Theoretical investigation on broadband THz deflectors for femtosecond electron beam diagnostics

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Abstract

Streak cameras are commonly used to measure duration and temporal structure of subpicosecond pulses. The resolution of a streak camera system strongly depends on the slew rate of the deflecting element, which approximately equals to the product of the amplitude and operation frequency of the device. As a combination of high frequency and high gradient, THz-driven deflector based streak camera system has been proposed as an attractive tool to reach femtosecond scale or even higher resolution. This study contributes to this growing area of research by investigating four promising designs of broadband THz deflectors with central frequencies at 0.3 THz. The basic properties of each structure, a systematical method to analyze their multipole fields and the derived fields are presented.

Keywords: THz streaking, longitudinal diagnostics, multipole modes, femtosecond resolution

1. Introduction

Particle accelerators proposed for TeV linear electron-positron colliders\textsuperscript{[1]} for high energy physics, used as drivers for femtosecond x-ray free electron lasers (FELs)\textsuperscript{[2]}, or involved in megavolt ultrafast electron diffraction (MeV-UED)\textsuperscript{[3]} must provide relativistic electron bunches with short lengths on a timescale of...
sub-picosecond or even femtosecond. Therefore, beam diagnostics with tem-
poral resolution down to femtosecond range is essential for these kinds of ma-
chines. Recently, using the strong fields existing in THz waves has attracted
researchers’ attention for realizing compact particle accelerators \[4\]–\[7\] as well as
small manipulators for particle beams \[8\]–\[17\].

A THz-driven deflector based streak camera concept has been proposed as
a promising tool for electron beam diagnostic with femtosecond resolution, as
shown schematically in Fig. \[8\],\[9\]. A laser beam illuminates the photo-cathode
and creates an electron beam, which is then accelerated to a designed energy
and usually compressed to below one picosecond. Using optical rectification
an intense quasi-single-cycle pulse can be generated \[18\]–\[21\], which excites the
deflector and produces a time-dependent deflecting (streaking) field. By letting
the electron bunch pass through the zero crossing of streaking field, a correlation
between longitudinal current density and the vertical beam offset is obtained.
This correlation is subsequently resolved by a detector downstream.

Figure 1: Typical setup of a THz-driven deflector based streak camera system.

In recent years, there has been an increasing interest in THz streak camera
setups. Several attempts were made in different energy scales. Sub-relativistic,
\(<100\) keV kinetic energy electron beams from DC sources were compressed us-
ing laser-driven THz fields to tens of femtoseconds, followed by temporal profile characterization with roughly 10 fs resolution via THz streaking [10, 11]. The arrival time of a roughly 3.4 MeV electron beam, which was compressed by a RF buncher cavity to around 6 fs, was determined by laser-driven THz streaking technique with 1.5 fs (rms) accuracy [12]. A similar setup was used to demonstrate the simultaneous determination of relative time-of-arrival and bunch duration of a 3.1 MeV (20 fs rms) electron beam by THz streaking, with a resolution below and approaching a single femtosecond respectively [13]. More recently, a 4.5 MeV electron beam with compressed rms length less than 100 fs was measured based on a double-slit split ring structure and 10 fs resolution was reached [14].

All these studies suggest the great potential of the THz streaking technique, where the deflector is the crucial element for the performance of the overall system. The typical THz deflectors can be simply divided into two types: resonant and nonresonant devices. For resonant structures used in Ref. [10, 12, 14], their resonant behavior indicates a more efficient use of the input THz power, because a larger field enhancement can be achieved in the interactive region, which is promising for streaking high-energy electrons. For nonresonant devices in Ref. [11, 13], the basic advantage is the relatively weaker derivative high-order modes, a cleaner streaking effect can be achieved.

This work investigates potential designs of THz deflectors in a generic way analyzing multipole parameters. Four broadband structures centered at 0.3 THz are presented: (i) a pure metallic device consisting of an inverse split ring resonator (ISRR), (ii) a derived ISRR array structure as another metallic device, (iii) a dielectric tube with metallic gratings as a hybrid dielectric/metal structure, and (iv) a simplified version of the segmented terahertz electron accelerator and manipulator (STEAM) [11] structure as the second hybrid device.

