# Generic loop effects of new scalars and fermions in $b \rightarrow s \ell^{+} \ell^{-}$and a vector-like $4^{\text {th }}$ generation 

Pere Arnan, ${ }^{a}$ Andreas Crivellin, ${ }^{b}$ Marco Fedele ${ }^{a}$ and Federico Mescia ${ }^{a}$<br>${ }^{a}$ Departament de Física Quàntica i Astrofísica (FQA), Institut de Ciències del Cosmos (ICCUB), Universitat de Barcelona (UB), Barcelona, Spain<br>${ }^{b}$ Paul Scherrer Institut, CH-5232, Villigen PSI, Switzerland<br>E-mail: arnan@ub.edu, andreas.crivellin@cern.ch, marco.fedele@icc.ub.edu, mescia@ub.edu

Abstract: In this article we investigate in detail the possibility of accounting for the $b \rightarrow s \ell^{+} \ell^{-}$anomalies via box contributions involving with new scalars and fermions. For this purpose, we first write down the most general Lagrangian which can generate the desired effects and then calculate the generic expressions for all relevant $b \rightarrow s$ Wilson coefficients. Here we extend previous analysis by allowing that the new particles can also couple to right-handed Standard Model (SM) fermions as preferred by recent $b \rightarrow s \ell^{+} \ell^{-}$ data and the anomalous magnetic moment of the muon.

In the second part of this article we illustrate this generic approach for a UV complete model in which we supplement the Standard Model by a $4^{\text {th }}$ generation of vector-like fermions and a real scalar field. This model allows one to coherently address the observed anomalies in $b \rightarrow s \ell^{+} \ell^{-}$transitions and in $a_{\mu}$ without violating the bounds from other observables (in particular $B_{s}-\bar{B}_{s}$ mixing) or LHC searches. In fact, we find that our global fit to this model, after the recent experimental updates, is very good and prefers couplings to right-handed SM fermions, showing the importance of our generic setup and calculation performed in the first part of the article.

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## 1 Introduction

While no particles beyond the ones of the Standard Model (SM) have been observed at the LHC (so far), $b \rightarrow s \ell^{+} \ell^{-}$data show a coherent pattern of deviations from the SM predictions with a significance of more than $4-5 \sigma[1-8]$. Recently, results of Belle and LHCb presented at Moriond EW 2019 [9, 10] confirmed these tensions, even though the significance for the new physics (NP) hypothesis, compared to the SM, did not change notably. ${ }^{1}$ In fact, including these new measurements, global fits of the Wilson coefficients governing $b \rightarrow s \ell^{+} \ell^{-}$transitions [12-17] still find that NP scenarios can describe data much better than the SM, even though the preferences between the different scenarios changed with respect to the previous experimental situation.

Concerning concrete NP models giving the desired pattern in the effective theory with a good fit to data, most analyses focused on scenarios in which the required NP effects are generated at tree-level, either by the exchange of $Z^{\prime}$ vector bosons [18-41] or via leptoquarks [42-73]. Nonetheless, since the size of the NP contribution required to account for current data is of the order of $20 \%$ compared to the (loop and CKM suppressed) amplitude of the SM, also new loop effects can in principle suffice for an explanation.

In this context, box contributions of heavy new scalars and fermions ${ }^{2}$ (also within multi Higgs doublet models with right-handed neutrinos [75-77]) have been shown to be a viable option [78-81]. ${ }^{3}$ Furthermore, an explanation of the anomalies in $b \rightarrow s \ell^{+} \ell^{-}$via loop effects allows for interesting connections to Dark Matter [86-92] and typically leads to correlated imprints on other observables like the anomalous magnetic moment of the muon $\left(a_{\mu}\right)$. However, the effect here is in most models too small since a quite large NP contribution is needed to account from the tantalizing tension between the measurement [93] and the SM prediction of around 3-4 $\sigma$. In fact, ref. [79] found that it is challenging to account for $\Delta a_{\mu}$ with TeV scale masses and not too large couplings to muons with a minimal particle content. In general, it has been argued [94] that one needs new sources of electroweak symmetry breaking (EWSB) if one aims at a high scale explanation of the anomalous magnetic moment of the muon. In the context of adding new scalars and fermions to the SM this can be achieved for example by a fourth generation of vector-like leptons coupling to the SM Higgs [94-102].

Therefore, we extend in this article the analysis of ref. [79] to include the possibility of new sources of EW symmetry breaking within the NP sector. For this purpose, an extension of the field content with respect to the minimal one of ref. [79] is necessary, i.e. more than three new fields need to be added to the SM particle content. In doing so, new couplings to right-handed quarks and leptons are introduced which do not only affect $a_{\mu}$ but also lead to different effects in $b \rightarrow s \mu^{+} \mu^{-}$(i.e. lead to solutions other than the purely left-handed $C_{9}=-C_{10}$ one obtained in ref. [79]). In fact, while before Moriond 2019 scenarios with

[^0]left-handed current were in general preferred, now including right-handed contributions (both in quark and leptonic sectors) can even give a better fit to data [12-14, 16, 17].

A UV complete example of such a setup with new scalars and fermions couplings to left- and right-handed SM fermions is a model with a vector-like $4^{\text {th }}$ generation. With respect to refs. [103, 104], also aiming at an explanation of the $b \rightarrow s \ell^{+} \ell^{-}$anomalies, we add not only a $4^{\text {th }}$ generation of leptons but also of quarks [105-108] to the SM. However, instead of adding a $Z^{\prime}$ boson we supplement the model by a neutral scalar to get the desired loop-contributions. Furthermore, one can forbid the dangerous mixing effect between the SM fermions and the new vector-like ones by assigning $U(1)$ changes to the new particles (resembling R-parity in the MSSM).

This article is organized as follows: in section 2 we define our generic setup, in which new scalars and fermions couple to SM quarks and leptons via Yukawa-like interactions. There, we also provide completely general expressions for the formulae of the relevant Wilson coefficients. We review the corresponding observables together with the current experimental situation in section 3 . Our generic approach of section 2 is then applied to a specific UV complete model in section 4, which contains a vector-like fourth generation of fermions and a neutral scalar. We study the phenomenology of this model in detail before we conclude in section 5 .

## 2 Generic setup and Wilson coefficients

In this section we define our generic setup and calculate completely general 1-loop expressions for contributions to $b \rightarrow s$ processes and the anomalous magnetic moment of the muon.

As outlined in the introduction, in the spirit of refs. [78, 79] we add to the SM particle content a NP sector with vector-like fermions $\Psi_{A}$ and new scalars $\Phi_{M}$ such that $b \rightarrow s \mu^{+} \mu^{-}$ transitions can be generated via box diagrams, as depicted in figure 1. In this respect, we generalize the previous analysis of ref. [79] by including in addition couplings of new particles to $\mathrm{SU}(2)$ singlet SM fermions. Moreover, we do not impose limitations on the number of fields added to the SM and allow for couplings of the new sector to the SM Higgs.

In order to generate box diagrams as the ones shown in figure 1 it is necessary that either the scalars $\Phi_{M, N}$ or the fermions $\Psi_{A, B}$ couple both to quarks and leptons, corresponding to case $a$ ) and $b$ ), respectively. This means that in diagrams of type $a$ ) the amplitudes (before using any Fierz identities) have the structure $(\bar{s} \Gamma b)(\bar{\mu} \Gamma \mu)$, while in type $b)$ amplitudes of the form $(\bar{\mu} \Gamma b)(\bar{s} \Gamma \mu)$ are generated. Here, $\Gamma$ denotes an arbitrary Dirac structure. Since semi-leptonic operators are commonly given in the form ( $\bar{s} \Gamma b$ ) ( $\bar{\mu} \Gamma \mu$ ), Fierz identities must be used in case $b$ ) in order to transform the expressions to this standard basis. We give the relevant Fierz identities in appendix A.

The Yukawa-like couplings of new scalars $\Phi_{M}$ and fermions $\Psi_{A}$ to bottom/strange quarks and muons can be parameterized completely generically (below the EWSB scale) by the Lagrangian

$$
\begin{align*}
\mathcal{L}_{\mathrm{int}}= & {\left[\bar{\Psi}_{A}\left(L_{A M}^{b} P_{L} b+L_{A M}^{s} P_{L} s+L_{A M}^{\mu} P_{L} \mu\right) \Phi_{M}\right.} \\
& \left.+\bar{\Psi}_{A}\left(R_{A M}^{b} P_{R} b+R_{A M}^{s} P_{R} s+R_{A M}^{\mu} P_{R} \mu\right) \Phi_{M}\right]+ \text { h.c. } \tag{2.1}
\end{align*}
$$



Figure 1. Box diagrams contributing to $b \rightarrow s \mu^{+} \mu^{-}$transitions. The diagram on the left is generated in models in which the fermions couples only to SM quarks or only to SM leptons, which corresponds to type $a$ ). The diagram on the right refers to models with scalars connecting $b$ to $s$ and $\mu$ to $\mu$, i.e. type $b$ ).

Here $\Psi_{A}$ and $\Phi_{M}$ have to be understood as generic lists containing in principle an arbitrary number of fields, meaning that $A$ and $M$ also include implicitly $\mathrm{SU}(2)$ and color indices. Therefore, the couplings $L_{A M}^{s, b}$ and $R_{A M}^{s, b}$ are generic matrices in $(A-M)$ space with the restriction that $\mathrm{U}(1)_{\mathrm{EM}}$ and $\mathrm{SU}(3)$ are respected. ${ }^{4}$

This Lagrangian will not only affect $b \rightarrow s \mu^{+} \mu^{-}$transitions but also unavoidably generate effects in $B_{s}-\bar{B}_{s}$ mixing, $b \rightarrow s \gamma$ decays, the anomalous magnetic moment of the muon $a_{\mu}$ as well as $Z$ couplings and decays to SM fermions. Furthermore, $b \rightarrow s \nu \bar{\nu}$ processes and $D_{0}-\bar{D}_{0}$ mixing can give relevant constraints once $\mathrm{SU}(2)$ invariance at the NP scale is imposed. Therefore, all these processes have to be taken into account in a complete phenomenological analysis. In order to perform such an analysis, the Wilson coefficients of the relevant effective Hamiltonian must be known. We will calculate them in the following subsections.

## 2.1 $b \rightarrow s \mu^{+} \mu^{-}$and $b \rightarrow s \gamma$ transitions

The dimension- 6 operators governing $b \rightarrow s \mu^{+} \mu^{-}$and $b \rightarrow s \gamma$ transitions are contained in the effective Hamiltonian:

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}^{\ell \ell}=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} \sum_{i}\left(C_{i}(\mu) \mathcal{O}_{i}(\mu)+C_{i}^{\prime}(\mu) \mathcal{O}_{i}^{\prime}(\mu)\right)+\text { h.c. } \tag{2.2}
\end{equation*}
$$

where

$$
\begin{array}{ll}
\mathcal{O}_{7}=\frac{e}{16 \pi^{2}} m_{b} \bar{s} \sigma^{\mu \nu} P_{R} b F_{\mu \nu}, & \mathcal{O}_{8}=\frac{g_{s}}{16 \pi^{2}} m_{b} \bar{s}_{\alpha} \sigma^{\mu \nu} P_{R} T_{\alpha \beta}^{a} b_{\beta} G_{\mu \nu}^{a}, \\
\mathcal{O}_{9}=\frac{\alpha_{\mathrm{EM}}}{4 \pi}\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\mu} \gamma^{\mu} \mu\right), & \mathcal{O}_{10}=\frac{\alpha_{\mathrm{EM}}}{4 \pi}\left(\bar{s} \gamma_{\mu} P_{L} b\right)\left(\bar{\mu} \gamma^{\mu} \gamma_{5} \mu\right), \\
\mathcal{O}_{S}=\frac{\alpha_{\mathrm{EM}}}{4 \pi}\left(\bar{s} P_{R} b\right)(\bar{\mu} \mu), & \mathcal{O}_{P}=\frac{\alpha_{\mathrm{EM}}}{4 \pi}\left(\bar{s} P_{R} b\right)\left(\bar{\mu} \gamma_{5} \mu\right), \\
\mathcal{O}_{T}=\frac{\alpha_{\mathrm{EM}}}{4 \pi}\left(\bar{s} \sigma_{\mu \nu} b\right)\left(\bar{\mu} \sigma^{\mu \nu} P_{R} \mu\right), & \tag{2.3}
\end{array}
$$

[^1]with $e$ being the electron charge, $\alpha_{\mathrm{EM}}$ the fine structure constant and $g_{s}$ the $\mathrm{SU}(3)$ gauge coupling. The primed operators are obtained by interchanging $L$ and $R$. NP contributions from box diagrams will generate effects in $C_{9,10}^{(\prime)}, C_{S, P}^{(\prime)}$ and $O_{T}^{(\prime)}$, while on-shell photon (gluon) penguins generate $C_{7(8)}^{(\prime)}$ and $C_{9}^{(\prime)}$, and $Z$-penguins $C_{9,10}^{(\prime)}$.

