Advanced Topics on RF Amplitude and Phase Detection for Low-Level RF Systems

Geng Zheqiao*, Roger Kalt

Paul Scherrer Institut, 5232 Villigen PSI, Switzerland

*Corresponding author. *E-mail address:* zheqiao.geng@psi.ch

Abstract: Low-Level RF (LLRF) systems stabilize the electromagnetic field in the RF cavities for beam acceleration in particle accelerators. Reliable, accurate and precise detection of RF amplitude and phase is particularly important to reach high field stability for pulsed accelerators of Free-Electron-Lasers (FEL). The digital LLRF systems employ Analog-to-digital Converters (ADC) to sample the frequency down-converted RF signal and use digital demodulation algorithms to calculate the RF amplitude and phase. Different sampling strategies and demodulation algorithms have been developed and will be introduced in this paper. This article will focus on some advanced topics for RF detection like the accurate RF transient measurement, wideband RF detection and the RF detection with asynchronous trigger, Local Oscillator (LO) or clock. The analysis will be based on the measurement from SwissFEL but the introduced algorithms are general for the RF signal detection in particle accelerators.

Key words: LLRF, RF Detector, RF Transient Detection, Asynchronous RF Detection

1 Introduction

Detection of the amplitude and phase of an RF signal is widely required in many subsystems of a particle accelerator, such as LLRF systems [1-8], beam diagnostics [9-11] and synchronization systems [12-14]. In LLRF systems, the RF detector is one of the key devices to diagnose and control the RF fields in cavities used for beam acceleration.

The major requirements for an RF detector include the accuracy, precision and response latency. With an accurate measurement of the RF amplitude and phase, we will be able to capture the exact changes in the RF field really felt by the beam. A precise RF detector will be able to detect small changes in the RF field or in other words, with high resolution in RF measurements. The latency of the RF detector is critical when using in an RF feedback loop for which the overall loop delay will limit the closed-loop bandwidth that the feedback loop can achieve [15],[16]. Generally, an RF detector with high accuracy, high precision and low latency is required in many LLRF systems.

Nowadays, digital RF detectors [17-19] are more and more adopted in LLRF systems. In this paper, we will assume the RF detector has a general architecture depicted in Fig. 1. The RF signal to be detected is firstly down-converted by mixing with a LO signal with a frequency different (normally lower) than the RF signal frequency. The lower side-band signal in the product of the mixer is picked up with a Low-Pass Filter (LPF), resulting in an Intermediate Frequency (IF) signal. The IF signal is sampled with an ADC and then demodulated with some digital algorithms to calculate the amplitude and phase of the RF signal. For pulsed RF signal, a trigger can be used to start the demodulation process. In this paper, we will focus on the digital demodulation algorithms [20-23] with the RF, LO and ADC clock frequencies fixed.

In most LLRF systems, the trigger, LO and clock are derived from the RF frequency to achieve synchronous sampling, which allows implementing some simple demodulation algorithms due to the deterministic relation in the clock and IF frequencies. With the synchronous sampling scheme, the RF phase measurement will have error if the relative phases of the clock and IF signals are changed, which is usually the case when power cycling the LO and clock generator. To overcome this problem, some

LLRF systems use asynchronous trigger, LO and clock signals generated by separate oscillators, resulting in the asynchronous sampling scheme. The asynchronous sampling scheme does not require deterministic phase relations between the trigger, LO, clock and RF signals but places difficulties in the design of demodulation algorithms.

Fig. 1. General architecture of an RF detector.

There are already quite a few articles to discuss the RF signal detection focusing on the accuracy, precision and latency requirements described above [24-28]. In this paper, some advanced topics for RF signal detection with special requirements will be discussed which can be met under different conditions when designing an RF detector. For these special requirements, different demodulation algorithms are needed based on the common RF detector architecture in Fig. 1. The following topics will be discussed:

- 1) *RF transient detection with non-IQ*: some RF signals may have transients with fast changing amplitude or phase which need to be measured accurately by the RF detector. This discussion will be based on the traditional non-IQ demodulation algorithm [21-23].
- 2) *Wideband RF detection*: in some applications, a large bandwidth of the digital demodulation is required to identify fast changes in the cavity field (e.g. beam transient measurements [29-32]).
- 3) *RF detection regardless of trigger*: in case a trigger is used to start the demodulation, the racing condition between the trigger and clock, which happens when the trigger rising edge is close to the clock rising edge in time, may introduce jumps in phase measurement. The RF detection should be independent to the trigger to avoid such problems.
- 4) *RF detection with asynchronous LO or clock*: in some applications, the phase locked LO and clock generator are not available, and this demodulation algorithm should allow using asynchronous LO or clock sources to perform RF amplitude and phase measurement.