In section 2, a detailed description of four structures are presented. We use 3D electromagnetic analysis based on finite integration technique (FIT) via CST microwave studio (MWS) [22] to predict the field behavior inside each structure. In section 3, the multipole fields of each device up to the 3rd order are analyzed.
based on particle tracking simulations (via CST particle-in-cell, PIC solver) and Panovsky-Wenzel theorem [23, 24]. In these simulations, a 1 MV/m quasi-single-cycle pulse (waveform as shown in Fig. 1) with a frequency range of 0.05 to 0.55 THz (3 dB bandwidth of 0.193 THz) was used as excitation.

2. THz-driven deflectors

2.1. ISRR

The original design of the THz streak camera was based on a sub-wavelength scale split ring resonator (SRR) with an active area of 20 µm × 20 µm and an interaction length of around 20 µm (shown as in Fig. 1). This structure poses some challenges for use in an accelerator environment. Mounting this resonator without perturbing the field pattern of the resonance would require a dielectric holder attached to the SRR. This approach causes two problems, one is the charging up of the SRR by halo electrons with subsequent break-downs and the second a strong heat up of the resonator by halo electron bombardment (or by an accidental hit with the main beam).

As an alternative, we developed an “inverse” SRR design (ISRR). In this design, the resonator is cut out of a solid 80 µm thick metallic plate, which simultaneously serves as a mount for the SRR to be fixed on a motorized stage for precise and reproducible positioning with micrometer resolution. The 80 µm thick plate not only improves mechanical stability and solves the problems of mounting the device. It also increases the kick strength, since the larger gap length increases the interaction region between the electron beam and the THz field.

Figure 2a) shows the ISRR model in conjunction with its simulation setup. The vertically polarized incident THz pulse will excite the fundamental resonance exhibiting strongly enhanced deflecting fields inside the gap. Figure 2b) shows the simulated electric field versus time inside the gap center with the corresponding normalized spectrum centered at 0.3 THz as shown in Fig. 2c). With the magnetic field parallel to the electron beam and a gap width small
compared to the wavelength, the electrons traversing the gap will get a time-
dependent transverse kick that closely follows the electric field curve as shown
in Fig. 2b).

Note that the ISRR model is principally similar to the structure in Ref. [14].

The main difference is that, here, the feeding mechanism of THz power from
the side enables the design of the ISRR array structure. Inside the ISRR array
structure, the longitudinal alignment problem between each ISRR can be solved
as presented in the following section.

2.2. ISRR array

For a higher kick strength, we propose an array structure as shown in
Fig. 3(a). Four ISRRs are placed along the electron beam direction at a dis-
tance of $\lambda$ between each other to ensure the phase-matching between electron
beam and THz field. The THz pulse reaches all gaps at the same time. Since
the oscillation in ISRR drops quickly due to the low quality factor, the array
would only show a moderate increase in the integrated kick if no additional
modifications are made. To improve, we introduced metallic reflector surfaces (marked as blue faces in Fig. 3a)) at defined distances (\(\lambda/4\), \(\lambda/4\), \(3\lambda/4\)) to the gaps. These surfaces reflect the incoming THz pulse back to the gaps well timed to maximize the kick seen by the beam as it passes the individual gap.

Adding these features also solves the alignment problem between the individual resonators, since the whole structure can be manufactured as a single device via 3D printing technology. The time-dependent electric fields inside each gap center are shown in Fig. 3b), where the marked thick lines indicate the expected region of interaction between field and beam for each ISRR. Comparing Fig. 2b) and Fig. 3b), the electrons passing through this device will gain an accumulated kick, which is approximately three times as large as a single ISRR.

![Figure 3: a) ISRR array structure designed at 0.3 THz, where L1 = L2 = L3 = \(\lambda\), D1 = D2 = \(\lambda/4\), D3 = \(3\lambda/4\); b) Electric field versus time inside each gap center, the electron-THz interaction windows are marked with thick blue, orange, green and purple lines for each gap.]