The box diagrams in figure 1, result in the following Wilson coefficients (here and in the remainder of the section, an implicit sum over all NP particles, i.e. $A, B, M, N$, is understood):

$$
\begin{align*}
& C_{9}^{\mathrm{box}, a)}=-\mathcal{N} \frac{\chi L_{A N}^{s *} L_{A M}^{b}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[L_{B M}^{\mu *} L_{B N}^{\mu}+R_{B M}^{\mu *} R_{B N}^{\mu}\right] F\left(x_{A M}, x_{B M}, x_{N M}\right), \\
& C_{9}^{\text {box }, b)}=-\mathcal{N} \frac{\chi L_{B M}^{s *} L_{A M}^{b}}{32 \pi \alpha_{E M} m_{\Phi_{M}}^{2}}\left[L_{A N}^{\mu *} L_{B N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.-R_{A N}^{\mu *} R_{B N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right],  \tag{2.4}\\
& C_{10}^{\mathrm{box}, a)}=\mathcal{N} \frac{\chi L_{A N}^{s *} L_{A M}^{b}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[L_{B M}^{\mu *} L_{B N}^{\mu}-R_{B M}^{\mu *} R_{B N}^{\mu}\right] F\left(x_{A M}, x_{B M}, x_{N M}\right), \\
& C_{10}^{\text {box }, b)}=\mathcal{N} \frac{\chi L_{B M}^{s *} L_{A M}^{b}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[L_{A N}^{\mu *} L_{B N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.+R_{A N}^{\mu *} R_{B N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right],  \tag{2.5}\\
& C_{S}^{\mathrm{box}, a)}=-\mathcal{N} \frac{\chi L_{A N}^{s *} R_{A M}^{b}}{16 \pi \alpha_{E M} m_{\Phi_{M}}^{2}}\left[R_{B M}^{\mu *} L_{B N}^{\mu}+L_{B M}^{\mu *} R_{B N}^{\mu}\right] \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right), \\
& C_{S}^{\mathrm{box}, b)}=\mathcal{N} \frac{\chi L_{B M}^{s *} R_{A M}^{b}}{16 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[R_{A N}^{\mu *} L_{B N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.+L_{A N}^{\mu *} R_{B N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{2 m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right],  \tag{2.6}\\
& C_{P}^{\mathrm{box}, a)}=\mathcal{N} \frac{\chi L_{A N}^{s *} R_{A M}^{b}}{16 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[R_{B M}^{\mu *} L_{B N}^{\mu}-L_{B M}^{\mu *} R_{B N}^{\mu}\right] \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right), \\
& C_{P}^{\mathrm{box}, b)}=-\mathcal{N} \frac{\chi L_{B M}^{s *} R_{A M}^{b}}{16 \pi \alpha_{E M} m_{\Phi_{M}}^{2}}\left[R_{A N}^{\mu *} L_{B N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.-L_{A N}^{\mu *} R_{B N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{2 m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right],  \tag{2.7}\\
& C_{T}^{\mathrm{box}, b)}=-\mathcal{N} \frac{\chi L_{B M}^{s *} R_{A M}^{b} L_{A N}^{\mu *} R_{B N}^{\mu}}{16 \pi \alpha_{E M} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right),  \tag{2.8}\\
& C_{9, S, T}^{\prime \mathrm{box}}=C_{9, S, T}^{\mathrm{box}}(L \leftrightarrow R), \quad C_{10, P}^{\prime \mathrm{box}}=-C_{10, P}^{\mathrm{box}}(L \leftrightarrow R), \tag{2.9}
\end{align*}
$$

where we have defined

$$
\begin{equation*}
x_{A M} \equiv\left(m_{\Psi_{A}} / m_{\Phi_{M}}\right)^{2}, \quad x_{B M} \equiv\left(m_{\Psi_{B}} / m_{\Phi_{M}}\right)^{2}, \quad x_{N M} \equiv\left(m_{\Phi_{N}} / m_{\Phi_{M}}\right)^{2}, \tag{2.10}
\end{equation*}
$$

| $\mathrm{SU}(3)$ | $b \rightarrow s \ell \bar{\ell}$ type $a)$ |  |  |  | $b \rightarrow s \ell \bar{\ell}$ type $b)$ |  |  |  | $\chi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\Psi_{A}$ | $\Psi_{B}$ | $\Phi_{M}$ | $\Phi_{N}$ | $\Psi_{A}$ | $\Psi_{B}$ | $\Phi_{M}$ | $\Phi_{N}$ |  |
| I | 3 | 1 | 1 | 1 | 1 | 1 | $\overline{3}$ | 1 | 1 |
| II | 1 | $\overline{3}$ | $\overline{3}$ | $\overline{3}$ | 3 | 3 | 1 | 3 | 1 |
| III | 3 | 8 | 8 | 8 | 8 | 8 | $\overline{3}$ | 8 | $4 / 3$ |
| IV | 8 | $\overline{3}$ | $\overline{3}$ | $\overline{3}$ | 3 | 3 | 8 | 3 | $4 / 3$ |
| V | $\overline{3}$ | 3 | 3 | 3 | $\overline{3}$ | $\overline{3}$ | 3 | $\overline{3}$ | 2 |

Table 1. Table of the possible $\mathrm{SU}(3)$ representations that can give an effect in $b \rightarrow s \ell^{+} \ell^{-}$or $b \rightarrow s \nu \nu$ transitions via box diagrams. $\chi$ denotes the resulting group factor appearing in eqs. (2.4)-(2.8) which also enters in $b \rightarrow s \nu \nu$ transitions.
and

$$
\begin{equation*}
\mathcal{N}^{-1}=\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*} . \tag{2.11}
\end{equation*}
$$

In the equations above, the labels $A, B, M$ and $N$ denote the particle (in case of several representations) and also include $\mathrm{SU}(2)$ components, while the sum over $\mathrm{SU}(3)$ indices is encoded in the group factors $\chi$. The dimensionless loop functions $F$ and $G$ are defined in appendix B.

Such box contributions are only possible if both color and electric charge are conserved. While the Wilson coefficients of $b \rightarrow s \ell \ell$ operators are insensitive to the electric charge of the particle in the box, concerning $\mathrm{SU}(3)$, the different possible representations of the new particles lead to distinct group factors $\chi$ in eqs. (2.4)-(2.8). These group factors are different for type $a$ ) and $b$ ) and are given for all the possible representations in table 1 . Furthermore, crossed box diagrams can be constructed in some particular cases. We give the corresponding expressions for such in appendix C. 1 for the real scalar (or Majorana fermion) case and in appendix D for the crossed diagrams arising with complex scalars.

On-shell photon penguins diagrams in figure 2 affect $C_{7}^{(\prime)}$ while off-shell ones enter $C_{9}^{\gamma(\prime)}$ :

$$
\begin{align*}
C_{7}= & \mathcal{N} \frac{\chi_{\gamma} L_{A M}^{b}}{2 m_{\Phi_{M}}^{2}}\left[L_{A M}^{s *}\left(Q_{\Phi_{M}} \widetilde{F}_{7}\left(x_{A M}\right)-Q_{\Psi_{A}} F_{7}\left(x_{A M}\right)\right)\right. \\
& \left.+R_{A M}^{s *} \frac{4 m_{\Psi_{A}}}{m_{b}}\left(Q_{\Phi_{M}} \widetilde{G}_{7}\left(x_{A M}\right)-Q_{\Psi_{A}} G_{7}\left(x_{A M}\right)\right)\right],  \tag{2.12}\\
C_{9}^{\gamma}= & \mathcal{N} \frac{\chi_{\gamma} L_{A M}^{s *} L_{A M}^{b}}{2 m_{\Phi_{M}}^{2}}\left[Q_{\Phi_{M}} \widetilde{F}_{9}\left(x_{A M}\right)-Q_{\Psi_{A}} \widetilde{G}_{9}\left(x_{A M}\right)\right],  \tag{2.13}\\
C_{7}^{\prime}= & C_{7}(L \leftrightarrow R), \quad C_{9}^{\gamma \prime}=C_{9}^{\gamma}(L \leftrightarrow R), \tag{2.14}
\end{align*}
$$

where $m_{b}$ is the $b$ quark mass. $Q_{\Phi_{M}}$ and $Q_{\Psi_{A}}$ are the electric charges of the NP fields $\Phi_{M}$ and $\Psi_{A}$, respectively. The conservation of electric charge imposes that $Q_{\Phi_{M}}+Q_{\Psi_{A}}=$ $Q_{d} \equiv-1 / 3$. The color factors $\chi_{\gamma}$, which depend on the $\mathrm{SU}(3)$ representations of the new particles in the loop, are given in table 2. The loop functions are defined in appendix B. Note that the terms proportional to $\widetilde{F}_{7}, \widetilde{G}_{7}$ and $\widetilde{F}_{9}$ in eqs. (2.12)-(2.14) stem from the


Figure 2. Photon-penguin diagrams contributing to $b \rightarrow s \gamma$ transitions and $a_{\mu}$.

| $\mathrm{SU}(3)$ | $\Psi_{A}$ | $\Phi_{M}$ | $\chi_{\gamma}$ | $\chi_{g}$ | $\tilde{\chi}_{g}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | 3 | 1 | 1 | 1 | 0 |
| II | 1 | $\overline{3}$ | 1 | 0 | 1 |
| III | 3 | 8 | $4 / 3$ | $-1 / 6$ | $3 / 2$ |
| IV | 8 | $\overline{3}$ | $4 / 3$ | $3 / 2$ | $-1 / 6$ |
| V | $\overline{3}$ | 3 | 2 | -1 | 1 |

Table 2. Table of the different $\mathrm{SU}(3)$ representations that can give non-zero effects via photonand gluon-penguin diagrams to $b \rightarrow s \mu^{+} \mu^{-}$transitions. $\chi_{\gamma}$ denotes the resulting group factor for the former contribution, while $\chi_{g}$ and $\tilde{\chi}_{g}$ represent the resulting group factors for the latter.
diagram where the photon couples to the scalar $\Phi_{M}$, while the terms proportional to $F_{7}$, $G_{7}$ and $\widetilde{G}_{9}$ stem from the diagram where the photon couples to the fermion $\Psi_{A}$.

Similarly, the gluon-penguin generates

$$
\begin{align*}
C_{8}= & \mathcal{N} \frac{L_{A M}^{b}}{2 m_{\Phi_{M}}^{2}}\left[L_{A M}^{s *}\left(\chi_{g} \widetilde{F}_{7}\left(x_{A M}\right)-\tilde{\chi}_{g} F_{7}\left(x_{A M}\right)\right)\right. \\
& \left.+R_{A M}^{s *} \frac{4 m_{\Psi_{A}}}{m_{b}}\left(\chi_{g} \widetilde{G}_{7}\left(x_{A M}\right)-\tilde{\chi}_{g} G_{7}\left(x_{A M}\right)\right)\right],  \tag{2.15}\\
C_{8}^{\prime}= & C_{8}(L \leftrightarrow R) \tag{2.16}
\end{align*}
$$

where the color factors $\chi_{g}$ and $\tilde{\chi}_{g}$ for the different possible $\mathrm{SU}(3)$ representations are given in table 2.

The contribution of $Z$-penguins to $C_{9,10}^{(\prime)}$ is given in section 2.6 together with a discussion of Z decays.

## $2.2 b \rightarrow s \nu \bar{\nu}$

As stated at the beginning of this section, $b \rightarrow s \nu_{\mu} \bar{\nu}_{\mu}$ processes have to be taken into account once $\mathrm{SU}(2)$ invariance at the NP scale is imposed. This implies that, in the generic description in eq. (2.1), one has to replace the left-handed muon fields with neutrinos. The box diagrams generating $b \rightarrow s \nu_{\mu} \bar{\nu}_{\mu}$ are therefore obtained from figure 1 by replacing muons with neutrinos.

The effective Hamiltonian describing this process reads (following the conventions of ref. [110])

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}^{\nu_{\mu}}=-\frac{4 G_{F}}{\sqrt{2}} V_{t b} V_{t s}^{*}\left(C_{L} \mathcal{O}_{L}+C_{R} \mathcal{O}_{R}\right)+\text { h.c. } \tag{2.17}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathcal{O}_{L(R)}=\frac{\alpha_{\mathrm{EM}}}{4 \pi}\left[\bar{s} \gamma^{\mu} P_{L(R)} b\right]\left[\bar{\nu}_{\mu} \gamma_{\mu}\left(1-\gamma^{5}\right) \nu_{\mu}\right] . \tag{2.18}
\end{equation*}
$$

The resulting WCs are:

$$
\begin{align*}
C_{L}^{a)} & =-\mathcal{N} \frac{\chi L_{A N}^{s *} L_{A M}^{b} L_{B M}^{\mu *} L_{B N}^{\mu}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}} F\left(x_{A M}, x_{B M}, x_{N M}\right) \\
C_{L}^{b)} & =\mathcal{N} \frac{\chi L_{B N}^{s *} L_{A M}^{b} L_{A N}^{\mu *} L_{B N}^{\mu}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}} F\left(x_{A M}, x_{B M}, x_{N M}\right)  \tag{2.19}\\
C_{R}^{a)} & =-\mathcal{N} \frac{\chi R_{A N}^{s *} R_{A M}^{b} L_{B M}^{\mu *} L_{B N}^{\mu}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}} F\left(x_{A M}, x_{B M}, x_{N M}\right) \\
C_{R}^{b)} & =-\mathcal{N} \frac{\chi R_{B N}^{s *} R_{A M}^{b} L_{A N}^{\mu *} L_{B N}^{\mu}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right) \tag{2.20}
\end{align*}
$$

where the normalization factor $\mathcal{N}$ has been introduced in eq. (2.11), and the loop functions $F(x, y, z)$ and $G(x, y, z)$ are defined in appendix B . The colour factor $\chi$ is the same as for $b \rightarrow s \mu^{+} \mu^{-}$transitions and is given in table 1 for the different representations.

## $2.3 \Delta B=\Delta S=2$ processes

The presence of $L_{A M}^{b, s}$ and $R_{A M}^{b, s}$ implies NP contributions to the $B_{s}-\bar{B}_{s}$ mixing which, using the conventions of refs. [111, 112], is governed by

$$
\begin{equation*}
\mathcal{H}_{\mathrm{eff}}^{B_{s} \bar{B}_{s}}=C_{i} \sum_{i=1}^{5} \mathcal{O}_{i}+\tilde{C}_{i} \sum_{i=1}^{3} \widetilde{\mathcal{O}}_{i}+\text { h.c. } \tag{2.21}
\end{equation*}
$$

with

$$
\begin{array}{ll}
\mathcal{O}_{1}=\left(\bar{s}_{\alpha} \gamma^{\mu} P_{L} b_{\alpha}\right)\left(\bar{s}_{\beta} \gamma^{\mu} P_{L} b_{\beta}\right), & \widetilde{\mathcal{O}}_{1}=\left(\bar{s}_{\alpha} \gamma^{\mu} P_{R} b_{\alpha}\right)\left(\bar{s}_{\beta} \gamma^{\mu} P_{R} b_{\beta}\right), \\
\mathcal{O}_{2}=\left(\bar{s}_{\alpha} P_{L} b_{\alpha}\right)\left(\bar{s}_{\beta} P_{L} b_{\beta}\right), & \widetilde{\mathcal{O}}_{2}=\left(\bar{s}_{\alpha} P_{R} b_{\alpha}\right)\left(\bar{s}_{\beta} P_{R} b_{\beta}\right), \\
\mathcal{O}_{3}=\left(\bar{s}_{\alpha} P_{L} b_{\beta}\right)\left(\bar{s}_{\beta} P_{L} b_{\alpha}\right), & \widetilde{\mathcal{O}}_{3}=\left(\bar{s}_{\alpha} P_{R} b_{\beta}\right)\left(\bar{s}_{\beta} P_{R} b_{\alpha}\right),  \tag{2.22}\\
\mathcal{O}_{4}=\left(\bar{s}_{\alpha} P_{L} b_{\alpha}\right)\left(\bar{s}_{\beta} P_{R} b_{\beta}\right), & \\
\mathcal{O}_{5}=\left(\bar{s}_{\alpha} P_{L} b_{\beta}\right)\left(\bar{s}_{\beta} P_{R} b_{\alpha}\right) . &
\end{array}
$$



Figure 3. Box diagrams contributing to $B_{s}-\bar{B}_{s}$ mixing. Both diagrams arise independently of the nature of the mediator involved in $b \rightarrow s \mu^{+} \mu^{-}$transitions.