2 Special RF Detection Algorithms

2.1 RF Transient Detection with non-IQ

The non-IQ algorithm is a popular demodulation algorithm for RF detection based on synchronous sampling of IF signal. The frequencies of the clock and IF signal satisfy

$$f_{CLK} = \frac{n}{m} f_{IF}, \quad (1)$$

where n and m are integers (usually $n > m$ for IF sampling and $n < m$ for sub-sampling) corresponding to sampling m IF periods with n samples. If the k th sample of the IF signal is denoted as x_k , the in-phase (I) and quadrature (Q) components of the RF envelop vector can be calculated with the last n samples as

$$\begin{aligned} I_i &= \frac{2}{n} \sum_{k=i-n+1}^i x_k \sin(k\Delta\varphi) \\ Q_i &= \frac{2}{n} \sum_{k=i-n+1}^i x_k \cos(k\Delta\varphi) \end{aligned}, \quad (2)$$

where the phase step $\Delta\varphi$ is the IF phase between two ADC samples which can be written as

$$\Delta\varphi = 2\pi m/n. \quad (3)$$

The non-IQ demodulation algorithm (2) can filter out the harmonics and DC offset in the IF samples [21],[22] with a reasonable latency if n is not too large. For example, the SwissFEL LLRF system [1],[2] uses a clock with a frequency 6 times the IF frequency, which corresponds to $n = 6$ and $m = 1$, resulting in a delay with only 3 clock cycles in the non-IQ demodulation. Due to such advantages, the non-IQ demodulation algorithm is widely used in LLRF systems. But when using it to measure the RF transients, such as the rising or falling edge of a pulse, big jumps are observed as shown by the solid lines in Fig. 2.

Fig. 2. Transient detection with non-IQ demodulation algorithm with an IF frequency 41.65 MHz and clock frequency 249.9 MHz. (a). amplitude waveform; (b). amplitude zoom-in at rising edge; (c). phase waveform; (d). phase zoom-in at rising edge.

The jumps shown in Fig. 2 are artifacts caused by the algorithm (2) when the changes of the IF amplitude or phase are fast. To understand the situation, we assume an IF signal with time-varying amplitude and phase:

$$x(t) = A(t) \sin[2\pi f_{IF}t + \varphi(t)]. \quad (4)$$

By sampling it at multiple of the clock period $T_s = 1/f_{CLK}$, equation (2) can be rewritten as

$$\begin{aligned} I_i &= \frac{2}{n} \sum_{k=i-n+1}^i A_k \sin(2\pi f_{IF}kT_s + \varphi_k) \cdot \sin(2\pi f_{IF}iT_s) \\ Q_i &= \frac{2}{n} \sum_{k=i-n+1}^i A_k \sin(2\pi f_{IF}kT_s + \varphi_k) \cdot \cos(2\pi f_{IF}iT_s) \end{aligned}, \quad (5)$$

where $A_k = A(kT_s)$ and $\varphi_k = \varphi(kT_s)$. Here we have used the relation $k\Delta\varphi = 2\pi f_{IF}kT_s$. The equation (5) can be rewritten as

$$\begin{aligned} I_i &= \frac{1}{n} \sum_{k=i-n+1}^i A_k \cos \varphi_k - \frac{1}{n} \sum_{k=i-n+1}^i A_k \cos(4\pi f_{IF}kT_s + \varphi_k) \\ Q_i &= \frac{1}{n} \sum_{k=i-n+1}^i A_k \sin \varphi_k + \frac{1}{n} \sum_{k=i-n+1}^i A_k \sin(4\pi f_{IF}kT_s + \varphi_k) \end{aligned}. \quad (6)$$

For both the I and Q outputs in (6), the first term are the average I and Q values of the IF signal with n samples which are the expected outputs of the demodulation algorithm; the second term are the double-IF-frequency components which will be automatically zeroed out by summing the n points if the amplitude and phase are constant. When the IF amplitude or phase change fast like in the transients of

an RF pulse, the second term will not vanish and appear as a systematic error with its spectrum centered at twice the IF frequency.

Fig. 3. (a). Spectrums of the baseband RF pulse; (b). Frequency responses of 3-tap and 6-tap moving-average FIR filters and the cascaded filter with non-IQ demodulation + 3-tap moving-average filter.

By checking the spectrum of the RF pulse in Fig. 2, higher noise Power Spectral Density (PSD) is visible around twice the IF frequency as predicted in equation (6), which is depicted with the solid-line spectrum in Fig. 3(a). The analysis above indicates that a notch filter at double of the IF frequency will be able to filter out the errors of the non-IQ demodulation algorithm for RF transient detection. Due to the fact that the clock frequency is synchronized with the IF frequency, a moving-average Finite Impulse Response (FIR) filter can also provide an accurate notch at double of the IF frequency. As shown in Fig. 3(b), a moving-average FIR filter with $n/2$ taps (with n an even number) or n taps (with n an odd number) can be used as a notch filter at double of the IF frequency. The moving-average FIR filter with 3 taps has been applied to SwissFEL LLRF system and resulted RF pulse baseband signal and spectrum have been shown as dotted-line plots in Fig. 2 and Fig. 3(a).

The moving-average FIR filter is easy to be implemented in digital processors like Digital Signal Processor (DSP) and Field-Programmable Gate Array (FPGA). Of course, it also generates extra magnitude attenuation at the frequencies near DC and a group delay for the RF detection, which is visible in the frequency response of the cascaded filter with non-IQ demodulation and 3-tap moving average filter in Fig. 3(b). When these issues are important concerns, a higher-order narrow-band notch filter can be used instead of the moving-average FIR filter.