2.3. Dielectric tube

In contrast to the previous deflectors, which were purely metallic structures, this device is a combination of dielectric and metal. The idea is the following: a dielectric tube of e.g. Alumina has multipole modes of dominantly TM (transverse magnetic) type, where the maximal longitudinal electric field is concentrated inside the dielectric wall. Interesting for use as a deflector is the dipole mode, whose electric field distribution is shown in Fig. 4b). In comparison with the ISRR based structures described above, this structure offers
a significantly larger aperture diameter of 120 µm for the electron beam. The kick comes mainly from the transverse magnetic field. For a coherent buildup of the kicker over several periods, we create a longitudinal modulation of the field by attaching a periodic metallic grating to the side of the tube (Fig. 4a)), Figure 4c) shows the effect. Concerning the dimensions of grating in Fig. 4a), $l_1 = l_2 = d_1 = d_2 = \lambda/4$ and $h = 3\lambda$, it’s worth to mention that $d_2$ and the height of grating $h$ could be adjusted in a wide range. In principle, this structure has the same problem of charging issues as for classic SRR, but the situation should be less critical due to the relatively larger aperture with less chance of electrons hitting.

Figure 4: a) Design of dielectric tube structure at 0.3 THz with a longitudinal length of $3\lambda$, the outer and inner diameters of tube are 240 µm and 120 µm respectively, the dielectric material is alumina ($\epsilon_r = 9.4$); the dimensions of grating are $l_1 = l_2 = d_1 = d_2 = \lambda/4$, $h = 3\lambda$; b) Electric field distribution in the transverse plane, as the maximal field is allocated inside the tube wall; c) The modulated magnetic field distribution inside the beam channel.
2.4. Simplified 'STEAM' streaker

A promising device with multiple applications in acceleration, manipulation and diagnostics is the segmented terahertz electron accelerator and manipulator (STEAM) structure [11]. Here, we concentrate on a simplified design of STEAM streaker at 0.3 THz for relativistic electron beams as shown in Fig. 5. The coupler and tapering sections used in [25] were excluded, as the main purpose here is to analyze the multipole fields inside the interaction region, which are not affected by these features.

The streaking is based on the superposed magnetic fields of two counter-propagating THz pulses. The layers are spaced at a period of $\lambda/2$ to ensure a continuous kick for electrons passing through the structure. The transverse dimension of the dielectric layers (quartz, $\epsilon_r = 4.41$) have an increasing length along the electron beam direction to match the arrival time of the THz pulse front to the relativistic electrons.

3. Multipole field analysis

Analysis of multipole field components in deflectors is essential, because the presence of multipoles kicks leads to perturbations of the beam dynamics, which
in turn influence the streaking behavior. Assuming that a particle of charge $e$ moves parallel to the z-axis with the speed of light $c$, the transverse Lorentz force acting on it and the transverse momentum $\vec{p}_\perp$ imparted to the particle after traversing the structure can be expressed as follows:

$$\vec{F}_\perp = e[\vec{E}_\perp + (\vec{v} \times \vec{B})_\perp]$$

$$\vec{p}_\perp = \int_{-\infty}^{+\infty} \vec{F}_\perp dt = \int_{-\infty}^{+\infty} e[\vec{E}_\perp + (\vec{v} \times \vec{B})_\perp] dt$$

Based on the Panofsky-Wenzel theorem \cite{23, 24} exploited in various wakefield calculation codes \cite{22, 26}, the transverse momentum is determined by the transverse variation of the longitudinal integrated acceleration after passing through a structure with length of $d$, as shown in Eq. \ref{eq:3}.

$$\vec{p}_\perp = \left(\frac{e}{w_0}\right) \int_0^d (-i)\nabla_\perp E_z dz$$

Equation \ref{eq:3} indicates the relation in frequency domain, the integration of $E_z$ along z returns the potential difference $V_z$ and the factor $(i/w_0)$ is equivalent to an integration in time domain. Therefore, the equivalent equation in time domain reads as:

$$\vec{p}_\perp(t) = -e \int_{-\infty}^t (\nabla_\perp V_z) dt$$

This equation indicates an easier way to analyze the multipole fields by calculating the accumulated potential difference $V_z$ at different transverse locations, rather than combining the transverse electric and magnetic fields inside a 3D region. In the following, a detailed explanation of the multipole field analysis process is presented:

(i) CST PIC solver \cite{22} simulation: tiny beamlets of 3 MeV electrons are emitted at various offsets in the x-y plane within the gap region. A PIC 2D monitor downstream the THz deflector is used to record the position, momentum, and arrival time of each electron.
(ii) Polynomial fitting: based on the results of step (i), the accumulated potential difference \( V_z(x, y, t) \) at different transverse coordinates \((x, y)\) can be deduced.