The box diagrams contributing to these above operators are shown in figure 3. Using the Lagrangian from eq. (2.1), one obtains the following results for the coefficients:

$$
\begin{align*}
C_{1}= & \left(\chi_{B B}+\tilde{\chi}_{B B}\right) \frac{L_{A N}^{s *} L_{A M}^{b} L_{B M}^{s *} L_{B N}^{b}}{128 \pi^{2} m_{\Phi_{M}}^{2}} F\left(x_{A M}, x_{B M}, x_{N M}\right)  \tag{2.23}\\
C_{2}= & \chi_{B B} \frac{R_{A N}^{s *} L_{A M}^{b} R_{B M}^{s *} L_{B N}^{b}}{64 \pi^{2} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)  \tag{2.24}\\
C_{3}= & \tilde{\chi}_{B B} \frac{R_{A N}^{s *} L_{A M}^{b} R_{B M}^{s *} L_{B N}^{b}}{64 \pi^{2} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right),  \tag{2.25}\\
C_{4}= & \chi_{B B} \frac{R_{A N}^{s *} L_{A M}^{b} L_{B M}^{s *} R_{B N}^{b}}{32 \pi^{2} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right) \\
& -\tilde{\chi}_{B B} \frac{R_{A N}^{s *} R_{A M}^{b} L_{B M}^{s *} L_{B N}^{b}}{32 \pi^{2} m_{\Phi_{M}}^{2}} F\left(x_{A M}, x_{B M}, x_{N M}\right)  \tag{2.26}\\
C_{5}= & \tilde{\chi}_{B B} \frac{R_{A N}^{s *} L_{A M}^{b} L_{B M}^{s *} R_{B N}^{b}}{32 \pi^{2} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right) \\
& -\chi_{B B} \frac{R_{A N}^{s *} R_{A M}^{b} L_{B M}^{s *} L_{B N}^{b}}{32 \pi^{2} m_{\Phi_{M}}^{2}} F\left(x_{A M}, x_{B M}, x_{N M}\right)  \tag{2.27}\\
\widetilde{C}_{1,2,3}= & C_{1,2,3}(L \leftrightarrow R) \tag{2.28}
\end{align*}
$$

The loop functions $F(x, y, z)$ and $G(x, y, z)$ are defined in appendix B and the colour factors $\chi_{B B}$ and $\tilde{\chi}_{B B}$ are given in table 3 for the different allowed representations. Again, in the presence of Majorana fermions or real scalars crossed diagrams can be constructed and the resulting expressions are given in appendix C.2.

## $2.4 \quad D_{0}-\bar{D}_{0}$ mixing

NP contributions to the $D_{0}-\bar{D}_{0}$ mixing can be obtained in complete generality (at the low scale) from eqs. (2.23)-(2.28) by making the substitutions $s \rightarrow u, b \rightarrow c$, introducing couplings $L_{A M}^{u, c}$ and $R_{A M}^{u, c}$ of new scalars and fermions to up-quarks in straightforward extension of eq. (2.1).

In the context a UV complete model, $\mathrm{SU}(2)$ invariance imposes at the high scale that couplings to left-handed up-type quarks are related to the couplings to left-handed downtype quarks via CKM rotations. Therefore, working in the down-basis, the "minimal"

| $\mathrm{SU}(3)$ | $\Psi_{A}$ | $\Psi_{B}$ | $\Phi_{M}$ | $\Phi_{N}$ | $\chi_{B B}$ | $\tilde{\chi}_{B B}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 3 | 3 | 1 | 1 | 1 | 0 |
| II | 1 | 1 | $\overline{3}$ | $\overline{3}$ | 0 | 1 |
| III | 3 | 3 | 8 | 8 | $1 / 36$ | $7 / 12$ |
| IV | 8 | 8 | $\overline{3}$ | $\overline{3}$ | $7 / 12$ | $1 / 36$ |
| V | 3 | 3 | $(1,8)$ | $(8,1)$ | $-1 / 6$ | $1 / 2$ |
| VI | $(1,8)$ | $(8,1)$ | $\overline{3}$ | $\overline{3}$ | $1 / 2$ | $-1 / 6$ |
| VII | $\overline{3}$ | $\overline{3}$ | 3 | 3 | 1 | 1 |

Table 3. Table of the different $\mathrm{SU}(3)$ representations that can give a non-zero effect via box diagrams to $B_{s}-\bar{B}_{s}$ mixing. $\chi_{B B}$ and $\tilde{\chi}_{B B}$ denote the resulting group factors.

| $\mathrm{SU}(3)$ | $\Psi_{A}$ | $\Phi_{M}$ | $\chi_{a_{\mu}}$ |
| :---: | :---: | :---: | :---: |
| I | 1 | 1 | 1 |
| II | $(3, \overline{3})$ | $(3, \overline{3})$ | 3 |
| III | 8 | 8 | 8 |

Table 4. Table of the different $\mathrm{SU}(3)$ representations that can give a non-zero effect to $a_{\mu}$. $\chi_{a_{\mu}}$ denotes the resulting group factor.
effect generated in $D_{0}-\bar{D}_{0}$ is induced by the couplings

$$
\begin{equation*}
L_{A M}^{u}=V_{u s}^{*} L_{A M}^{s}+V_{u b}^{*} L_{A M}^{b}, \quad L_{A M}^{c}=V_{c s}^{*} L_{A M}^{s}+V_{c b}^{*} L_{A M}^{b} . \tag{2.29}
\end{equation*}
$$

### 2.5 Anomalous magnetic moment of the muon

The anomalous magnetic moment of the muon $\left(a_{\mu} \equiv(g-2)_{\mu} / 2\right)$ and its electric dipole moments ( $d_{\mu}$ ) we find from the diagrams in figure 2

$$
\begin{align*}
\Delta a_{\mu}= & \frac{\chi_{a_{\mu}} m_{\mu}^{2}}{8 \pi^{2} m_{\Phi_{M}}^{2}}\left[\left(L_{A M}^{\mu *} L_{A M}^{\mu}+R_{A M}^{\mu *} R_{A M}^{\mu}\right)\left(Q_{\Phi_{M}} \widetilde{F}_{7}\left(x_{A M}\right)-Q_{\Psi_{A}} F_{7}\left(x_{A M}\right)\right)\right.  \tag{2.30}\\
& \left.+\left(L_{A M}^{\mu *} R_{A M}^{\mu}+R_{A M}^{\mu *} L_{A M}^{\mu}\right) \frac{2 m_{\Psi_{A}}}{m_{\mu}}\left(Q_{\Phi_{M}} \widetilde{G}_{7}\left(x_{A M}\right)-Q_{\Psi_{A}} G_{7}\left(x_{A M}\right)\right)\right], \\
d_{\mu}= & \frac{\chi_{a_{\mu}} m_{\Psi_{A}}}{8 \pi^{2} m_{\Phi_{M}}^{2}} e\left(L_{A M}^{\mu *} R_{A M}^{\mu}-R_{A M}^{\mu *} L_{A M}^{\mu}\right)\left(Q_{\Phi_{M}} \widetilde{G}_{7}\left(x_{A M}\right)-Q_{\Psi_{A}} G_{7}\left(x_{A M}\right)\right), \tag{2.31}
\end{align*}
$$

where $m_{\mu}$ is the muon mass, $\chi_{a_{\mu}}$ is the colour factor given in table 4 , and $Q_{\Phi_{M}}$ and $Q_{\Psi_{A}}$ are the electric charges of the NP fields $\Phi_{M}$ and $\Psi_{A}$, respectively. Analogously to photonpenguin contributions to $b \rightarrow s$ transitions, the conservation of electric charge imposes that $Q_{\Phi_{M}}+Q_{\Psi_{A}}=Q_{\mu} \equiv-1$. Finally, the loop functions $F_{7}(x), \widetilde{F}_{7}(x), G_{7}(x)$ and $\widetilde{G}_{7}(x)$ are defined in appendix $B$.

### 2.6 Modified $Z$ couplings

Here, we study the effects of our new particles on modified $Z$ couplings, i.e. on $Z \bar{\mu} \mu, Z \bar{b} b$, $Z \bar{s} s$ and $Z \bar{s} b$ couplings, both for off- and on-shell $Z$ bosons. ${ }^{5}$ We define the form-factors governing $Z \bar{f} f$ interactions as [114]

$$
\begin{equation*}
-\frac{g_{2}}{c_{W}} \bar{f}^{\prime} \gamma^{\mu}\left[g_{L}^{f^{\prime} f}\left(q^{2}\right) P_{L}+g_{R}^{f^{\prime} f}\left(q^{2}\right) P_{R}\right] f Z_{\mu}+\text { h.c. } \tag{2.32}
\end{equation*}
$$

where $f=\{b, s, \mu\}, g_{2}$ is the $\operatorname{SU}(2)$ gauge coupling, $\theta_{W}$ the Weinberg angle and $q$ is the $Z$ momentum. Moreover,

$$
\begin{equation*}
g_{L(R)}^{f^{\prime} f}\left(q^{2}\right)=g_{f_{L}}^{\mathrm{SM}} \delta_{f^{\prime} f}+\Delta g_{L(R)}^{f^{\prime} f}\left(q^{2}\right) \tag{2.33}
\end{equation*}
$$

with $g_{f_{L}}^{\mathrm{SM}}=\left(T_{3}^{f}-Q_{f} s_{W}^{2}\right)$ and $g_{f_{R}}^{\mathrm{SM}}=-Q_{f} s_{W}^{2}$ being the $Z$ couplings to SM fermions at tree-level. The relevant Feynman diagrams are shown in figure 4. We write the coupling of the $Z$ boson to the new scalars and fermions as

$$
\begin{equation*}
\mathcal{L}^{Z}=-\frac{g_{2}}{c_{W}} Z_{\mu}\left(\bar{\Psi}_{A} \gamma^{\mu}\left[g_{A B}^{\Psi, L} P_{L}+g_{A B}^{\Psi, R} P_{R}\right] \Psi_{B}+g_{M N}^{\Phi} \Phi_{M}^{\dagger} i \grave{\partial}^{\mu} \Phi_{N}\right)+\text { h.c. } \tag{2.34}
\end{equation*}
$$

where we have introduced the notation $a \overleftrightarrow{\partial^{\mu}} b=a\left(\partial^{\mu} b\right)-\left(\partial^{\mu} a\right) b$, and with generic couplings $g_{A B}^{\Psi, L, R}$ and $g_{M N}^{\Phi}$ which can only be determined in a UV complete model in which also the couplings of the new particles to the SM Higgs are known. Using the generic Lagrangian from eq. (2.1), one obtains the following results for the coefficients

$$
\begin{align*}
\Delta g_{L}^{f^{\prime} f}\left(q^{2}\right)= & \frac{\chi_{Z} L_{B N}^{f^{\prime}} L_{A M}^{f *}}{32 \pi^{2}} \\
& {\left[2 g_{A B}^{\Psi, L} \delta_{M N} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G_{Z}\left(x_{A M}, x_{B M}\right)-g_{A B}^{\Psi, R} \delta_{M N} F_{Z}\left(x_{A M}, x_{B M}, m_{\Phi_{M}}\right)\right.} \\
& +g_{M N}^{\Phi} \delta_{A B} H_{Z}\left(x_{A M}, x_{A N}, m_{\Psi_{A}}\right)-\frac{1}{2}\left(g_{f_{L}}^{S M}+g_{f_{L}^{\prime}}^{S M}\right) \delta_{A B} \delta_{M N} I_{Z}\left(x_{A M}, m_{\Phi_{M}}\right) \\
& +q^{2}\left(g_{A B}^{\Psi, L} \delta_{M N} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{4}} \widetilde{G}_{Z}\left(x_{A M}, x_{B M}\right)-\frac{2}{3} \frac{g_{A B}^{\Psi, R} \delta_{M N}}{m_{\Phi_{M}}^{2}} \widetilde{F}_{Z}\left(x_{A M}, x_{B M}\right)\right. \\
& \left.\left.-\frac{1}{3} \frac{g_{M N}^{\Phi} \delta_{A B}}{m_{\Psi_{A}}^{2}} \widetilde{H}_{Z}\left(x_{A M}, x_{A N}\right)\right)\right]  \tag{2.35}\\
\Delta g_{R}^{f^{\prime} f}\left(q^{2}\right)= & \Delta g_{L}^{f^{\prime} f}\left(q^{2}\right)(L \leftrightarrow R) \tag{2.36}
\end{align*}
$$

where the loop functions are defined in appendix B , and the colour factor $\chi_{Z}=\chi_{\gamma}$ for $f, f^{\prime}=b, s$ (see table 2) and $\chi_{Z}=\chi_{a_{\mu}}$ (see table 4) for $f=\mu$. Here we have set the masses and momenta of the external fermions to 0 and expanded up to first order in $q^{2}$ over the NP scale. If one is considering data from $Z$ decays, eq. (2.35) has to be evaluated to $q^{2}=m_{Z}^{2}$ while for processes with an off-shell $Z$ (like $b \rightarrow s \ell^{+} \ell^{-}$) one has to set $q^{2}=0$.

Note that in the absence of EW symmetry breaking in the NP sector, the contribution of the self-energies cancel the one of the genuine vertex correction and eq. (2.35) vanishes

[^2]

Figure 4. Feynman diagrams modifying the $Z \bar{f}_{i} f_{j}$ vertex with $f_{i}=s, b, \mu$.
for $q^{2}=0$. Therefore, as noted above, $g_{A B}^{\Psi, L}, g_{A B}^{\Psi, R}$ and $g_{M N}^{\Phi}$ are only meaningful after EWSB and it is not possible to relate them purely to $\mathrm{SU}(2) \times \mathrm{U}(1)$ quantum numbers. In a specific UV model with a known pattern of EWSB, rotation matrices can be used to relate the couplings before and after the breaking. Consequently, the cancellation of UV divergences (present in some of the loop functions in eq. (2.35)) is only manifest after summation over $\mathrm{SU}(2)$ indices, due to a GIM-like cancellation originating from the unitarity of the rotation matrices. We will give a concrete example of this in section 4.