2.2 Wideband RF Detection

The non-IQ demodulation algorithm requires n samples of the IF signal to do the demodulation which reduces the bandwidth of the sampling. From equation (6), the non-IQ demodulation with n samples corresponds to an n -tap moving average FIR filter for the baseband I and Q signals. In some applications, it is needed to detect a fast transient (e.g. beam transient in a cavity) which requires maximizing the detection bandwidth with the same time series of the IF signal. In principle, with less

number of samples to calculate the I and Q components, the larger bandwidth can be achieved. Here we will still consider the synchronous sampling with a deterministic IF phase lag $\Delta\varphi$ between two ADC samples. Let's assume that the ADC data is collected after a trigger and the first sample corresponds to a phase φ_0 of the IF signal, see Fig. 4 below.

Fig. 4. Synchronous sampling of IF signal.

The I and Q components of the IF samples in Fig. 4 are represented by the amplitude A_0 and initial phase φ_0 of the IF signal:

$$I = A_0 \cos \varphi_0, \quad Q = A_0 \sin \varphi_0. \quad (7)$$

In order to get the I and Q components, at least two samples are needed, resulting in the possible maximum RF detection bandwidth. We need to work out a demodulation algorithm which allows calculating the I and Q components from two adjacent samples. Counting from the first sample after the trigger, the k th and $(k+1)$ th sample of an ideal IF signal (single-tone without amplitude or phase noises) can be written as

$$\begin{aligned} x_k &= A_0 \sin[\varphi_0 + (k-1)\Delta\varphi] \\ x_{k+1} &= A_0 \sin(\varphi_0 + k\Delta\varphi) \end{aligned} \quad (8)$$

From (7) and (8), we can get

$$\begin{aligned} I &= \frac{x_{k+1} \cos[(k-1)\Delta\varphi] - x_k \cos(k\Delta\varphi)}{\sin \Delta\varphi} \\ Q &= \frac{x_k \sin(k\Delta\varphi) - x_{k+1} \sin[(k-1)\Delta\varphi]}{\sin \Delta\varphi} \end{aligned} \quad (9)$$

The phase-lag $\Delta\varphi$ between ADC samples should not be close to integer times of π to avoid singularities in the division in equation (9). A phase-lag close to π may cause the IF sampling close to the zero-crossing points of the IF signal, resulting in a low signal-to-noise ratio in ADC samples. This situation can happen if the phases of the IF and clock signals are aligned at some time point, when the clock samples a point close to the zero-crossing of the IF signal. Due to the close-to- π phase-lag, quite a few samples afterward will be still around the zero-crossing points of the IF signal.

The demodulation algorithm above has been applied to the same ADC samples in section 2.1 and the results are plot in Fig. 5 together with the results from non-IQ demodulation without moving-average filtering. Compared to the non-IQ demodulation algorithm, the wideband detection algorithm (9) captures faster rising edge of the RF pulse. This can also be explained by the spectrums of the baseband signals in Fig. 6(a) where more high-frequency components are kept with the wideband detection algorithm. The frequency responses of the non-IQ demodulation algorithm with $n = 6$ and the wideband detection algorithm (9) are compared in Fig. 6(b). It can be seen that the wideband detection algorithm has larger bandwidth without notches at the IF frequency and double of the IF frequency.

Fig. 5. Results of wideband detection algorithm and non-IQ demodulation algorithm for the same data in Fig. 2. (a). amplitude waveform; (b). amplitude zoom-in at rising edge; (c). phase waveform; (d). phase zoom-in at rising edge.

Fig. 6. (a). Spectrums of the baseband RF pulse; (b). Frequency responses of non-IQ demodulation and wideband detection algorithms.

The wideband detection algorithm can detect faster transients in the RF pulse with a cost of keeping all unwanted frequency components introduced in the down-conversion mixing and ADC sampling processes, such as the DC offset in ADC samples, leakage of IF frequency in demodulation and all higher-order harmonics of the IF frequency due to the non-linearity in the RF detector. When handling the results of the wideband detection algorithm, proper filtering is required to remove the unwanted frequency components or pick up the wanted frequency components. For example, a narrow-band notch filter may be needed to remove the harmonics of the IF frequency.

Another advantage of the fast detection algorithm (9) is that we can demodulate the samples of IF signal only if the relationship between the IF and clock frequencies are known. The non-IQ demodulation algorithm (2) requires the IF and clock frequencies satisfy (1). In some cases, the IF and clock frequencies may not have a simple relation as (1) or the n or m values become too large, therefore the algorithm (2) become not practical for implementation but the algorithm (9) can still be applied.

2.3 RF Detection Regardless of Trigger

For RF systems working in pulsed mode, the measurement of an RF pulse is usually started with a trigger as shown in Fig. 4. The phase of the first sample after the trigger is defined as the reference phase of the demodulation and the phase variations of the later samples are detected by removing the expected phase lags compared to the reference phase. Usually the trigger period, clock and IF frequencies are chosen in a proper combination (e.g. trigger period always covers integer times of the IF period and clock period) so that the first sample after each trigger locates at the same reference phase, resulting in a stable phase measurement for all pulses. In the practice, the trigger and clock signals are noisy with timing jitters and are distributed via cables with unknown delays. If the rising edges of the trigger and clock signals are very closely aligned in time, the trigger may fall randomly into two adjacent clock cycles and select different ADC samples. This is the so-called racing between clock and trigger, resulting in a random phase jump by $\pm\Delta\phi$. Such racing condition has been observed in the operation of SwissFEL LLRF system. Even if tuning the cable length to remove the racing condition, a change of clock phase after power cycling the master oscillator or clock generator may make the racing condition happens again. A demodulation algorithm independent to trigger is expected to overcome this problem.

