

# OPAL-MITHRA: Self-consistent Software for Start-to-End Simulation of Undulator-based Facilities

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**Abstract**—We present the recently started effort and progress towards the development of a software for start-to-end simulation of accelerator facilities employing undulator radiation. The core of this effort aims at the combination of two numerical solvers, the Object Oriented Parallel Accelerator Library (OPAL), a parallel open source tool for charged-particle optics in linear accelerators and rings, including 3D space charge, and the full-wave simulation tool for free electron lasers MITHRA, which solves the electromagnetic potential equations in free-space for radiating particles propagating along an undulator using an FDTD/PIC scheme. As an example, we present the application of the OPAL-MITHRA platform for the simulation of an experiment that will take place at the Argonne Wakefield Accelerator, in which the capability of a magnetic wiggler to induce energy modulation starting from a micro-bunched electron beam will be investigated.

**Keywords**—*accelerator physics, undulator radiation, numerical simulation, electron beam cooling*

## I. INTRODUCTION

The Electron-Ion Collider (EIC) being designed at Brookhaven National Laboratory (BNL) will have to operate at high luminosities of  $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  for a large range of center-of-mass energies, which will require achieving and preserving a small phase-space volume for several hours [1]. Hence a new method known as Micro-Bunched Electron Cooling (MBEC) is being considered for reducing the energy spread and emittance of hadrons and ions [2]. This technique employs space charge effects in drifts to induce energy modulation in electron bunches starting from density modulation. Based on results from the literature [3, 4], it was recently suggested that the cooling section could be shortened by replacing these drifts in the MBEC scheme by shorter magnetic wigglers, which are expected to yield stronger energy modulation. An experiment to assess the feasibility of this replacement will be carried out at the Argonne Wakefield Accelerator (AWA), where micro-bunched electron beams will be accelerated to relativistic speeds, and passed through a 1 m wiggler. A numerical study on this experiment, and on the effects of a wiggler in the MBEC scheme, will require a code that includes space-charge and radiation effects computed from first-principles. The recently developed code MITHRA [5] offers such possibilities, by solving the inhomogeneous Maxwell equations in an undulator using Finite-Differences in Time-Domain (FDTD) and Particle-in-Cell (PIC). On the other

hand, OPAL (Object Oriented Parallel Accelerator Library) [6] is a 3D tracking code for accelerators that offers a framework that can simulate a variety of machines, in which the accelerator's lay-out is defined by the user. In particular, its package OPAL-t models beam lines, linacs and rf-photo injectors, but not wigglers and undulators since its solver only includes space-charge, not radiation. By combining the strengths of these codes, MITHRA and OPAL can be combined into a single code that can simulate the start-to-end MBEC experiment.

## II. NUMERICAL APPROACH

### A. OPAL static solver

To compute space-charge forces in the bunch, OPAL solves the static Maxwell equations at each time step, which are

$$\frac{\partial}{\partial t} \rightarrow 0 \Rightarrow \begin{cases} \nabla \cdot \mathbf{E} = \rho / \epsilon_0 \\ \nabla \times \mathbf{B} = \mu_0 \mathbf{j} \end{cases} \quad (1)$$

By solving the equations in a Lorentz-boosted frame at the average speed of the bunch, the magnetic field vanishes and one is simply left with the Poisson equation

$$\begin{cases} \nabla^2 \varphi = -\rho / \epsilon_0 \\ \text{Boundary conditions} \end{cases} \quad (2)$$

which is efficiently solved on a mesh using Fast Fourier Transform (FFT) techniques, with computational cost scaling as  $(2N \log(2N))^3$ , where  $N$  is the number of mesh-points in one dimension. Note that, in equation (1) the coupling between electric and magnetic fields is lost due to the static approximation, thus OPAL cannot account for the propagation of electromagnetic waves.

### B. MITHRA

The inhomogeneous Maxwell equations are solved in MITHRA by solving the potential wave equations using finite differences in time and space. One can then explicitly update the scalar and vector potentials on a mesh. At each time step the charge and current density from the particles are deposited on the discrete mesh-points, and the fields are interpolated from the mesh-points to the particle positions. To reduce the computational load, MITHRA performs the entire simulation in a Lorentz-boosted frame moving at the average speed of the

bunch. In this way, the mesh can be smaller since it only needs to encapsulate the bunch rather than the whole undulator.