The form-factors in eq. (2.32) includes $Z \bar{s} b$ couplings generating contributions to $C_{9,10}^{(\prime)}$

$$
\begin{align*}
C_{9}^{(\prime) Z} & =\mathcal{N} \frac{\pi g^{2}}{\alpha_{\mathrm{EM}}^{2} c_{W}^{2} m_{Z}^{2}} g_{L(R)}^{s b}\left(q^{2}=0\right)\left(1-4 s_{W}^{2}\right),  \tag{2.37}\\
C_{10}^{(\prime) Z} & =-\mathcal{N} \frac{\pi g^{2}}{\alpha_{\mathrm{EM}} c_{W}^{2} m_{Z}^{2}} g_{L(R)}^{s b}\left(q^{2}=0\right) . \tag{2.38}
\end{align*}
$$

Note that these contributions are lepton flavour universal and therefore cannot account for $R_{K}$ and $R_{K^{*}}$. However, a mixture of lepton flavour universal and violating contributions is phenomenologically interesting [115], especially in the light of the recant Belle and LHCb measurements $[12,15]$. In a similar fashion, $Z \bar{s} b$ couplings will also generate the following contributions to $b \rightarrow s \nu \bar{\nu}$ contained in $C_{L(R)}$

$$
\begin{equation*}
C_{L(R)}^{Z}=-\mathcal{N} \frac{\pi g^{2}}{\alpha_{\mathrm{EM} c_{W}^{2}} m_{Z}^{2}} g_{L(R)}^{s b}\left(q^{2}=0\right) . \tag{2.39}
\end{equation*}
$$

Finally, if $\operatorname{SU}(2)$ invariance at the NP scale is imposed, the new scalars and fermions couple also the neutrinos. Hence, contributions to $Z \rightarrow \nu \bar{\nu}$ and $W \rightarrow \mu \bar{\nu}$ will arise as well. Concerning $Z \rightarrow \nu \bar{\nu}, g_{\nu_{L}}\left(q^{2}=m_{Z}^{2}\right)$ can be straightforwardly extracted from eq. (2.35) by appropriate replacements and the same is true concerning $W \mu \bar{\nu}$ couplings.

## 3 Experimental constraints on Wilson coefficients

In this section we review the experimental situation and the resulting constraints on the Wilson coefficients calculated in the previous section.

## 3.1 $b \rightarrow s$ transitions

The semileptonic operators $\mathcal{O}_{9,10}^{(\prime)}, \mathcal{O}_{S, P}^{(\prime)}$ and $\mathcal{O}_{T}^{(\prime)}$, together with magnetic operators $\mathcal{O}_{7}^{(\prime)}$, contribute to a plethora of $b \rightarrow s \ell^{+} \ell^{-}$observables. The corresponding measurements include total branching ratios of $B_{s} \rightarrow \ell^{+} \ell^{-}$[11], of the exclusive decays $B \rightarrow K^{*} \gamma[11], B \rightarrow$ $\phi \gamma$ [116], the inclusive decay $B \rightarrow X_{s} \gamma$ [11], the angular analyses of $B \rightarrow K^{(*)} \ell^{+} \ell^{-}[117-123]$ (proposed in refs. [124-126]) and $B_{s} \rightarrow \phi \ell^{+} \ell^{-}$[127], and also the ratios $R_{K}$ [9] and $R_{K^{*}}[10,128]$ measuring lepton flavour universality violation.

First of all, the contributions of scalar operators are helicity-enhanced in the $B_{s} \rightarrow$ $\mu^{+} \mu^{-}$branching ratio with respect to the $O_{10}$ contribution of the SM. This results in the bound [129]

$$
\begin{align*}
\frac{\mathcal{B}^{\exp }\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)}{\mathcal{B}^{\mathrm{SM}}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)}-1= & \left|1+\frac{C_{10}-C_{10}^{\prime}}{C_{10}^{\mathrm{SM}}}+\frac{m_{B_{s}}^{2}}{2 m_{\mu}\left(m_{b}+m_{s}\right)} \frac{C_{P}-C_{P}^{\prime}}{C_{10}^{\mathrm{SM}}}\right|^{2} \\
& +\frac{m_{B_{s}}^{2}\left(m_{B_{s}}^{2}-4 m_{\mu}^{2}\right)}{4 m_{\mu}^{2}\left(m_{b}+m_{s}\right)^{2}}\left|\frac{C_{S}-C_{S}^{\prime}}{C_{10}^{\mathrm{SM}}}\right|^{2}-1=-0.13 \pm 0.20 \tag{3.1}
\end{align*}
$$

which excludes sizable contributions to scalar operators (unless there is a purely scalar quark current) and leads to

$$
\begin{equation*}
\left|C_{S, P}^{(\prime)}\right| \lesssim 0.03 \quad(2 \sigma) \tag{3.2}
\end{equation*}
$$

from the updated one-parameter fit of ref. [130]. Therefore, we neglect the effects of scalar operators in semi-leptonic $B$ since they anyway cannot explain the corresponding anomalies.

Moreover, the inclusive $b \rightarrow s \gamma$ decay strongly constrains the magnetic operators. From [131, 132], in the limit of vanishing $C_{7,8}^{\prime}{ }^{6}$ we have

$$
\begin{equation*}
\frac{\mathcal{B}^{\exp }(b \rightarrow s \gamma)}{\mathcal{B}^{\mathrm{SM}}(b \rightarrow s \gamma)}-1=-2.87\left[C_{7}+0.19 C_{8}\right]=(-0.7 \pm 8.2) \times 10^{-2} \tag{3.3}
\end{equation*}
$$

leading to

$$
\begin{equation*}
\left|C_{7}+0.19 C_{8}\right| \lesssim 0.06 \quad(2 \sigma) \tag{3.4}
\end{equation*}
$$

Here, we used $C_{7,8}$ at a matching scale of 1 TeV as input. Again, these constraints are so stringent that the effect of $C_{7,8}$ on the flavour anomalies can be mostly neglected.

On the other hand, vector operators can explain the $b \rightarrow s \ell^{+} \ell^{-}$anomalies. We therefore refer to global fits to constrain $C_{9,10}^{(\prime)}$, where all the relevant observables have been taken into account [1-8]. The results of the most recent fits find at the $2 \sigma$ level

$$
\begin{array}{ll}
-1.48 \leq C_{9} \leq-0.71, & -0.12 \leq C_{10} \leq 0.61 \\
-0.56 \leq C_{9}^{\prime} \leq 1.14, & -0.57 \leq C_{10}^{\prime} \leq 0.34
\end{array}
$$

according to ref. [12] (which is compatible with refs. [13, 14, 16, 17]).

[^3]As explained in section 2.2, $\mathrm{SU}(2)$ invariance implies the presence of contributions to $B \rightarrow K^{(*)} \nu \bar{\nu}$ decays as well. Since there is no experimental way to distinguish different neutrino flavours in these decays, one measures the total branching ratio which we normalize to its SM prediction [110]:

$$
\begin{equation*}
R_{K^{(*)}}^{\nu \bar{\nu}}=\frac{\mathcal{B}^{\exp }\left(B \rightarrow K^{(*)} \nu \bar{\nu}\right)}{\mathcal{B}^{\mathrm{SM}}\left(B \rightarrow K^{(*)} \nu \bar{\nu}\right)}=\frac{2\left(C_{L}^{\mathrm{SM}}\right)^{2}+\left(C_{L}^{\mathrm{SM}}+C_{L}\right)^{2}-\kappa\left(C_{L}^{\mathrm{SM}}+C_{L}\right) C_{R}+C_{R}^{2}}{3\left(C_{L}^{\mathrm{SM}}\right)^{2}} . \tag{3.5}
\end{equation*}
$$

In the case of a $K$ in the final state one has $\kappa \equiv-2$, while for the $K^{*}$ one gets $\kappa=$ $1.34(4)$ [110]. The current experimental limits at $90 \%$ C.L. are [134]

$$
\begin{equation*}
R_{K}^{\nu \bar{\nu}}<3.9, \quad \quad R_{K^{*}}^{\nu \bar{\nu}}<2.7 \tag{3.6}
\end{equation*}
$$

### 3.2 Neutral meson mixing

The experimental constraint on the Wilson coefficients in eqs. (2.23)-(2.28) comes from the mass difference $\Delta M_{s}$ of neutral $B_{s}$ mesons, see e.g. ref. [111] (in the case of real Wilson coefficients). To compare our results with experiments we use

$$
\begin{align*}
\frac{\Delta M_{s}^{\exp }}{\Delta M_{s}^{\mathrm{SM}}} & =\left|1+\sum_{i, j=1}^{3} R_{i}\left(\mu_{b}\right) \frac{\eta_{i j}\left(\mu_{b}, \mu_{H}\right)}{C_{1}^{\mathrm{SM}}\left(\mu_{b}\right)}\left(C_{j}+\widetilde{C}_{j}\right)+\sum_{i, j=4}^{5} R_{i}\left(\mu_{b}\right) \frac{\eta_{i j}\left(\mu_{b}, \mu_{H}\right)}{C_{1}^{\mathrm{SM}}\left(\mu_{b}\right)} C_{j}\right| \\
& =\left|1+\frac{0.8\left(C_{1}+\widetilde{C}_{1}\right)-1.9\left(C_{2}+\widetilde{C}_{2}\right)+0.5\left(C_{3}+\widetilde{C}_{3}\right)+5.2 C_{4}+1.9 C_{5}}{C_{1}^{\mathrm{SM}}\left(\mu_{b}\right)}\right|, \tag{3.7}
\end{align*}
$$

where $R_{i}\left(\mu_{b}\right)$ is related to the matrix element of the operators $Q_{i}$ in eq. (2.22) at the scale $\mu_{b}$ by the relation

$$
\begin{equation*}
R_{i}\left(\mu_{b}\right)=\frac{\left\langle\bar{B}_{s}\right| Q_{i}\left(\mu_{b}\right)\left|B_{s}\right\rangle}{\left\langle\bar{B}_{s}\right| Q_{1}\left(\mu_{b}\right)\left|B_{s}\right\rangle} . \tag{3.8}
\end{equation*}
$$

The coefficients $C_{i}$ and $\widetilde{C}_{i}$ are the ones in eqs. (2.23)-(2.28), computed at the NP scale $\mu_{H}$. The matrix in operator space $\eta_{i j}\left(\mu_{b}, \mu_{H}\right)$ encodes the QCD evolution from the high scale $\mu_{H}$ to $\mu_{b}$, which we calculated numerically for a reference scale $\mu_{H}=1 \mathrm{TeV}$ [135]. The matrix elements in eqs. (3.7)-(3.8) have been computed by a $N_{f}=2+1$ lattice simulation [136], which found values consistent with the $N_{f}=2$ calculation [137] and recent sum rules results [138]. It is worth mentioning that FLAG-2019 [139] only provides a lattice average for $\left\langle\bar{B}_{s}\right| Q_{1}\left(\mu_{b}\right)\left|B_{s}\right\rangle$, which is however dominated by the $N_{f}=2+1$ results from ref. [136]. Therefore, we decided to employ the results from ref. [136] in eqs. (3.7)-(3.8). The SM value for the Wilson coefficient is $C_{1}^{\mathrm{SM}}\left(\mu_{b}\right)=\frac{G_{F}^{2} M_{W}^{2}}{4 \pi^{2}} \lambda_{t}^{2} \eta_{11}\left(\mu_{b}, m_{t}\right) S_{0}\left(x_{t}\right) \simeq 7.2 \times 10^{-11} \mathrm{GeV}^{-2}$.

The experimental constraint therefore reads [140]

$$
\begin{equation*}
R_{\Delta M_{s}}=\frac{\Delta M_{s}^{\exp }}{\Delta M_{s}^{\mathrm{SM}}}-1=-0.09 \pm 0.08 \tag{3.9}
\end{equation*}
$$

computed with the values from ref. [136] for $\left\langle\bar{B}_{s}\right| Q_{1}\left(\mu_{b}\right)\left|B_{s}\right\rangle$. This value shows a slight tension with the SM as first outlined in refs. [141, 142]. The tension would be reduced if one considered the results for the matrix element from ref. [138]: however, in this case
one should rely on a separate computation for the decay constant, while in ref. [136] both quantities are computed together.

Analogously to the $B_{s}$ system, $D_{0}-\bar{D}_{0}$ mixing is constrained by the mass difference of neutral $D_{0}$ mesons [140]:

$$
\begin{equation*}
\Delta M_{D_{0}}^{\exp }=\left(0.63_{-0.29}^{+0.27}\right) \times 10^{-11} \mathrm{MeV} . \tag{3.10}
\end{equation*}
$$

Unfortunately, a precise SM prediction is still lacking in this sector but one can constrain the NP contribution by assuming that not more than the total mass difference is generated by it.

### 3.3 Anomalous magnetic moment of the muon

From the experimental side, this quantity has been already measured quite precisely [93], but further improvements by experiments at Fermilab [143] and J-PARC [144] (see also [145]) are expected in the future. On the theory side, the SM prediction has been improved continuously [146-169]. The current tension between the two determinations accounts to

$$
\begin{equation*}
\Delta a_{\mu}=a_{\mu}^{\exp }-a_{\mu}^{\mathrm{SM}} \sim 270(85) \times 10^{-11} . \tag{3.11}
\end{equation*}
$$

## 3.4 $Z$ decays

The main experimental measurements of $Z$ couplings have been performed at LEP [170] (at the $Z$ pole). We extract from the model independent analysis of ref. [171] the values for the NP contributions ${ }^{7}$

$$
\begin{array}{rlrl}
\Delta g_{\mu_{L}}\left(m_{Z}^{2}\right) & =-(0.1 \pm 1.1) \times 10^{-3}, & \Delta g_{\mu_{R}}\left(m_{Z}^{2}\right)=(0.0 \pm 1.3) \times 10^{-3}, \\
\Delta g_{b_{L}}\left(m_{Z}^{2}\right) & =-(0.33 \pm 0.16) \times 10^{-2}, & \Delta g_{b_{R}}\left(m_{Z}^{2}\right)=-(2.30 \pm 0.82) \times 10^{-3}, \\
\Delta g_{\nu_{L}}\left(m_{Z}^{2}\right) & =(0.40 \pm 0.21) \times 10^{-2}, & & \tag{3.12}
\end{array}
$$

neglecting cancellations and correlations.

## $4 \quad 4^{\text {th }}$ generation model

In this section we propose a model with a vector-like $4^{\text {th }}$ generation of fermions and a new complex scalar. This will also allow us to apply and illustrate the generic findings of the previous section to a UV complete model and study the effects in $b \rightarrow s \ell^{+} \ell^{-}$data and $a_{\mu}$.