As mentioned above, the trigger defines the reference phase, so a new reference is needed if the algorithm does not refer to the trigger. Usually the Master Oscillator (MO) signal, which is the reference signal to the entire RF station, can be down-converted and sampled with the same LO and clock. The measurement of the RF signal can be referred to the MO reference signal with the double-channel strategy in Fig. 7. Here we still use the synchronous sampling with the RF, LO and clock signals synchronized to the MO.

Fig. 7. Strategy of RF detection regardless of trigger.

In the digital signal processing part in firmware or software, the ADC samples of the reference signal is firstly filtered with a Band-Pass Filter (BPF) to remove the DC offset and higher-order harmonics of the IF frequency. The time series of the reference IF signal is then split into two signals with in-phase (0° phase shifted with proper delay) and quadrature (90° phase shifted) phases. Then the IF samples of the RF signal is multiplied with the two resulted reference signals and the I and Q components in baseband can be calculated with low-pass filtering. The LPFs in the I and Q output paths are used to remove the leakage of the IF frequency and the higher-order (especially the second-order) harmonics of the IF frequency.

With synchronous sampling, the $0^\circ/90^\circ$ splitter in Fig. 7 can be implemented in a simple way according to equation (9). The reference IF signal and its quadrature signal with 90° phase shifted are

denoted as $x_{ref,k} = A_{ref} \sin(2\pi f_{IF} k T_s)$ and $\tilde{x}_{ref,k} = A_{ref} \cos(2\pi f_{IF} k T_s)$ respectively for the k th sample. In order to generate the quadrature signal from the time series of the reference IF signal, the last two ADC samples denoted as $x_{ref,k-1}$ and $x_{ref,k}$ can be used. If we define the IF phase of the sample $x_{ref,k}$ as reference phase $\varphi_{0k} = 2\pi f_{IF} k T_s$, the latest two samples of the reference IF signal can be represented as

$$\begin{aligned} x_{ref,k} &= A_{ref} \sin \varphi_{0k}, \\ x_{ref,k-1} &= A_{ref} \sin(\varphi_{0k} - \Delta\varphi) \\ &= A_{ref} \sin \varphi_{0k} \cos \Delta\varphi - A_{ref} \cos \varphi_{0k} \sin \Delta\varphi \end{aligned} \quad (10)$$

As mentioned before, the k th sample of the quadrature signal of the reference IF signal is given by

$$\tilde{x}_{ref,k} = A_{ref} \cos \varphi_{0k}. \quad (11)$$

By comparing with (10), it can be calculated as

$$\tilde{x}_{ref,k} = (x_{ref,k} \cos \Delta\varphi - x_{ref,k-1}) / \sin \Delta\varphi. \quad (12)$$

Similar as the discussion for equation (9), the phase-lag $\Delta\varphi$ between two ADC samples shouldn't be close to integer times of π .

It can be seen that each point in the quadrature signal can be derived with the last two samples of the reference IF signal. Fig. 8 shows an example of the quadrature signal derived from the reference IF signal at SwissFEL, where the clock frequency is 6 times the IF frequency.

Fig. 8. Reference IF signal and derived quadrature signal with synchronous sampling at SwissFEL.

The LPFs in the I and Q output paths in Fig. 7 can also be implemented as simple moving-average FIR filters for synchronous sampling. As discussed in section 2.1, a moving-average FIR filter with n taps, where n is the minimum number of samples to cover full IF periods, will provide notches at different harmonics of the IF frequency.

Because the RF detection strategy in Fig. 7 always compares the RF phase and the MO phase regardless to the trigger, the phase jumps caused by the racing between trigger and clock can be avoided. For a pulsed system, the trigger is still needed to start firing the RF pulse or collecting baseband I and Q data for data acquisition. Fig. 9 shows the detection of the same RF pulse as in section 2.1 with the algorithm described in this section. The non-IQ demodulation results are also plotted as a comparison. The absolute phases are not equal in subplot (c) and (d) because the MO phase is also included in the phase calculation.

Fig. 9. Results of trigger independent detection algorithm and non-IQ demodulation algorithm for the same data in Fig. 2. (a). amplitude waveform; (b). amplitude zoom-in at rising edge; (c). phase waveform; (d). phase zoom-in at rising edge and the phase difference between two cases.