As an example, the simulated \( V_z(x, y, t) \) at \( x = 0 \) in terms of \( y \) is depicted in Fig. 6a) and additionally the variation of \( V_z(x, y, t) \) versus \( x \) at \( y = 7 \, \mu m \) is shown in Fig. 6b). Figure 6 indicates two important laws governing the variation of \( V_z(x, y, t) \): a) it almost has no change with varying horizontal position \( x \) for a fixed vertical position \( y \) within the gap region of ISRR; and b) it only includes the odd powers of the vertical position \( y \).

According to the results, the polynomial fitting equation (up to 3rd order) of \( V_z(x, y, t) \) for the ISRR structure is:

\[
V_z(x, y, t)_{ISRR} = b_1(t)y + c_1(t)xy + d_1(t)(x^2y - \frac{1}{3}y^3)
\]

where \( b_1(t) \), \( c_1(t) \) and \( d_1(t) \) corresponds to dipole, quadrupole and sextupole in the time domain respectively.

The ISRR array structure has the same multipole components as that of the ISRR structure. For the dielectric tube structure, the polynomial fitting
equation is shown in Eq. (6), where \( a_2(t) \), \( b_2(t) \) and \( c_2(t) \) corresponds to monopole, dipole and quadrupole mode.

\[
V_z(x, y, t)_{\text{Ttube}} = a_2(t) + b_2(t)x + c_2(t)(y^2 - x^2) \tag{6}
\]

Concerning the simplified 'STEAM' streaker, mainly the dipole mode \( b_3(t) \) is included as shown in Eq. (7):

\[
V_z(x, y, t)_{\text{STEAM}} = b_3(t)x \tag{7}
\]

(iii) Deducing frequency response: to further understand these devices, the frequency response of their multipole modes with respect to the incident THz spectrum are calculated.

As a result, we obtain the multipole acceleration and deflection components of each device. These components in time domain are presented in the left column of Fig. and the corresponding frequency response are shown in the right column of Fig. 7, each device is shown in a different color as noted in legend.

The dielectric tube has a monopole mode, but the peak resonance is offset to the target streaking frequency of 0.3 THz and the strength is minor (maximal 0.5 kV) compared to the particle energy of 3 MeV.

For the deflecting dipole mode, the ISRR array shows the strongest peak resonance, roughly 7.3 times that of the STEAM model, 4 times that of the dielectric tube structure and 3 times that of the ISRR structure. The central frequency \( f_0 \), the 3 dB bandwidth \( \Delta f_{3dB} \) and the quality factor \( Q_0 \) of each structure are listed in Table 1.

In terms of the quadrupole and sextupole modes, which might cause beam size growth over bunch coordinates and thereby a worse temporal resolution, ISRR and ISRR array show similar properties: (i) Skew quad moments, which are caused by asymmetries in the transverse plane, (ii) The resonances are located near the design frequency of 0.3 THz and more or less in counter phase (180 degree phase difference) with the main dipole modes. The dielectric tube
Figure 7: Multipole modes of all devices with an input amplitude of 1 MV/m within [0.05, 0.55] THz: a) Monopole; b) Dipole; c) Quadrupole; and d) Sextupole. The left part shows the behavior in time domain and the right part shows the corresponding frequency response normalized to the incident THz pulse.

structure has a complex quadrupole spectrum mainly located at frequencies higher than 3 dB bandwidth of dipole mode, which will typically not get ex-
Table 1: Dipole mode of each THz device.

<table>
<thead>
<tr>
<th></th>
<th>ISRR array</th>
<th>ISRR tube</th>
<th>Dielectric tube</th>
<th>STEAM streaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_0)/THz</td>
<td>0.315</td>
<td>0.311</td>
<td>0.300</td>
<td>0.333</td>
</tr>
<tr>
<td>(\Delta f_{\mathrm{3dB}})/THz</td>
<td>0.100</td>
<td>0.072</td>
<td>0.053</td>
<td>0.178</td>
</tr>
<tr>
<td>(Q_0)</td>
<td>3.15</td>
<td>4.32</td>
<td>5.64</td>
<td>1.87</td>
</tr>
</tbody>
</table>

The sextupole components of the dielectric tube, the quadrupole and sextupole modes of the STEAM streaker are all negligible.

Figure 8: Time-dependent streaked momentum of charged particles after passing through each device. The marked asterisks indicate the zero-crossing of each curve and the slope of nearby parts are shown as slew rate of each structure.