### 4.1 Lagrangian

The Lagrangian for our $4^{\text {th }}$ generation model is obtained from the SM one by adding a $4^{\text {th }}$ vector-like generation $[103,104]$ and a neutral scalar

$$
\begin{align*}
L^{4 \mathrm{th}}= & \sum_{i}\left(\Gamma_{q_{i}}^{L} \bar{\Psi}_{q} P_{L} q_{i}+\Gamma_{\ell_{i}}^{L} \bar{\Psi}_{\ell} P_{L} \ell_{i}+\Gamma_{u_{i}}^{R} \bar{\Psi}_{u} P_{R} u_{i}+\Gamma_{d_{i}}^{R} \bar{\Psi}_{d} P_{R} d_{i}+\Gamma_{e_{i}}^{R} \bar{\Psi}_{e} P_{R} e_{i}\right) \Phi+\text { h.c. } \\
& +\sum_{C=L, R}\left(\lambda_{C}^{U} \bar{\Psi}_{q} P_{C} \tilde{h} \Psi_{u}+\lambda_{C}^{D} \bar{\Psi}_{q} P_{C} h \Psi_{d}+\lambda_{C}^{E} \bar{\Psi}_{\ell} P_{C} h \Psi_{e}\right)+\text { h.c. } \\
& +\sum_{F=q, \ell, u, d, e} M_{F} \bar{\Psi}_{F} \Psi_{F}+\kappa h^{\dagger} h \Phi^{\dagger} \Phi+m_{\Phi}^{2} \Phi^{\dagger} \Phi \tag{4.1}
\end{align*}
$$

[^4]where $i$ is a family index and $h$ the SM Higgs doublet. The charge assignments for the new vector-like fermions $\Psi=\Psi_{L}+\Psi_{R}$ with $P_{L, R} \Psi=\Psi_{L, R}$ and the new scalar $\Phi$ are
\[

$$
\begin{array}{c|cccc} 
& \mathrm{SU}(3) & \mathrm{SU}(2) & \mathrm{U}(1) & \mathrm{U}^{\prime}(1)  \tag{4.2}\\
\hline \Psi_{q} & 3 & 2 & 1 / 6 & Z \\
\Psi_{u} & 3 & 1 & 2 / 3 & Z \\
\Psi_{d} & 3 & 1 & -1 / 3 & Z \\
\Psi_{\ell} & 1 & 2 & -1 / 2 & Z \\
\Psi_{e} & 1 & 1 & -1 & Z \\
\Phi & 1 & 1 & 0 & -Z
\end{array}
$$ .
\]

The SM fermions have the same $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ charge assignments of the relative NP fermion partner, and the higgs transforms as a ( $1,2,1 / 2$ ). Here we assigned to the new particles also charges under a new $\mathrm{U}(1)$ group in order to forbid mixing with the SM particles, giving a similar effect as R-parity in the MSSM. ${ }^{8}$ From table 4.2 we see that concerning $\mathrm{SU}(3)$ we are dealing with cases I of our generic analysis in tables 1-4. In particular, concerning $b \rightarrow s \mu^{+} \mu^{-}$, this model would generate diagrams of type a) in figure 1.

After EWSB, mass matrices for the new fermions are generated

$$
\begin{align*}
L_{\mathrm{mass}}^{4 \mathrm{th}}= & \binom{\bar{\Psi}_{q, 1}}{\bar{\Psi}_{u}}^{T} \mathbf{M}_{U} P_{L}\binom{\Psi_{q, 1}}{\Psi_{u}}+\binom{\bar{\Psi}_{q, 2}}{\bar{\Psi}_{d}}^{T} \mathbf{M}_{D} P_{L}\binom{\Psi_{q, 2}}{\Psi_{d}} \\
& +\binom{\bar{\Psi}_{\ell, 2}}{\bar{\Psi}_{e}}^{T} \mathbf{M}_{E} P_{L}\binom{\Psi_{\ell, 2}}{\Psi_{e}}+\text { h.c. }, \tag{4.3}
\end{align*}
$$

where $\mathbf{M}_{U, D, E}$ are non-diagonal mass matrices

$$
\mathbf{M}_{D(U)}=\left(\begin{array}{cc}
M_{q} & \sqrt{2} v \lambda_{R}^{D(U)}  \tag{4.4}\\
\sqrt{2} v \lambda_{L}^{D(U) *} & M_{d(u)}
\end{array}\right), \quad \mathbf{M}_{E}=\left(\begin{array}{cc}
M_{\ell} & \sqrt{2} v \lambda_{R}^{E} \\
\sqrt{2} v \lambda_{L}^{E *} & M_{e}
\end{array}\right) .
$$

Here the subscripts 1 and 2 denote the $\operatorname{SU}(2)$ component of the doublet. We diagonalize these mass matrices by performing the field redefinitions

$$
\begin{align*}
P_{L}\binom{\Psi_{q, 1}}{\Psi_{u}}_{I} & \rightarrow W_{I J}^{U_{L}} \Psi_{J}^{U_{L}}, & & P_{L}\binom{\Psi_{q, 2}}{\Psi_{d}}_{I} \rightarrow W_{I J}^{D_{L}} \Psi_{J}^{D_{L}}, \quad+L \rightarrow R \\
P_{L} \Psi_{L, 1} & \rightarrow \Psi^{N_{L}}, & & P_{L}\binom{\Psi_{\ell, 2}}{\Psi_{e}}_{I} \rightarrow W_{I J}^{E_{L}} \Psi_{J}^{E_{L}}, \quad+\quad L \rightarrow R \tag{4.5}
\end{align*}
$$

leading to

$$
\begin{equation*}
\left(W^{F_{L} \dagger} \mathbf{M}_{F} W^{F_{R}}\right)_{I J}=m_{F_{I}} \delta_{I J}, \quad \text { with } F=U, D, E . \tag{4.6}
\end{equation*}
$$

Therefore, after EWSB we have the mass eigenstates $\Psi_{I}^{U_{L, R}}, \Psi_{I}^{D_{L, R}}, \Psi_{I}^{E_{L, R}}$ and $\Psi^{N_{L, R}}$, with $I=\{1,2\}$. In particular, $\Psi_{I}^{U_{L, R}}$ and $\Psi_{I}^{D_{L, R}}\left(\Psi_{I}^{E_{L, R}}\right.$ and $\left.\Psi^{N_{L, R}}\right)$ are $\mathrm{SU}(3)$ triplets (singlets)

[^5]with the same electric charges as up-type and down-type quarks (charged-leptons and neutrinos), respectively.

The rotations introduced at eq. (4.5) lead to the following Lagrangian for the interactions in the broken phase

$$
\begin{align*}
L_{\mathrm{int}}^{4 \mathrm{th}}= & \left(L_{I}^{d_{i}} \bar{\Psi}_{I}^{D} P_{L} d_{i}+L_{I}^{e_{i}} \bar{\Psi}_{I}^{E} P_{L} e_{i}+R_{I}^{d_{i}} \bar{\Psi}_{I}^{D} P_{R} d_{i}+R_{I}^{e_{i}} \bar{\Psi}_{I}^{E} P_{R} e_{i}\right) \Phi \\
& +\left(L_{I}^{u_{i}} \bar{\Psi}_{I}^{U} P_{L} u_{i}+L^{\nu_{i}} \bar{\Psi}^{N} P_{L} \nu_{i}+R_{I}^{u_{i}} \bar{\Psi}_{I}^{U} P_{R} u_{i}\right) \Phi+\text { h.c. } \tag{4.7}
\end{align*}
$$

which resembles eq. (2.1) for the special case of our 4th generation model. Thus identify

$$
\begin{array}{llll}
L_{I}^{d_{i}}=\Gamma_{q_{i}}^{L} W_{1 I}^{D_{R}{ }^{*}}, & L_{I}^{e_{i}}=\Gamma_{\ell_{i}}^{L} W_{I I}^{E_{R}{ }^{*}}, & L_{I}^{u_{i}}=\Gamma_{q_{j}}^{L} V_{i j}^{*} W_{I I}^{U_{R}{ }^{*}}, \quad L^{\nu_{i}}=\Gamma_{\ell_{i}}^{L}, \\
R_{I}^{d_{i}}=\Gamma_{d_{i}}^{R} W_{2 I}^{D_{L}^{*}}, & R_{I}^{e_{i}}=\Gamma_{e_{i}}^{R} W_{2 I}^{E_{L}^{*}}, & R_{I}^{u_{i}}=\Gamma_{u_{i}}^{R} W_{2 I}^{U_{L}^{*}} . \tag{4.8}
\end{array}
$$

Here we worked in the down-basis for the SM quarks which means that CKM matrices $V_{i j}$ appear in vertices involving up-type quarks. The first two columns of the above Lagrangian involves couplings with down-type quarks and charged leptons and can be directly matched on the Lagrangian in eq. (2.1) for the case of only one scalar, i.e. $\Phi_{M} \equiv \Phi$ and $\Psi_{A} \equiv$ $\left\{\Psi_{I}^{D}, \Psi_{I}^{E}\right\}$. The presence of $L_{I}^{u_{i}}\left(L^{\nu_{i}}\right)$ resembles the fact, mentioned in section 2, that left-handed couplings to down-quarks (leptons) lead via $\mathrm{SU}(2)$ to couplings to left-handed up-quarks (neutrinos). In addition couplings to right-handed up-quarks $R_{I}^{u_{i}}$ appear in our model which are however not relevant for our phenomenology.

### 4.2 Wilson coefficients

With these conventions we can now easily derive the Wilson coefficients within our model which can be directly obtained from the results of section 2 . In order to simplify the expressions, we will assume $M_{Q}=M_{d} \equiv m_{D}$ and $M_{L}=M_{e} \equiv m_{E}$ and only take into account couplings to $b, s$ and $\mu$ in eq. (4.1):

$$
\begin{equation*}
\left\{\Gamma_{s}^{L}, \Gamma_{b}^{L}, \Gamma_{\mu}^{L}, \Gamma_{s}^{R}, \Gamma_{b}^{R}, \Gamma_{\mu}^{R}\right\}, \tag{4.9}
\end{equation*}
$$

Concerning $\operatorname{SU}(2)$ breaking effects the couplings $\lambda_{L, R}^{D}$ and $\lambda_{L, R}^{E}$ related to the down and charged leptons sector, respectively, can be relevant. However, concerning $\lambda_{L, R}^{D}$ recall that from section 3.1 that experimental data suggests very small values for $C_{S, P}$ and $C_{7,8}$. In our model this can be achieved by assuming $\lambda_{L, R}^{D}=0 .{ }^{9}$ In this limit the mass matrix $\mathbf{M}^{D}$ in eq. (4.4) is diagonal and the corresponding rotation matrices $W^{D_{R(L)}}$ in eq. (4.6) are equal to the identity, which implies

$$
\begin{equation*}
C_{S, P} \propto L_{A}^{s *} R_{A}^{b} \propto W_{1 A}^{D_{R}} W_{2 A}^{D_{L}{ }^{*}}=\delta_{1 A} \delta_{2 A}=0 \tag{4.10}
\end{equation*}
$$

With this setup, we obtain the following non-vanishing couplings in the quark sector of the Lagrangian in eq. (4.7):

$$
\begin{array}{ll}
L_{1}^{s}=\Gamma_{s}^{L}, \quad L_{1}^{b}=\Gamma_{b}^{L}, \quad R_{2}^{s}=\Gamma_{s}^{R}, \quad R_{2}^{b}=\Gamma_{b}^{R}, \\
L_{1}^{u}=V_{u s}^{*} \Gamma_{s}^{L}+V_{u b}^{*} \Gamma_{b}^{L}, \quad L_{1}^{c}=V_{c s}^{*} \Gamma_{s}^{L}+V_{c b}^{*} \Gamma_{b}^{L} . \tag{4.11}
\end{array}
$$

[^6]with
\[

$$
\begin{equation*}
\Gamma^{L} \equiv L_{1}^{b} L_{1}^{s *}, \quad \Gamma^{R} \equiv R_{2}^{b} R_{2}^{s *}, \quad x_{D(E)} \equiv \frac{m_{D(E)}^{2}}{m_{\Phi}^{2}} \tag{4.12}
\end{equation*}
$$

\]

The expressions of Wilson coefficients for $b \rightarrow s$ processes simplify to:

- $b \rightarrow s \mu^{+} \mu^{-}$and $b \rightarrow s \gamma$ (see eqs. (2.4)-(2.16))

$$
\begin{align*}
C_{9}^{\mathrm{box}} & =-\mathcal{N} \frac{\Gamma^{L}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi}^{2}}\left(\left|\Gamma_{\mu}^{L}\right|^{2}+\left|\Gamma_{\mu}^{R}\right|^{2}\right) F\left(x_{D}, x_{E}\right),  \tag{4.13}\\
C_{10}^{\mathrm{box}} & =\mathcal{N} \frac{\Gamma^{L}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi}^{2}}\left(\left|\Gamma_{\mu}^{L}\right|^{2}-\left|\Gamma_{\mu}^{R}\right|^{2}\right) F\left(x_{D}, x_{E}\right),  \tag{4.14}\\
C_{9}^{\gamma} & =\mathcal{N} \frac{\Gamma^{L}}{6 m_{\Phi}^{2}} \widetilde{G}_{9}\left(x_{D}\right), \quad C_{7}=\mathcal{N} \frac{\Gamma^{L}}{6 m_{\Phi}^{2}} F_{7}\left(x_{D}\right), \quad C_{8}=-\mathcal{N} \frac{\Gamma^{L}}{2 m_{\Phi}^{2}} F_{7}\left(x_{D}\right),  \tag{4.15}\\
C_{9}^{\prime \text { box }} & =C_{9}^{\text {box }}(L \leftrightarrow R), \quad C_{10}^{\prime \text { box }}=-C_{10}^{\text {box }}(L \leftrightarrow R),  \tag{4.16}\\
C_{9}^{\prime \gamma} & =C_{9}^{\gamma}(L \leftrightarrow R), \quad C_{7,8}^{\prime}=C_{7,8}(L \leftrightarrow R) . \tag{4.17}
\end{align*}
$$

- $b \rightarrow s \nu \bar{\nu}$ (see eqs. (2.19)-(2.20))

$$
\begin{equation*}
C_{L}=-\mathcal{N} \frac{\Gamma^{L}\left|\Gamma_{\mu}^{L}\right|^{2}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi}^{2}} F\left(x_{D}, x_{E}\right), \quad C_{R}=-\mathcal{N} \frac{\Gamma^{R}\left|\Gamma_{\mu}^{L}\right|^{2}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi}^{2}} F\left(x_{D}, x_{E}\right) . \tag{4.18}
\end{equation*}
$$

- $B_{s}-\bar{B}_{s}$ (see eqs. $\left.(2.23)-(2.28)\right)$

$$
\begin{equation*}
C_{1}=\frac{\left|\Gamma^{L}\right|^{2}}{128 \pi^{2} m_{\Phi}^{2}} F\left(x_{D}\right), \quad C_{5}=-\frac{\Gamma^{L} \Gamma^{R}}{32 \pi^{2} m_{\Phi}^{2}} F\left(x_{D}\right), \quad \widetilde{C}_{1}=\frac{\left|\Gamma^{R}\right|^{2}}{128 \pi^{2} m_{\Phi}^{2}} F\left(x_{D}\right), \tag{4.19}
\end{equation*}
$$

where the (simplified) loop function are defined in appendix B. In addition there are contributions to the $C_{1}$ analogue in $D^{0}-\overline{D^{0}}$ mixing obtained by substituting $L_{1}^{b} \rightarrow L_{1}^{c}$ and $L_{1}^{s} \rightarrow L_{1}^{u}$ within $\Gamma^{L}$.