By using the MO signal as reference in Fig. 7, the amplitude and phase noises in the MO signal and the corresponding RF detector channel will be also transferred to the detection of the RF signal. This could reduce the signal-to-noise ratio (SNR) of the RF signal measurement and should be understood explicitly. Assume both the MO and RF signals are with noises and their IF signal samples can be written as

$$\begin{aligned} x_{ref,k} &= A_{ref} (1 + \Delta\alpha_{ref,k}) \sin(2\pi f_{IF} kT_s + \Delta\varphi_{ref,k}) \\ x_{rf,k} &= A_{rf} (1 + \Delta\alpha_{rf,k}) \sin(2\pi f_{IF} kT_s + \varphi_0 + \Delta\varphi_{rf,k}) + A_{rf0} \end{aligned} \quad (13)$$

where A_{ref} , $\Delta\alpha_{ref}$ and $\Delta\varphi_{ref}$ are the amplitude, relative amplitude noise and phase noise of the reference signal; A_{rf} , $\Delta\alpha_{rf}$, φ_0 , $\Delta\varphi_{rf}$ and A_{rf0} are the amplitude, relative amplitude noise, initial phase, phase noise and DC offset of the RF signal. Here k denotes the values of the parameters at time kT_s where T_s is the clock period and k is an integer. The reference signal does not have DC offset due to the BPF in Fig. 7. With equation (12), the quadrature signal can be written as

$$\begin{aligned} \tilde{x}_{ref,k} &= (x_{ref,k} \cos \Delta\varphi - x_{ref,k-1}) / \sin \Delta\varphi \\ &\approx A_{ref} (1 + \Delta\alpha_{ref,k}) \cos(2\pi f_{IF} kT_s + \Delta\varphi_{ref,k}) \end{aligned} \quad (14)$$

Here we have assumed the amplitude and phase noises of the reference signal are approximately constants for the two adjacent samples, which means, we only consider the noises within a bandwidth much smaller than the clock frequency. The outputs of the multipliers in Fig. 7 contain baseband signals, IF frequency signals and double IF frequency signals. The IF frequency outputs are generated by the DC offset in the RF signal while the double IF frequency outputs come from the multiplication of two signals both at IF frequency. After the LPFs, the I and Q baseband outputs can be written as

$$\begin{aligned} I &= \frac{A_{ref} A_{rf}}{2} (1 + \Delta\alpha_{ref,k} + \Delta\alpha_{rf,k}) \cos(\varphi_0 + \Delta\varphi_{rf,k} - \Delta\varphi_{ref,k}) \\ Q &= \frac{A_{ref} A_{rf}}{2} (1 + \Delta\alpha_{ref,k} + \Delta\alpha_{rf,k}) \sin(\varphi_0 + \Delta\varphi_{rf,k} - \Delta\varphi_{ref,k}) \end{aligned} \quad (15)$$

It can be seen that the baseband outputs contain amplitude and phase noises from both the reference signal and RF signal. The noises can come from the common LO and clock signals which result in

correlated noises or from the RF detector electronics like mixers and ADCs, resulting in uncorrelated noises. The noises in (15) are summarized below:

- 1) The amplitude noises of the reference and RF signals are summed up. When the two amplitude noises are correlated, they will be summed up in magnitude because the amplitude drifts in the two measurement channels change mostly in the same direction. When uncorrelated, the two amplitude noises will be summed up randomly.
- 2) The phase noises of the reference and RF signals are subtracted. When the two phase noises are correlated, they will be cancelled out if they change in the same direction, which is true within a certain bandwidth because they are mostly caused by the same phase drifts in the LO and clock signals. When uncorrelated, the two phase noises will be also summed up randomly because a minus sign does not influence the total power of two independent noises.

The low-frequency noises, which are the drifts in the amplitudes and phases of the reference and RF signals, are mostly correlated while the high-frequency noises are mostly uncorrelated. Equation (15) shows that we will always get worse amplitude noise with the RF detection strategy in Fig. 7, while for phase noise, we will get lower noise at low frequencies by tracking the reference phase drift and will get worse phase noise at high frequencies due to the sum of random noises in the two channels. This implies two updates of the RF detection strategy in Fig. 7 to reduce the noises. Firstly we need to narrow down the bandwidth of the BPF for the reference signal to only keep the low frequency noises which are correlated to the noises in the RF signal. Secondly, the amplitude of the reference signal should be replaced with a constant so that the amplitude noise in the reference signal does not appear in the I and Q outputs, leading to an upgrade of the reference signal path in Fig. 7 as in Fig. 10.

Fig. 10. Generation of the reference quadrature signal with amplitude noise reduction.

Compared to Fig. 7, two blocks are inserted between the $0^\circ/90^\circ$ splitter and the multipliers. The in-phase and quadrature versions of the reference signal are converted into amplitude and phase. Then the amplitude is dropped (can be used for diagnostics) and the phase is converted back to in-phase and quadrature signals with a constant amplitude, like 2 here to normalize the amplitude of (15) to A_{rf} . The conversions between I/Q and amplitude and phase, which are usually implemented with the Coordinate Rotation Digital Computer (CORDIC) algorithm in FPGA [33],[34], will increase the latency of the reference signal path. Practically, the latency in the reference signal path is not a big issue because we usually care only about slow drifts in the reference signal.

2.4 RF Detection with Asynchronous LO or Clock

In many situations we do not have frequency synthesizers to generate LO and clock from the reference MO signal. Instead, it might be easier to get separate oscillators to provide the LO and clock signals, which are not synchronized with the RF signal to be detected. In this case, the RF detection can still be performed with two channels as shown in Fig. 11, in which the MO signal is still used as the reference for the measurement of the RF signal (the RF signal is synchronized with the MO signal).

