \frac{\phi}{2} \cos \frac{\phi + \rho}{2} \sin \frac{\phi + \rho}{2} \\ \phi_s &= 2 \arctan \frac{K_{z0}}{K_{x0}} \quad \text{q.e.d..} \end{aligned} \quad (12)$$

Introducing ϕ_s in equation 10 shows the linear angle variation:

$$\begin{aligned} \rho &= 2 \arctan \left(\frac{K_{z0}}{K_{x0}} \frac{1}{\tan \arctan \frac{K_{z0}}{K_{x0}}} \cot \alpha \right) - 2 \arctan \frac{K_{z0}}{K_{x0}} \\ &= 2 \arctan(\cot \alpha) - 2 \arctan \frac{K_{z0}}{K_{x0}} = 2 \left(\pm \frac{\pi}{2} \mp \alpha \right) - \phi_s, \end{aligned} \quad (13)$$

with $\cot x = \tan(\pi/2 - x)$, respectively $\arctan(\cot x) = \pi/2 - x$ if $0 < x < \pi$ or $\arctan(\cot x) = -\pi/2 + x$ if $-\pi < x < 0$.

At the symmetry phase ϕ_s , in circular mode the polarization stays constant while in linear mode the energy remains constant and the angle changes linearly with ρ . This is a beautiful symmetry which, however, can be fully exploited only in a 6 motor APPLE II. The energy has to be set by adjusting the gap.

CONCLUSION

A common description of all kind of APPLE II undulators has been presented based on a semianalytical model with lowest effort on in situ calibration measurements at the beamlines. Operation recipes are given for standard APPLE II undulators with 2 shift arrays as well as for a fixed gap APPLE II and an advanced operation mode for a 6 motor APPLE II. Work is ongoing in considering the energy shift due to emittance and aperture at the standard APPLE II at SLS. So far, we correct the measured energies by calculated red-shifts to the corresponding zero emittance on-axis energies. Energy settings vary up to about 3%, which at the moment are corrected by adjusting the energy.

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