In the charged-lepton sector $\operatorname{SU}(2)$ breaking effects (encoded in $\lambda_{L, R}^{E}$ ) can give a sizable chiral enhancement of the NP effect in $a_{\mu}$ (see eq. (2.30)) such that the long-standing anomaly in this channel can be addressed. In general one can parametrize the rotation matrices as

$$
W^{E_{L, R}}=\left(\begin{array}{cc}
\cos \left(\theta_{L, R}\right) & -\sin \left(\theta_{L, R}\right)  \tag{4.20}\\
\sin \left(\theta_{L, R}\right) & \cos \left(\theta_{L, R}\right)
\end{array}\right),
$$

leading to

$$
\begin{align*}
L_{1}^{\mu}=\Gamma_{\mu}^{L} \cos \theta_{L}, & L_{2}^{\mu}=-\Gamma_{\mu}^{L} \sin \theta_{L}, \quad L^{\nu}=\Gamma_{\mu}^{L}, \\
R_{1}^{\mu}=\Gamma_{\mu}^{R} \sin \theta_{R}, & R_{2}^{\mu}=\Gamma_{\mu}^{R} \cos \theta_{R} . \tag{4.21}
\end{align*}
$$

In our analysis we will consider a simplified setup with $\lambda_{R}^{E}=-\lambda_{L}^{E} \equiv \lambda^{E}$ that maximizes the effect in $a_{\mu}$ (which at leading order in $v$ is proportional to $\lambda_{R}^{E}-\lambda_{L}^{E}$ ). In this approximation we have for

- $a_{\mu}$ (see eq. (2.30))

$$
\begin{equation*}
\Delta a_{\mu}=\frac{m_{\mu}^{2}}{8 \pi^{2} m_{\Phi}^{2}}\left[\left(\left|\Gamma_{\mu}^{L}\right|^{2}+\left|\Gamma_{\mu}^{R}\right|^{2}\right) F_{7}\left(x_{E}\right)+\frac{8}{\sqrt{2}} \frac{v \lambda^{E}}{m_{\mu}} \Gamma_{\mu}^{L} \Gamma_{\mu}^{R} G_{7}\left(x_{E}\right)\right] \tag{4.22}
\end{equation*}
$$

where we have assumed real values for the couplings, implying a vanishing $d_{\mu}$. Let us stress that the contributions proportional to $v \lambda^{E}$, coming from $\mathrm{SU}(2)$ breaking terms, is chirally enhanced can give a sizable effect that can explain the $a_{\mu}$ anomaly.

- $Z \rightarrow \mu^{+} \mu^{-}$(see eqs. (2.35)-(2.36))

$$
\begin{align*}
& \Delta g_{\mu_{L}}\left(m_{Z}^{2}\right)=-\frac{\left|\Gamma_{\mu}^{L}\right|^{2}}{32 \pi^{2}}\left[\frac{m_{Z}^{2}}{m_{\Phi}^{2}}\left(\left(1-2 s_{W}^{2}\right) \widetilde{G}_{9}\left(x_{E}\right)+\frac{2}{3}\left(\frac{v \lambda^{E}}{m_{E}}\right)^{2} F_{9}\left(x_{E}\right)\right)+\left(\frac{v \lambda^{E}}{m_{E}}\right)^{2} F_{Z}\left(x_{E}\right)\right] \\
& \Delta g_{\mu_{R}}\left(m_{Z}^{2}\right)=\frac{\left|\Gamma_{\mu}^{R}\right|^{2}}{32 \pi^{2}}\left[\frac{m_{Z}^{2}}{m_{\Phi}^{2}}\left(2 s_{W}^{2} \widetilde{G}_{9}\left(x_{E}\right)+\frac{2}{3}\left(\frac{v \lambda^{E}}{m_{E}}\right)^{2} F_{9}\left(x_{E}\right)\right)+\left(\frac{v \lambda^{E}}{m_{E}}\right)^{2} F_{Z}\left(x_{E}\right)\right] \tag{4.23}
\end{align*}
$$

where the simplified loop function $F_{Z}\left(x_{E}\right)$ has been defined in appendix B. The results for $Z \rightarrow b \bar{b}$ couplings can be easily obtained by suitable substitutions. Note that in our approximation of $\lambda_{L, R}^{D}=0$ the correction to the $Z \bar{s} b$ vertex vanishes at $q^{2}=0$. Note that the UV divergences cancel as required, once for the couplings in eq. (2.34) the relations

$$
g^{\Psi, L(R)}=W^{E_{L(R)} \dagger}\left(\begin{array}{cc}
g_{\Psi_{L, 2}} & 0  \tag{4.25}\\
0 & g_{\Psi_{e}}
\end{array}\right) W^{E_{L(R)}}=W^{E_{L(R)} \dagger}\left(\begin{array}{cc}
g_{\mu_{L}}^{\mathrm{SM}} & 0 \\
0 & g_{\mu_{R}}^{\mathrm{SM}}
\end{array}\right) W^{E_{L(R)}}
$$

and $g_{\Phi}=0$ are used. Thus the finiteness of the result can be traced by to the unitarity of the matrices $W$.

### 4.3 Phenomenology

We are now ready to consider the phenomenology of our $4^{\text {th }}$ generation model. For this purpose we will perform a combined fit to all the relevant and available experimental data, as briefly reviewed in section 3. We perform this fit using the publicly available HEPfit package [172], performing a Markov Chain Monte Carlo (MCMC) analysis employing the Bayesian Analysis Toolkit (BAT) [173].

Let us first choose specific values for the masses of the scalar $\Phi$ and the fermions $\Psi$. As observed in ref. [81] a large splitting between the scalar mass and the vector-like lepton mass with respect to the vector-like quark masses is welcome to suppress the relative effect in $\Delta m_{B_{s}}$. Since the vector-like quarks should not be too light anyway because of direct LHC searches $[174,175]$ we choose $m_{\Phi} \simeq m_{E} \simeq 450 \mathrm{GeV}^{10}$ and $m_{D}=3.15 \mathrm{TeV}$, corresponding to $x_{E, L} \simeq 1$ and $x_{D} \simeq 50$. These values are well beyond the reach of direct searches at LHC: concerning $m_{E, L}$ the bounds come from Drell-Yan production of the new

[^7]fermions which are subsequently decaying in the neutral scalar and SM leptons. Therefore, the collider signature is similar to the one of MSSM slepton [176, 177]. ${ }^{11}$

Turning to the coupling of the new scalars and fermions to quarks and muons, we assume a flatly distributed priors within the range $|\Gamma| \leq 1.5$ such that perturbativity is respected. The marginalized posterior probability distribution for all NP couplings, together with the correlations among them, can be found in appendix E. In the rest of this section we will focus on one particular benchmark point, that we selected because it lies within all the $1 \sigma$ regions of the combined posterior distributions for the NP couplings (see figure 10). The benchmark values are

$$
\begin{equation*}
\left|\Gamma_{\mu}^{L}\right|=1.5, \quad\left|\Gamma_{\mu}^{R}\right|=1.4, \quad \lambda^{E}=0.0015, \quad \Gamma^{L}=-1.0, \quad \Gamma^{R}=-0.12 \tag{4.26}
\end{equation*}
$$

assuming real values for all couplings. Note that the small value for $\lambda^{E}$ is obtained from the fit due to its correlation with $\left|\Gamma_{\mu}^{R}\right|$. As can be seen from the combined posterior distribution of these 2 parameters shown in figure 10 , higher values of $\lambda^{E}$ would require lower values of $\left|\Gamma_{\mu}^{R}\right|$, which is disfavored by the current fit to $b \rightarrow s \ell^{+} \ell^{-}$data.

We observe that it is extremely important to allow for a right-handed coupling $\Gamma_{\mu}^{R}$ together a mixing coupling $\lambda^{E}$ in the muon sector such that $a_{\mu}$ can be explained. This can be seen from the fit in the $\left(\left|\Gamma_{\mu}^{L}\right|,\left|\Gamma_{\mu}^{R}\right|\right)$-plane from the left panel of figure 5 . In the case with $\lambda^{E}=0.0015$, corresponding to the benchmark point reported in eq. (4.26), one can see that it is possible to explain the deviation in $a_{\mu}$ by means of couplings of order unity. However, the situation changes significantly if one did not allow the presence of a coupling of the vector-like leptons to the SM Higgs. As shown in the right panel of figure 5, with $\lambda^{E}=0$, it is not possible to obtain couplings that are perturbative and capable to give a satisfactory explanation of the anomalous magnetic moment of the muon at the same time. The presence of $\Gamma_{\mu}^{R}$ ameliorates the tension, but it is still not sufficient by itself to address the anomaly.

Also in the quark sector right-handed couplings are needed to address the $B$ anomalies without spoiling at the same time the measurement for $\Delta M_{s}$. This is particularly evident by looking at the left panel of figure 6 , where the region allowed by both $b \rightarrow s \mu^{+} \mu^{-}$ transitions and $B_{s}-\bar{B}_{s}$ is shown. Indeed, if one performs a separate fit to $b \rightarrow s \mu^{+} \mu^{-}$ transitions and $\Delta M_{s}$ as shown in the right panel of figure 6 , it is evident that the two channels are incompatible as long as one assumes a vanishing coupling to right-handed bottom and strange quarks, i.e. $\Gamma^{R}=0$.

The preference for non-zero couplings (i.e. beyond the SM effects) is in general driven by $\Delta a_{\mu}$, the angular analyses of $B \rightarrow K^{*} \mu^{+} \mu^{-}$and $B_{s} \rightarrow \phi \mu^{+} \mu^{-}$, the branching fraction of $B_{s} \rightarrow \mu^{+} \mu^{-}$and the ratios $R_{K}$ and $R_{K^{*}}$. On the other hand, the experimental constraints coming from $b \rightarrow s \gamma$ and $B \rightarrow K^{(*)} \nu \bar{\nu}$ and $\Delta M_{B_{s}}$ set bounds on $\Gamma^{L, R}$ and $\left|\Gamma_{\mu}^{L}\right|$ that are less stringent than the ones obtained by the inclusion of the aforementioned channels involving $b \rightarrow s$ transitions in our setup with $\lambda_{L, R}^{D}=0$. Analogously, the constraints from $Z \rightarrow \mu^{+} \mu^{-}$are found to give negligible constraints on $\left|\Gamma_{\mu}^{L, R}\right|$. Concerning $D_{0}-\bar{D}_{0}$ mixing,

[^8]

Figure 5. Left panel: allowed region for the coupling strength to the muon $\left|\Gamma_{\mu}^{L}\right|$ from the muon anomalous magnetic moment as a function of $\left|\Gamma_{\mu}^{R}\right|$, assuming $m_{\phi}=450 \mathrm{GeV}, x^{E}=1$ and $\lambda^{E}=$ 0.0015 . The excluded region due to the requirement of perturbativity for $\left|\Gamma_{\mu}^{L, R}\right|$ is given in gray. Dark (light) green corresponds to $1 \sigma(2 \sigma)$ region. The red star marks the benchmark point (1.5, 1.4). Right panel: same as the left panel, but assuming $\lambda^{E}=0$.
we recall that eq. (4.11) implies a relation between $\Gamma^{L} \equiv L_{1}^{b} L_{1}^{s *}$ and $L_{1}^{u, c}$. Exploiting the fact that only the product of $L_{1}^{b} L_{1}^{s *}$ enters $b \rightarrow s \ell^{+} \ell^{-}$, together with the suppression of the $L_{1}^{b}$ in $L_{1}^{u, c}$ by small CKM factors $\left(\mathcal{O}\left(\lambda^{3}\right)\right.$ and $\mathcal{O}\left(\lambda^{2}\right)$, respectively), it is possible arrange the contributions to $\Gamma^{L}$ in such a way that the constraint imposed by $D_{0}-\bar{D}_{0}$ mixing is automatically satisfied.

We conclude this section by giving the results for some important observables (within our model) obtained from the global fit, namely

$$
\begin{align*}
R_{K}[1.1,6] & =0.781(45), \quad R_{K^{*}}[1.1,6]=0.885(39), \quad \overline{\mathcal{B}}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)=3.30(21) \cdot 10^{-9}, \\
P_{5}^{\prime}[4,6] & =-0.454(69), \quad \quad P_{5}^{\prime}[6,8]=-0.626(59) \\
\Delta a_{\mu} & =235(87) \cdot 10^{-11}, \quad R_{\Delta M_{s}}=-0.02(8) . \tag{4.27}
\end{align*}
$$

All the predictions for these observables are compatible at the $1 \sigma$ level with their experimental measurements described in section 3 , except for $R_{K^{*}}$ which is compatible only at the $\sim 2 \sigma$ level. However, this is expected both from the global fit and from our specific model: as can be seen from the right panel of figure 6 , there is no overlap of the $1 \sigma$ regions from $b \rightarrow s \mu^{+} \mu^{-}$data and $\Delta M_{s}$ in our model. Furthermore, since our model only allows for NP in muons (neglecting $Z, \gamma$ penguin effects), some small tensions are generated since flavour conserving $b \rightarrow s \mu^{+} \mu^{-}$data prefers a smaller value of $R_{K}$ than the one recently measured.


Figure 6. Left panel: allowed region for the coupling $\Gamma^{L} \equiv \Gamma_{b}^{L} \Gamma_{s}^{L *}$ and $\Gamma^{R} \equiv \Gamma_{b}^{R} \Gamma_{s}^{R *}$ from $B_{s}-\bar{B}_{s}$ mixing and $b \rightarrow s \mu^{+} \mu^{-}$data for $m_{\phi}=m_{E}=450 \mathrm{GeV}, m_{D}=3.15 \mathrm{TeV},\left|\Gamma_{\mu}^{L}\right|=1.5$ and $\left|\Gamma_{\mu}^{R}\right|=1.4$. The red star marks the values at our benchmark point $(-1,-0.12)$. The dark (light) purple regions is preferred at the $1 \sigma(2 \sigma)$ level. Right panel: same as the left panel, but showing separately the allowed regions coming from $B_{s}-\bar{B}_{s}$ mixing (in blue) and $b \rightarrow s \mu^{+} \mu^{-}$data (in red). The upper branch allowed by $B_{s}-\bar{B}_{s}$ mixing corresponds to the one shown on the left.