Fig. 11. RF detection with asynchronous LO or clock signals.

The blocks in the firmware or software side of Fig. 11 are the same as the blocks in Fig. 7. The major differences are the implementations of the $0^\circ/90^\circ$ splitter and the LPF due to asynchronous sampling. With asynchronous sampling, the IF phase lag $\Delta\hat{\varphi}$ between two adjacent ADC samples is unknown so we cannot use the simple algorithm in (12) to calculate the quadrature signal of the reference signal. Instead, the Hilbert transform algorithm can be used to generate the expected 90° phase shift for the quadrature signal, see Fig. 12.

Fig. 12. Implementation of $0^\circ/90^\circ$ splitter with Hilbert transform.

For a real time-domain signal $x(t)$, its Hilbert transform is defined as

$$H[x(t)] := x^H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau. \quad (16)$$

The Hilbert transform $x^H(t)$ is also a real time-domain signal which is the convolution of $x(t)$ with a function $1/(\pi t)$. In frequency domain, the Hilbert transform introduces a phase shift of 90° to every Fourier component of $x(t)$. A full implementation of the Hilbert transform for a time series adopts Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) [35] which is not suitable for real-time processing of the reference signal. Practically we can use an FIR filter to approximately implement the Hilbert transform. Fig. 13 shows the frequency response of a 16-tap Hilbert transform FIR filter designed with the “Filter Designer” of Matlab.

Fig. 13. Frequency response of an FIR filter for Hilbert transform. The clock frequency is 249.9 MHz.

The phase lag caused by group delay has been removed from the phase response.

It can be seen from Fig. 13 that the FIR-based implementation of the Hilbert transform has perfect phase response but big ripples in amplitude response which should be 0 dB for all frequencies. The error in the Hilbert transform implementation at the IF frequency can be modeled as an amplitude imbalance g and phase imbalance φ as in Fig. 12. These imbalances are systematic errors at a fixed IF frequency which can be compensated by a matrix [36] shown in Fig. 12 which can be represented as

$$\begin{bmatrix} x_{ref,k} \\ \tilde{x}_{ref,k} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \tan \varphi & \frac{1}{g \cos \varphi} \end{bmatrix} \begin{bmatrix} x'_{ref,k} \\ \tilde{x}'_{ref,k} \end{bmatrix}, \quad (17)$$

where $x_{ref,k}$ and $\tilde{x}_{ref,k}$ are the k th sample of the expected in-phase and quadrature signals of the reference IF signal and $x'_{ref,k}$ and $\tilde{x}'_{ref,k}$ are the outputs from the delay stage and Hilbert transform FIR filter with errors caused by the amplitude and phase imbalances. Their expressions have been shown in Fig. 12.

The calibration in (17) requires the knowledge of the imbalances g and φ which can be estimated from experiments. With a time series of a stable reference IF signal, the in-phase and quadrature signals $x'_{ref,k}$ and $\tilde{x}'_{ref,k}$ can be calculated with the delay stage and the FIR filter in Fig. 12. They can be combined to a complex signal

$$\mathbf{x}'_{ref,k} = x'_{ref,k} + j\tilde{x}'_{ref,k}, \quad (18)$$

with j the imaginary unit. Due to the asynchronous sampling, the phase lag $\Delta\hat{\varphi}$ between two samples is unknown, but it can be estimated with the phase slope of the complex signal in equation (18) calculated with linear fitting of the phases of each sample. Generally, the expressions of $x'_{ref,k}$ and $\tilde{x}'_{ref,k}$ can be written as

$$\begin{aligned} x'_{ref,k} &= A_{ref} \sin(k\Delta\hat{\varphi} + \varphi_{ref}) + A_{ref0} \\ \tilde{x}'_{ref,k} &= \tilde{A}_{ref} \cos(k\Delta\hat{\varphi} + \tilde{\varphi}_{ref}) + \tilde{A}_{ref0} \end{aligned}, \quad (19)$$

where A_{ref} , φ_{ref} , A_{ref0} , $\tilde{A}_{ref}$, $\tilde{\varphi}_{ref}$, $\tilde{A}_{ref0}$ are unknown constant parameters which can be estimated with multi-variable linear fitting. Let's use the in-phase signal $x'_{ref,k}$ as example to explain the fitting algorithm. As described above, the samples of $x'_{ref,k}$ are derived from the ADC samples of the reference IF signal with a delay stage and the phase lag $\Delta\hat{\varphi}$ is estimated with the phase slope of the complex signal, then for the k th sample, we get

$$x'_{ref,k} = a \sin(k\Delta\hat{\varphi}) + b \cos(k\Delta\hat{\varphi}) + c, \quad k = 0, 1, \dots, N-1, \quad (20)$$

where

$$a = A_{ref} \cos \varphi_{ref}, \quad b = A_{ref} \sin \varphi_{ref}, \quad c = A_{ref0}. \quad (21)$$

Here we have used N samples ($N > 3$) resulting in N equations about the unknown parameters a , b and c . This is a typical multi-variable linear fitting problem and can be solved with Matlab. Then the amplitude and phase of the in-phase signal can be calculated as

$$A_{ref} = \sqrt{a^2 + b^2}, \quad \varphi_{ref} = \text{tg}^{-1}(b/a). \quad (22)$$

The same algorithm can be applied to the quadrature signal $\tilde{x}'_{ref,k}$ to estimate its amplitude and phase.