### C. OPAL-MITHRA

In addition to being a standalone code, MITHRA has been adapted into a static library such that OPAL can use its functions and classes. Now OPAL can simulate an entire beam line using the static solver and, when it reaches the fringe fields of a wiggler, it performs a Lorentz transformation and uses the FDTD solver from MITHRA to simulate from that point onward. A schematic of how this works to model the AWA beam line can be seen in figure 1.

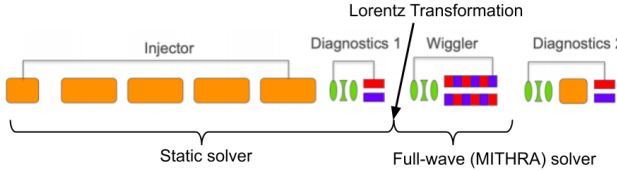


Fig. 1: Lay-out of the AWA beam line and usage of the solvers [4].

## III. EXPERIMENT AND SIMULATIONS

### A. Experimental Setup

The AWA accelerator has photo-cathode emitting an electron-bunch formed by 4 Gaussian distributions separated longitudinally by  $\lambda$ , hence giving a micro-bunched bunch. The electrons are then accelerated through 4 or 6 cavities, each adding roughly 10 MeV to the average bunch energy, followed by a 13 m drift with diagnostics and focusing elements, and finally a 1.1 m wiggler with a period  $\lambda_u = 8.5$  cm and a strength parameter  $K = 10.81$ . A transverse deflecting cavity followed by a spectrometer will sample the longitudinal phase-space, and hence the induced energy modulation around the micro-bunches. A more detailed explanation on the numerical study of the AWA experiment can be found in [7].

### B. Start-to-End Simulations

Using the OPAL-MITHRA combination as described in figure 1, the entire AWA experiment can be modelled. During the preparation of the experiment the start-to-end simulations guided the selection of bunch parameters that would yield measurable increases in energy modulation, and that could preserve the micro-bunches up to the wiggler. Figures 2 and 3 show some of these simulations, using 4 cavities for accelerating, a bunch charge  $Q = 300$  pC, and a micro-bunch separation  $\lambda = 600 \mu\text{m}$ . Beams with smaller transverse sizes show a larger gain in energy modulation.

## IV. CONCLUSIONS

The combination of OPAL and MITHRA provides a unique tool for FEL community, which can simulate start-to-end machines including the photo-cathode, linear accelerator, compression stages, and undulator, in a single run. Additionally his code can accurately model a wider regime of beams and wigglers than most other FEL codes, which use approximations

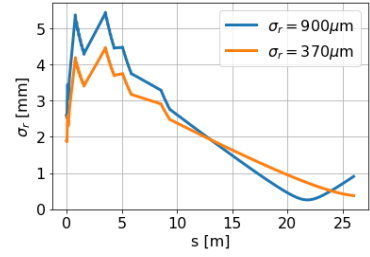


Fig. 2: RMS transverse size of two example bunches along the AWA beam line from photo-cathode to wiggler. The values indicated in the legend are the RMS transverse sizes at 26 m, when entering the wiggler.

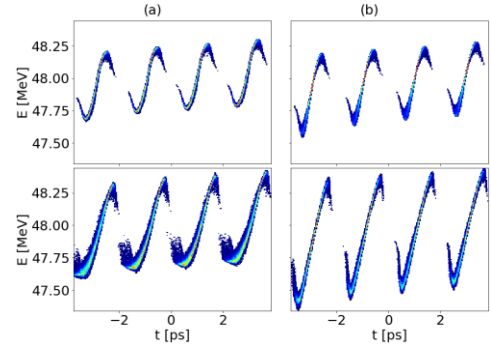


Fig. 3: Longitudinal phase-space before (top) and after (bottom) the wiggler. (a)  $\sigma_r = 900 \mu\text{m}$  and (b)  $\sigma_r = 370 \mu\text{m}$  (Figure 2).

to reduce computational load. This new tool was successfully used to study and guide the MBEC experiments that will be taking place at AWA. It was found that a wiggler induces stronger energy modulation in a bunch with density modulation than a drift. Further analysis of the wiggler on Gaussian bunches and on the emitted radiation could give deeper insight on its effects and could be better compared to the theory.

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