## 5 Conclusions and outlook

In this article we have studied in details the possibility that the intriguing anomalies in $b \rightarrow s \ell^{+} \ell^{-}$processes are explained via box diagrams involving new scalars and fermions. Within this setup we have generalized previous analysis [78, 79, 81] to include couplings of the new particles to right-handed SM fermion and calculated the completely general expressions for the Wilson coefficients governing $b \rightarrow s$ processes $\left(b \rightarrow s \ell^{+} \ell^{-}, b \rightarrow s \nu \bar{\nu}\right.$, $b \rightarrow s \gamma$ and $B_{s}-\bar{B}_{s}$ mixing). In addition, we have computed the effects in $a_{\mu}$ and $Z \rightarrow \mu^{+} \mu^{-}$which unavoidably arise in such scenarios.

Furthermore, we have proposed a UV complete model containing a $4^{\text {th }}$ vector-like generation of fermions and a new scalar, which is capable of explaining $b \rightarrow s \ell^{+} \ell^{-}$data and $a_{\mu}$. We applied the formula derived in our generic setup (see section 2) to this model, illustrating their usefulness. In the following phenomenological analysis of our $4^{\text {th }}$ generation model (see section 4.3) we came to the conclusion that the $b \rightarrow s \ell^{+} \ell^{-}$anomalies and $a_{\mu}$ can be explained simultaneously. As a benchmark point which can achieve this, and is consistent with direct LHC searches, we have used 450 GeV for the vector-like fermions and for the new scalar and $m_{D}=3.15 \mathrm{TeV}$ for the vector like quarks (with order one couplings). We have observed that right-handed couplings in the muon sector allow to address the long-standing anomaly in $a_{\mu}$ without having to require too large lepton couplings, if one allows for interactions of the vector-like fermions with the SM Higgs. Interestingly,
due to the new results from LHCb and BELLE [9, 10] the global fit to $b \rightarrow s \ell^{+} \ell^{-}$data now prefers non-zero right-handed couplings to quarks and leptons as well, justifying the importance of our generalization of previous analysis performed in this work.

Our $4^{\text {th }}$ generation model is also interesting since $\Phi$ is a viable (stable) Dark Matter candidate. We briefly showed that if the mass of $\Phi$ is close to the one of the vectorlike leptons, the correct relic density can be obtained while respecting the limits from Dark Matter direct detection. However, a more detailed investigation in the future seems worthwhile.

We conclude observing that our formalism can be directly applied to $b \rightarrow d \ell^{+} \ell^{-}$ transitions. In fact, it leads to correlated effects in Kaon physics [178] if one aims at explaining the slight tensions in $B \rightarrow \pi \mu^{+} \mu^{-}$[179] simultaneously with the ones in $b \rightarrow$ $s \ell^{+} \ell^{-}$data. Furthermore, our setup and results can also be used for addressing the tension between theory and experiment in $\epsilon^{\prime} / \epsilon[180,181] .{ }^{12}$ Here, as a special case of our generic approach, the MSSM has already been studied with the conclusion that it can provide a valid explanation of the anomaly [184-186].

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## A Fierz identities

Here we list the Fierz identities for spinors used in the computations. With $i, j, k$ and $l$ representing Dirac indices here we find

$$
\begin{align*}
\left(\gamma_{\mu} P_{L, R}\right)_{i j}\left(\gamma_{\mu} P_{L, R}\right)_{k l} & =-\left(\gamma_{\mu} P_{L, R}\right)_{i l}\left(\gamma_{\mu} P_{L, R}\right)_{k j}  \tag{A.1}\\
\left(\gamma_{\mu} P_{L, R}\right)_{i j}\left(\gamma_{\mu} P_{R, L}\right)_{k l} & =2\left(P_{R, L}\right)_{i l}\left(P_{L, R}\right)_{k j}  \tag{A.2}\\
\left(P_{L, R}\right)_{i j}\left(P_{L, R}\right)_{k l} & =\frac{1}{2}\left(P_{L, R}\right)_{i l}\left(P_{L, R}\right)_{k j}+\frac{1}{8}\left(\sigma_{\mu \nu}\right)_{i l}\left(\sigma_{\mu \nu} P_{L, R}\right)_{k j}  \tag{A.3}\\
\left(P_{L, R}\right)_{i j}\left(P_{R, L}\right)_{k l} & =\frac{1}{2}\left(\gamma_{\mu} P_{R, L}\right)_{i l}\left(\gamma_{\mu} P_{L, R}\right)_{k j}  \tag{A.4}\\
\left(\sigma_{\mu \nu}\right)_{i j}\left(\sigma_{\mu \nu} P_{L, R}\right)_{k l} & =6\left(\gamma_{\mu} P_{L, R}\right)_{i l}\left(\gamma_{\mu} P_{L, R}\right)_{k j}-\frac{1}{2}\left(\sigma_{\mu \nu}\right)_{i l}\left(\sigma_{\mu \nu} P_{L, R}\right)_{k j}, \tag{A.5}
\end{align*}
$$

where $P_{L, R}=\left(1 \mp \gamma_{5}\right) / 2$ and $\sigma_{\mu \nu}=\frac{i}{2}\left[\gamma_{\mu}, \gamma_{\nu}\right]$. When dealing with diagrams with crossed fermion lines, one needs Fierz identities involving charge conjugation matrices. Here, ex-

[^9]changing the second and the third Dirac index we find
\[

$$
\begin{align*}
\left(\gamma_{\mu} P_{L, R} C\right)_{i j}\left(C \gamma_{\mu} P_{L, R}\right)_{k l} & =-2\left(P_{R, L}\right)_{i k}\left(P_{L, R}\right)_{j l}  \tag{A.6}\\
\left(\gamma_{\mu} P_{L, R} C\right)_{i j}\left(C \gamma_{\mu} P_{R, L}\right)_{k l} & =-\left(\gamma_{\mu} P_{L, R}\right)_{i k}\left(\gamma_{\mu} P_{R, L}\right)_{j l}  \tag{A.7}\\
\left(P_{L, R} C\right)_{i j}\left(C P_{L, R}\right)_{k l} & =\frac{1}{2}\left(P_{L, R}\right)_{i k}\left(P_{L, R}\right)_{j l}-\frac{1}{8}\left(\sigma_{\mu \nu}\right)_{i k}\left(\sigma_{\mu \nu} P_{L, R}\right)_{j l}  \tag{A.8}\\
\left(P_{L, R} C\right)_{i j}\left(C P_{R, L}\right)_{k l} & =-\frac{1}{2}\left(\gamma_{\mu} P_{R, L}\right)_{i k}\left(\gamma_{\mu} P_{R, L}\right)_{j l}, \tag{A.9}
\end{align*}
$$
\]

with the charge conjugation matrix defined as $C=i \gamma_{0} \gamma_{2}$.

## B Loop functions

Here we list the dimensionless loop functions introduced in sections 2 and 4. The loop functions appearing in box diagrams that involve four different masses are defined as

$$
\begin{align*}
& F(x, y, z)=\frac{x^{2} \log (x)}{(x-1)(x-y)(x-z)}+\frac{y^{2} \log (y)}{(y-1)(y-x)(y-z)}+\frac{z^{2} \log (z)}{(z-1)(z-x)(z-y)}, \\
& G(x, y, z)=2\left(\frac{x \log (x)}{(x-1)(x-y)(x-z)}+\frac{y \log (y)}{(y-1)(y-x)(y-z)}+\frac{z \log (z)}{(z-1)(z-x)(z-y)}\right), \tag{B.1}
\end{align*}
$$

which in the equal mass limit read

$$
\begin{equation*}
F(1,1,1)=-G(1,1,1)=\frac{1}{3} \tag{B.2}
\end{equation*}
$$

In the presence of only three different masses in the loop, one gets the functions

$$
\begin{align*}
& F(x, y) \equiv F(x, y, 1)=\frac{1}{(1-x)(1-y)}+\frac{x^{2} \log (x)}{(1-x)^{2}(x-y)}+\frac{y^{2} \log (y)}{(1-y)^{2}(y-x)} \\
& G(x, y) \equiv G(x, y, 1)=2\left(\frac{1}{(1-x)(1-y)}+\frac{x \log (x)}{(1-x)^{2}(x-y)}+\frac{y \log (y)}{(1-y)^{2}(y-x)}\right) \tag{B.3}
\end{align*}
$$

while, in the presence of only two different masses in the loop, one gets

$$
\begin{align*}
& F(x) \equiv F(x, x)=\frac{x+1}{(x-1)^{2}}-\frac{2 x \log (x)}{(x-1)^{3}} \\
& G(x) \equiv G(x, x)=\frac{2}{(x-1)^{2}}-\frac{(x+1) \log (x)}{(x-1)^{3}} \tag{B.4}
\end{align*}
$$

The loop functions appearing in photon- and gluon-penguin diagrams are defined as

$$
\begin{array}{ll}
F_{7}(x)=\frac{x^{3}-6 x^{2}+3 x+2+6 x \log x}{12(x-1)^{4}}, & \widetilde{F}_{7}(x)=x^{-1} F_{7}\left(x^{-1}\right), \\
G_{7}(x)=\frac{x^{2}-4 x+3+2 \log x}{8(x-1)^{3}}, & \widetilde{G}_{7}(x)=\frac{x^{2}-2 x \log x-1}{8(x-1)^{3}}, \\
F_{9}(x)=\frac{-2 x^{3}+9 x^{2}-18 x+11+6 \log x}{36(x-1)^{4}}, & \widetilde{F}_{9}(x)=x^{-1} F_{9}\left(x^{-1}\right), \\
G_{9}(x)=\frac{-16 x^{3}+45 x^{2}-36 x+7+6(2 x-3) x^{2} \log x}{36(x-1)^{4}}, & \widetilde{G}_{9}(x)=x^{-1} G_{9}\left(x^{-1}\right), \quad(\mathrm{B}
\end{array}
$$

which in the equal mass limit read

$$
\begin{equation*}
F_{7}(1)=\widetilde{F}_{7}(1)=\frac{G_{7}(1)}{2}=\widetilde{G}_{7}(1)=-F_{9}(1)=-\widetilde{F}_{9}(1)=\frac{G_{9}(1)}{3}=\frac{\widetilde{G}_{9}(1)}{3}=\frac{1}{24} \tag{B.6}
\end{equation*}
$$

Finally, the loop functions for the calculation of $Z$-penguins are defined as

$$
\begin{align*}
G_{Z}(x, y) & =x F_{V}(x, y)+x \leftrightarrow y \\
F_{Z}(x, y, m) & \equiv \bar{F}_{Z}(x, y)-\overline{\operatorname{div}}_{\varepsilon}=\left(x^{2} F_{V}(x, y)+x \leftrightarrow y\right)-\overline{d i v}_{\varepsilon} \\
H_{Z}(x, y, m) & \equiv \bar{H}_{Z}(x, y)+\overline{\operatorname{div}}_{\varepsilon}=\left(y F_{V}(x, y)+x \leftrightarrow y\right)+1+\overline{d i v}_{\varepsilon} \\
I_{Z}(x, m) & \equiv \bar{I}_{Z}(x)+\overline{\operatorname{div}}_{\varepsilon}=\frac{x}{x-1}-x^{2} F_{V}(x, 1)+\overline{\operatorname{div}}_{\varepsilon} \\
\widetilde{G}_{Z}(x, y) & =x K_{V}(x, y)+x \leftrightarrow y \\
\widetilde{F}_{Z}(x, y) & =\left(x^{2} K_{V}(x, y)-\frac{x^{2}}{x-y} F_{V}(x, y)\right)+x \leftrightarrow y \\
\widetilde{H}_{Z}(x, y) & =\left(\frac{x^{2} y}{(y-1)(x-y)^{2}}-\frac{x^{2} y^{2}(3 x-y-2) \log (x)}{(x-1)^{2}(x-y)^{3}}\right)+x \leftrightarrow y \tag{B.7}
\end{align*}
$$

where we have defined $\overline{\operatorname{div}}_{\varepsilon}=\Delta_{\varepsilon}-\log \left(\frac{m^{2}}{\mu^{2}}\right)$ and

$$
\begin{equation*}
F_{V}(x, y)=\frac{\log (x)}{(x-1)(x-y)}, \quad K_{V}(x, y)=\frac{\left(x^{2}+x y-2 y\right) \log (x)}{(x-1)^{2}(x-y)^{3}}-\frac{1}{(x-1)(x-y)^{2}} \tag{B.8}
\end{equation*}
$$

It is interesting to notice that the following relations hold between particular limits of the penguin induced functions:

$$
\begin{align*}
& H_{Z}\left(\frac{m^{2}}{n^{2}}, \frac{m^{2}}{n^{2}}, m\right)=I_{Z}\left(\frac{m^{2}}{n^{2}}, n\right) \\
&-\frac{m^{2}}{n^{2}} G_{Z}\left(\frac{m^{2}}{n^{2}}, \frac{m^{2}}{n^{2}}\right)+\frac{1}{2} F_{Z}\left(\frac{m^{2}}{n^{2}}, \frac{m^{2}}{n^{2}}, n\right)+\frac{1}{4} H_{Z}\left(\frac{m^{2}}{n^{2}}, \frac{m^{2}}{n^{2}}, m\right)+\frac{1}{4} I_{Z}\left(\frac{m^{2}}{n^{2}}, n\right)=0 \\
& \frac{1}{2} x \widetilde{G}_{Z}(x, x)-\frac{1}{3} \widetilde{F}_{Z}(x, x)=\widetilde{G}_{9}(x) \\
& \frac{1}{6} \widetilde{F}_{Z}(x, x)=F_{9}(x)  \tag{B.9}\\
& \frac{1}{6 x} \widetilde{H}_{Z}(x, x)=\widetilde{F}_{9}(x) \tag{B.10}
\end{align*}
$$

Moreover, it is useful to define the limit

$$
\begin{equation*}
F_{Z}(x) \equiv \bar{F}_{Z}(x, x)=\frac{x}{x-1}+\frac{(x-2) x \log x}{(x-1)^{2}} \tag{B.11}
\end{equation*}
$$

Finally, the equal mass limits read

$$
\begin{equation*}
G_{Z}(1,1)=\frac{\bar{F}_{Z}(1,1)}{3}=-\bar{H}_{Z}(1,1)=-\bar{I}_{Z}(1)=6 \widetilde{G}_{Z}(1,1)=-2 \widetilde{F}_{Z}(1,1)=-2 \widetilde{H}_{Z}(1,1)=\frac{1}{2} . \tag{1}
\end{equation*}
$$



Figure 7. Crossed box diagrams contributing to $b \rightarrow s \mu^{+} \mu^{-}$transitions. The diagram on the left appears in models with real scalars, while the one on the right can be constructed in models with Majorana fermions.

| $\mathrm{SU}(3)$, type $a)$ | $\Psi_{A}$ | $\Psi_{B}$ | $\Phi_{M}$ | $\Phi_{N}$ | $\mathrm{SU}(3)$, type $b)$ | $\Psi_{A}$ | $\Psi_{B}$ | $\Phi_{M}$ | $\Phi_{N}$ | $\chi$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 3 | 1 | 1 | 1 | I | 1 | 1 | 3 | 1 | 1 |
| III | 3 | 8 | 8 | 8 | III | 8 | 8 | 3 | 8 | $4 / 3$ |

Table 5. Table of $\mathrm{SU}(3)$-factors entering the box induced Wilson coefficients involved in $b \rightarrow$ $s$ transitions for real scalars, type $a$ ), and Majorana fermions, type $b$ ). The numbers of each representation refer to the ones in table 1.