Then the imbalances used in equation (17) can be calculated as

$$g = \tilde{A}_{ref} / A_{ref}, \quad \varphi = \tilde{\varphi}_{ref} - \varphi_{ref}. \quad (23)$$

The algorithm described above was applied to the samples of the reference signal of an RF station in SwissFEL and the results of the calibrated in-phase and quadrature signals are shown in Fig. 14. The estimated amplitude and phase imbalances are $g = 1.02$ (or 0.17 dB) and $\varphi = -1.52\text{e-}4^\circ$, which agree well with the frequency responses of the FIR-based Hilbert transform at the IF frequency (41.65 MHz) in Fig. 13.

Fig. 14. Generation of in-phase and quadrature signals of the reference IF signal with FIR-based Hilbert transform and the calibration of imbalances.

Due to the asynchronous sampling, the moving-average FIR filter is no longer suitable to implement the LPFs at the firmware or software side in Fig. 11. We should use the general formats of low-pass filters like the FIR filter, Infinite Impulse Response (IIR) filter or Cascaded Integrator-Comb (CIC) filter [22],[37]. The low-pass filter design is out of the scope of this paper and can be referred to the digital signal processing books [38].

The algorithms described in this section are applied to the same data as in Fig. 2 and the results are shown in Fig. 15. The results from non-IQ demodulation algorithm are also shown as a comparison.

Fig. 15. Results of asynchronous detection algorithm and non-IQ demodulation algorithm for the same data in Fig. 2. (a). amplitude waveform; (b). amplitude zoom-in at rising edge; (c). phase waveform; (d). phase zoom-in at rising edge and the phase difference between two cases.

4 Conclusions and Outlook

The digital RF amplitude and phase detector is one of the key components in LLRF systems. Based on the same hardware, different digital demodulation algorithms should be implemented to address different requirements on the RF detection. This paper describes several RF detection algorithms to satisfy different requirements that the author has met when working on different LLRF systems, which can be used in other machines with similar conditions. New algorithms will be developed for newly emerging requirements on the performance, reliability and robustness of RF detectors. If the requirements (e.g. bandwidth requirements) exceed the limits of the existing hardware, a modification of the RF detector hardware could be also needed together with the demodulation algorithm development.

References

- [1] A. Hauff. SwissFEL C-Band LLRF Prototype System, in Proceedings of LINAC2014, Geneva, Switzerland (2014), pp. 683-685.
- [2] Z. Geng. Architecture Design for the SwissFEL LLRF System, in Proceedings of LINAC2014, Geneva, Switzerland (2014), pp. 1114-1116.
- [3] J. Branlard, G. Ayvazyan, V. Ayvazyan et al. The European XFEL LLRF System, in Proceedings of IPAC2012, New Orleans, Louisiana, USA (2012), pp. 55-57.
- [4] C. Serrano, K. Campbell, L. Doolittle et al. Design and Implementation of the LLRF System for LCLS-II, in Proceedings of ICALEPCS2017, Barcelona, Spain (2017), pp. 1969-1974.
- [5] T. Matsumoto, S. Michizono, Z. Geng et al. Low-Level RF System for STF, in Proceedings of LINAC2006, Knoxville, Tennessee, USA (2006), pp. 586-588.
- [6] Z. Mu, J. Li, X.A. Xu et al. Overview of the CSNS Linac LLRF and Operational Experiences during Beam Commissioning, in Proceedings of HB2016, Malmoe, Sweden (2016), pp. 409-412.
- [7] S. Michizono, S. Anami, Z. Fang et al. Performance of a Digital LLRF Field Control System for the J-PARC Linac, in Proceedings of LINAC2006, Knoxville, Tennessee, USA (2006), pp. 574-576.
- [8] Z. Zhang, Y. Zhao, K. Xu et al. Digital LLRF Controller for SSRF Booster RF System Upgrade. Nuclear Science and Techniques, 2015, 26: 030106. DOI: [10.13538/j.1001-8042/nst.26.030106](https://doi.org/10.13538/j.1001-8042/nst.26.030106)