Figure 8 presents the time-dependent streaked momentum and the derived
slew rate of each structure corresponding to the 1 MV/m input THz pulse. The
temporal resolution depends on the ratio of the initial angular spread of beam to
the slew rate of deflectors. Assuming a 3 MeV electron beam with initial angular
spread of 10 µrad and a reasonable input THz amplitude of 10 MV/m, the ISRR
array could reach a slew rate of 126 (keV/c)/ps and thereby an intrinsic temporal
resolution of 0.3 fs; the other three structures can also reach intrinsic resolutions
of roughly 1 fs. It should be noted that the initial angular spread of beam is
limited by the achievable beam emittance, which makes it more challenging for
structures with smaller beam apertures (i.e. ISRR) to obtain such a resolution.

Different THz pulse energies are needed for these four structures to get a
peak field strength of 1 MV/m, due to their different dimensions. We assume a
Gaussian beam with a full width at half maximum (FWHM) pulse duration $\tau$
of 3.3 ps, that is, a single cycle of 0.3 THz. Then the THz pulse energy $E_{THz}$
can be expressed as below:

$$E_{THz} = \frac{\epsilon_0 c E_0^2 \tau A}{0.94}$$

where $\epsilon_0 \approx 8.854 \times 10^{-12}$ F/m is the vacuum permittivity, $E_0 = 1$ MV/m is the
peak electric field strength and $A$ is the focal size of THz pulse. The focused spot
size and the required THz pulse energy of each structure to obtain a peak field
strength of 1 MV/m are listed in table 2. It is worth to mention that the actual
area of ISRR structure is 0.08 mm$^2$, while the spot size listed here is limited
by the theoretical minimum focal diameter of $\lambda/2$. Note that it is relatively
challenging to create a focal diameter of $\lambda/2$ in an accelerator environment due
to the existence of vacuum window and the nonadjustable distance from the
window to the reaction site. The figure of merit $Q$ in the last line of table 2
is defined as the ratio of slew rate (in Fig. 8) to the square root of THz pulse
energy $E_{THz}$ in the table, to measure the utilization efficiency of each THz
device for a given THz pulse energy. Based on the information, ISRR array has
the highest efficiency to transfer the incident THz pulse energy into streaking
strength.
Table 2: The focused spot size and the required energy of THz pulse to obtain a 1 MV/m peak field strength in each THz device, as well as a figure of merit to measure the efficiency to utilize THz energy.

<table>
<thead>
<tr>
<th></th>
<th>ISRR array</th>
<th>ISRR tube</th>
<th>Dielectric tube</th>
<th>STEAM streaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focused spot size $A$ / mm$^2$</td>
<td>0.50</td>
<td>3.08</td>
<td>0.72</td>
<td>3.10</td>
</tr>
<tr>
<td>THz pulse energy $E_{THz}$ / nJ</td>
<td>4.71</td>
<td>28.98</td>
<td>6.78</td>
<td>29.17</td>
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<tr>
<td>Figure of merit $Q$</td>
<td>2.07</td>
<td>2.34</td>
<td>1.23</td>
<td>0.56</td>
</tr>
</tbody>
</table>

4. Conclusion and Outlook

This paper discussed four broadband THz devices and presented a generic approach to systematically analyze the multipole modes in each device type. The optimum streaking strength can be obtained by the ISRR array at the price of smaller aperture for electron beam, which is only 20 µm × 20 µm. Taking into account multipole kicks, electron beam sizes should be even smaller at values of less than 15 µm. The ISRR is similar to the ISRR array with respect to these two properties, but is able to work with strongly focused THz pulses and less critical alignment requirements due to its small thickness of 80 µm. The other two hybrid structures have weaker dipole strength with the advantage of generous aperture. Their multipole kicks are negligible (STEAM device) or can be ignored for a sufficiently narrow band THz pulses (dielectric tube). Based on previous evaluation and the reported strong-field THz of more than 50 MV/m [13], these structures are promising for femtosecond or even sub-femtosecond scale resolution.

Author contribution

X. Y. Liu: Methodology, Validation, Writing - original draft, Data curation.
M. Dehler: Conceptualization, Methodology, Writing - review & editing.
A. Fallahi: Methodology, Writing - review & editing.
Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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