## C Real scalars and Majorana fermions

If the NP fields have the appropriate quantum numbers they can be either real scalars or Majorana fermions. If this is the case, crossed diagrams as shown in figure 7 can be constructed and contribute to $b \rightarrow s \mu^{+} \mu^{-}$transitions in addition to the ones shown in figure 1. Similarly, there are contributions from crossed boxes to $B_{s}-\bar{B}_{s}$ mixing (in addition to the ones in figure 3) arising due to the diagrams in figure 8 .

## C. $1 \quad b \rightarrow s \mu^{+} \mu^{-}$

In $b \rightarrow s \mu^{+} \mu^{-}$the possible representations that give rise to additional crossed diagrams with real scalars or Majorana fermions are listed in table 5 . For type $a$ ) the only possibility is to have real scalars, while for type $b$ ) one can only have crossed diagrams in the presence of Majorana fermions.

The contribution to the Wilson coefficients stemming from the diagrams in figure 7a) corresponds to the ones listed for $a$ )-type in eqs. (2.4)-(2.7), after inverting $M \leftrightarrow N$ in the muon couplings and changing $F\left(x_{A M}, x_{B M}, x_{N M}\right) \rightarrow-F\left(x_{A M}, x_{B M}, x_{N M}\right)$. For case $\left.b\right)$ (see right diagram in figure 7) the Wilson coefficients are given by

$$
\begin{align*}
C_{9}^{\text {box } b)}= & -\mathcal{N} \frac{\chi L_{B M}^{s *} L_{A M}^{b}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[L_{B N}^{\mu *} L_{A N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.-R_{B N}^{\mu *} R_{A N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right], \tag{C.1}
\end{align*}
$$

| $\mathrm{SU}(3)$ | $\Psi_{A}$ | $\Psi_{B}$ | $\Phi_{M}$ | $\Phi_{N}$ | $\chi_{B B}^{M}$ | $\tilde{\chi}_{B B}^{M}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| I | 3 | 3 | 1 | 1 | 1 | 0 | Real $\Phi$ |
| II | 1 | 1 | 3 | 3 | 0 | 1 | Majorana $\Psi$ |
| III | 3 | 3 | 8 | 8 | $5 / 18$ | $-1 / 6$ | Real $\Phi$ |
| IV | 8 | 8 | 3 | 3 | $-1 / 6$ | $5 / 18$ | Majorana $\Psi$ |
| V | 3 | 3 | $(1,8)$ | $(8,1)$ | $1 / 6$ | $-1 / 2$ | Real $\Phi$ |
| VI | $(1,8)$ | $(8,1)$ | 3 | 3 | $-1 / 2$ | $1 / 6$ | Majorana $\Psi$ |

Table 6. Table of $\mathrm{SU}(3)$-factors entering the box induced Wilson coefficients involved in $B_{s}-\bar{B}_{s}$ mixing for real scalars and Majorana fermions.

$$
\begin{align*}
C_{10}^{\text {box } b)}= & \mathcal{N} \frac{\chi L_{B M}^{s *} L_{A M}^{b}}{32 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[L_{B N}^{\mu *} L_{A N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.+R_{B N}^{\mu *} R_{A N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right],  \tag{C.2}\\
C_{S}^{\text {box } b)}= & \mathcal{N} \frac{\chi L_{B M}^{s *} L_{A M}^{b}}{16 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[R_{B N}^{\mu *} L_{A N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.+L_{B N}^{\mu *} R_{A N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{2 m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right],  \tag{C.3}\\
C_{P}^{\text {box } b)}= & \mathcal{N} \frac{\chi L_{B M}^{s *} L_{A M}^{b}}{16 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}}\left[R_{B N}^{\mu *} L_{A N}^{\mu} F\left(x_{A M}, x_{B M}, x_{N M}\right)\right. \\
& \left.-L_{B N}^{\mu *} R_{A N}^{\mu} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{2 m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)\right],  \tag{C.4}\\
C_{T}^{\text {box } b)}= & -\mathcal{N} \frac{\chi L_{B M}^{s *} R_{A M}^{b} L_{B N}^{\mu *} R_{A N}^{\mu}}{16 \pi \alpha_{\mathrm{EM}} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right),  \tag{C.5}\\
C_{9, S}^{\prime \text { box }}= & C_{9, S}^{\text {box }}(L \leftrightarrow R), \quad C_{P, 10}^{\prime \text { box }}=-C_{P, 10}^{\text {box }}(L \leftrightarrow R), \tag{C.6}
\end{align*}
$$

## C. $2 \quad \boldsymbol{B}_{s}-\bar{B}_{s}$ mixing

For $B_{s}$ mixing we can either have real scalars or Majorana fermions. In table 6 we list the possible representations of the diagrams in figure 8 writing explicitly if we have a real scalar contribution (diagrams on the left side of the figure) or a Majorana fermion (diagrams on the right). The WCs for real scalar crossed diagrams correspond to the ones listed in eqs. (2.4)-(2.7), after inverting $M \leftrightarrow N$ in two of the four couplings and changing $F\left(x_{A M}, x_{B M}, x_{N M}\right) \rightarrow-F\left(x_{A M}, x_{B M}, x_{N M}\right)$, whereas matching to the generic Lagrangian from eq. (2.1) with the crossed fermion contributions, one obtains the following results for


Figure 8. Box diagrams contributing to $B_{s}-\bar{B}_{s}$ mixing. The diagram on the left is relative to models with real scalars, while the one on the right refers to models with Majorana fermions.
the coefficients:

$$
\begin{align*}
C_{1} & =\left(\chi_{B B}^{M}+\tilde{\chi}_{B B}^{M}\right) \frac{L_{A N}^{s *} L_{B M}^{b} L_{A M}^{s *} L_{B N}^{b}}{128 \pi^{2} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right)  \tag{C.7}\\
C_{2,3} & =-\left(\chi_{B B}^{M}+\tilde{\chi}_{B B}^{M}\right) \frac{R_{A N}^{s *} L_{B M}^{b} R_{A M}^{s *} L_{B N}^{b}}{64 \pi^{2} m_{\Phi_{M}}^{2}} \frac{m_{\Psi_{A}} m_{\Psi_{B}}}{m_{\Phi_{M}}^{2}} G\left(x_{A M}, x_{B M}, x_{N M}\right),  \tag{C.8}\\
C_{4} & =\frac{L_{A M}^{b} R_{A N}^{b}}{32 \pi^{2} m_{\Phi_{M}}^{2}}\left[\chi_{B B}^{M} L_{B M}^{s *} R_{B N}^{s *}-\tilde{\chi}_{B B}^{M} R_{B M}^{s *} L_{B N}^{s *}\right] F\left(x_{A M}, x_{B M}, x_{N M}\right),  \tag{C.9}\\
C_{5} & =\frac{L_{A M}^{b} R_{A N}^{b}}{32 \pi^{2} m_{\Phi_{M}}^{2}}\left[\tilde{\chi}_{B B}^{M} L_{B M}^{s *} R_{B N}^{s *}-\chi_{B B}^{M} R_{B M}^{s *} L_{B N}^{s *}\right] F\left(x_{A M}, x_{B M}, x_{N M}\right)  \tag{C.10}\\
\widetilde{C}_{i} & =C_{i}(L \rightarrow R), \quad \text { for } \quad i=\{1,2,3\} \tag{C.11}
\end{align*}
$$

The corresponding contributions to $D_{0}-\bar{D}_{0}$ mixing are obtained from eqs. (2.22)-(2.28) via the replacements $s \rightarrow u$ and $b \rightarrow c$.

## D Crossed diagrams with complex scalars

There is also the possibility that a complex scalar couples to the down-type quarks whereas its hermitian conjugated version couples to muons. This means that the Lagrangian in


Figure 9. Crossed box diagrams contributing to $b \rightarrow s \mu^{+} \mu^{-}$transitions. The diagram appears when a complex scalar couples to $b, s$ quarks and its conjugate couples to the muons.

| $\mathrm{SU}(3)$ | $\Psi_{A}$ | $\Psi_{B}$ | $\Phi_{M}$ | $\Phi_{N}$ | $\chi$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | $(1,3)$ | $(3,1)$ | $(\overline{3}, 1)$ | $(1, \overline{3})$ | 1 |
| II | $(8,3)$ | $(3,8)$ | $(\overline{3}, 8)$ | $(8, \overline{3})$ | $4 / 3$ |
| III | $\overline{3}$ | $\overline{3}$ | 3 | 3 | 2 |

Table 7. Table of $\mathrm{SU}(3)$-factors entering the box induced Wilson coefficients involved in $b \rightarrow s$ transitions for crossed diagrams with complex scalars.
eq. (2.1) takes a slightly different form, namely

$$
\begin{align*}
\mathcal{L}_{\mathrm{int}}= & {\left[\bar{\Psi}_{A}\left(L_{A M}^{b} P_{L} b+L_{A M}^{s} P_{L} s+R_{A M}^{b} P_{R} b+R_{A M}^{s} P_{R} s\right) \Phi_{M}\right.} \\
& \left.+\bar{\Psi}_{A}\left(L_{A M}^{\mu} P_{L}+R_{A M}^{\mu} P_{R} \mu\right) \Phi_{M}^{\dagger}\right]+ \text { h.c. } \tag{D.1}
\end{align*}
$$

Also this Lagrangian generates a contribution to $b \rightarrow s \mu^{+} \mu^{-}$via the diagram shown in figure 9. The possible representations under the $\mathrm{SU}(3)$ of the new scalars and fermions in the loop are listed in table 7. The corresponding Wilson Coefficients can be obtained from the ones calculated for the type $b$ ) diagrams in eqs. (2.4)-(2.7) by exchanging $M \leftrightarrow N$ in the couplings $R, L$ and replacing $F\left(x_{A M}, x_{B M}, x_{N M}\right) \rightarrow-F\left(x_{A M}, x_{B M}, x_{N M}\right)$.

## E Posterior distributions

Here we show the $1 D$ marginalized posterior distributions of the parameters from the global fit described in section 4.2 , together with the $2 D$ combined correlations between these parameters. The results are summarized in figure 10 . We recall that, for all couplings $\Gamma$, we imposed $|\Gamma| \leq 1.5$ such that perturbativity is satisfied. The consequences of such this choice are evident in the posterior distributions of $\Gamma^{L},\left|\Gamma_{\mu}^{L}\right|$ and $\left|\Gamma_{\mu}^{R}\right|$, which are truncated because of this reason.


Figure 10. The $1 D$ marginalized posterior distributions of the parameters from the fit described in section 4.2 , together with the $2 D$ correlations between them. The green, red and orange regions correspond to $68 \%, 95 \%$ and $99 \%$ probability regions, respectively.

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[^0]:    ${ }^{1}$ Note that deviations from the SM predictions have been observed in $b \rightarrow c \tau \nu$ transitions as well [11]. However, since these tensions cannot be explained by loop effects, we do not discuss them in this article.
    ${ }^{2}$ Box contributions of new vectors and fermions were studied in the context of $Z^{\prime}$ models with vector-like quarks in ref. [74].
    ${ }^{3}$ Alternatively, models with large couplings to right-handed top quarks can give the desired effect via a $W$-loop [82-84], as first shown in the EFT context in ref. [85].

[^1]:    ${ }^{4}$ Here we only consider coupling to muons in order to explain the anomalies in $b \rightarrow s \ell^{+} \ell^{-}$. The reason for this is that in our setup sizable couplings to electrons would in general generate effects in $\mu \rightarrow e \gamma$, which would contradict experimental bounds [109] by orders of magnitude.

[^2]:    ${ }^{5}$ Expressions for $Z$ couplings in generic gauge theories can be found in ref. [113].

[^3]:    ${ }^{6}$ Note that $C_{7,8}^{\prime}$ are less constrained since they do not interfere with the SM. For a more detailed analysis including primed operators see e.g. ref. [133].

[^4]:    ${ }^{7} Z$ couplings in ref. [171] are defined with opposite sign with respect to our conventions [114].

[^5]:    ${ }^{8}$ We did not assume a $Z_{2}$ symmetry because this would allow the scalar $\Phi$ to be real and lead to crossed boxes in $b \rightarrow s \ell^{+} \ell^{-}$, canceling the desired effect there.

[^6]:    ${ }^{9}$ Note that the effect in scalar and magnetic operators can also be suppressed if $\Gamma_{b, s}^{R}=0$ or very small. However, we decided to focus on option with $\lambda_{L, R}^{D}$ being very small.

[^7]:    ${ }^{10}$ Nearly degenerate masses $m_{\Phi} \simeq m_{E}$ are also welcome in the light of the dark matter relic density since the stable $\Phi$ is a suitable DM candidate. In fact, for $m_{\Phi}=450 \mathrm{GeV}, 450 \leq m_{E} \leq 520 \mathrm{GeV}$ the model allows for an efficient annihilation such that one does not over-shoot the matter density of the universe for order one $\Gamma$ couplings.

[^8]:    ${ }^{11}$ A detailed study recasting these MSSM analysis for our model has been performed in refs. [101, 102], finding $m_{E} \gtrsim m_{\Phi}=450 \mathrm{GeV}$ as an allowed solution.

[^9]:    ${ }^{12}$ Note that calculations using chiral perturbation theory [182, 183] instead are consistent with both the experimental measurement and the SM results of refs. [180, 181].