- [9] B. Keil, R. Baldinger, R. Ditter et al. Design of the SwissFEL BPM System, in Proceedings of IBIC2013, Oxford, UK (2013), pp. 427-430.
- [10] A. Young, R. Claus, J. M. D'Ewart et al. Design of LCLS-II ATCA BPM System, in Proceedings of IPAC2017, Copenhagen, Denmark (2017), pp. 477-479.
- [11] H. Maesaka, H. Ego, T. Ohshima et al. Performance of the RF Cavity BPM at XFEL/Spring-8 SACLA, in Proceedings of FEL2011, Shanghai, China (2011), pp. 539-542.
- [12] S. Hunziker, V. Arsov, F. Buechi et al. Reference Distribution and Synchronization System for SwissFEL: Concept and First Results, in Proceedings of IBIC2014, Monterey, CA, USA (2014), pp. 29-33.
- [13] E. Janas, K. Czuba, M. Felber et al. MTCA.4 Phase Detector for Femtosecond-Precision Laser Synchronization, in Proceedings of FEL2015, Daejeon, Korea (2015), pp. 474-477.
- [14] J. M. Byrd, L. Doolittle, G. Huang et al. Femtosecond Synchronization of Laser Systems for the LCLS, SLAC-PUB-15226, 2010.
- [15] R.C. Dorf, R.H. Bishop. Modern Control Systems. London, UK: Pearson Education, Inc., 2016.
- [16] S. Skogestad, I. Postlethwaite. Multivariable Feedback Control: Analysis and Design. New York, USA: John Wiley & Sons, Inc., 2007.
- [17] R. Liu, Z. Wang, W. Pan et al. FPGA-based Amplitude and Phase Detection in DLLRF. Chinese Physics C, 2009, 33(7): 594-598. [DOI: 10.1088/1674-1137/33/7/017](https://doi.org/10.1088/1674-1137/33/7/017)
- [18] Y. Xu, G. Huang, L. Doolittle et al. High-precision Phase Detection in Femtosecond Timing and Synchronization System for TXGLS. Meas. Sci. Technol., 2018, 29: 065011. [DOI: 10.1088/1361-6501/aab6f7](https://doi.org/10.1088/1361-6501/aab6f7)
- [19] D. Zhao, G. Wang, W. Pan et al. High Accuracy Amplitude and Phase Measurements based on a Double Heterodyne Architecture. Chinese Physics C, 2015, 39(1): 017002-1. [DOI: 10.1088/1674-1137/39/1/017002](https://doi.org/10.1088/1674-1137/39/1/017002)
- [20] C. Ziomek, P. Corredoura. Digital I/Q Demodulator, SLAC-PUB-95-6857, 1996.
- [21] L. Doolittle, H. Ma, M. Stuard. Digital Low-Level RF Control using Non-IQ Sampling, in Proceedings of LINAC2006, Knoxville, Tennessee, USA (2006), pp. 568-570.
- [22] T. Schilcher. Digital Signal Processing in RF Applications, CERN Accelerator School, 2007.
- [23] Z. Geng, S. N. Simrock. Evaluation of Fast ADCs for Direct Sampling RF Field Detection for the European XFEL and ILC, in Proceedings of LINAC08, Victoria, BC, Canada (2008), pp. 1030-1032.
- [24] F. Ludwig. Tutorial on RF Receiver Fundamentals, LLRF2011 workshop, Hamburg, Germany, 2011.
- [25] M. Hoffmann. Ph.D. Thesis, Hamburg University of Technology, 2008.
- [26] U. Mavric, B. Chase, J. Branlard et al. A 96 Channel Receiver for the ILCTA LLRF System at FermiLab, FERMILAB-CONF-07-299-AD, 2007.
- [27] U. Mavric, M. Vidmar, B. Chase. Design and Evaluation of a Low-Level RF Control System Analog/Digital Receiver for the ILC Main Linacs, FERMILAB-PUB-08-157-AD, 2008.
- [28] L. Doolittle. Tutorial on Analog Signal Processing and Design for Low Noise LLRF Front Ends, LLRF2017 workshop, Barcelona, Spain, 2017.
- [29] P. Pawlik, M. Grecki, S. Simrock. Single Bunch Transient Detection for the Beam Phase Measurement in Superconducting Accelerators, in Proceedings of DIPAC2005, Lyon, France (2005), pp. 108-110.
- [30] P. Pawlik. Ph.D. Thesis, Technical University of Lodz, 2007.
- [31] G. Penco, M. Svandrlík. Experimental Studies on Transient Beam Loading Effects in the Presence of a Superconducting Third Harmonic Cavity. Physical Review Special Topics – Accelerators and Beams, 2006, 9: 044401. [DOI: 10.1103/PhysRevSTAB.9.044401](https://doi.org/10.1103/PhysRevSTAB.9.044401)

- [32] R. Zeng, O. Troeng. Transient Beam Loading based Calibration for Cavity Phase and Amplitude Setting, in Proceedings of HB2016, Malmoe, Sweden (2016), pp. 250-253.
- [33] K. H. Park, Y. G. Jung, D. E. Kim et al. The Development of LLRF System at PAL, in Proceedings of IPAC2013, Shanghai, China (2013), pp. 3004-3006.
- [34] B. T. Du, H. X. Lin, W. Liu et al. DSP Frame and Algorithm of LLRF of IR-FEL, in Proceedings of IPAC2017, Copenhagen, Denmark (2017), pp. 4017-4019.
- [35] X. Wang. Master Thesis, University of Saskatchewan, 2006.
- [36] Z. Geng, B. Hong. Design and Calibration of an RF Actuator for Low-Level RF Systems. IEEE Transactions on Nuclear Science, 2016, 63: 281-287. [DOI: 10.1109/TNS.2015.2507204](https://doi.org/10.1109/TNS.2015.2507204)
- [37] A. Engelbert, C. Hallqvist. Master Thesis, Chalmers University of Technology, 2008.
- [38] S. M. Alessio. Digital Signal Processing and Spectral Analysis for Scientists: Concepts and Applications. New York, USA: Springer, 2016.