# Partitioning the Two-Leg Spin Ladder in $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ : From Magnetic Order through Spin-Freezing to Paramagnetism 

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Cite This: Chem. Mater. 2023, 35, 2752-2761


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#### Abstract

Ba}_{2} \mathrm{CuTeO}_{6}\) has attracted significant attention as it contains a two-leg spin ladder of $\mathrm{Cu}^{2+}$ cations that lies in close proximity to a quantum critical point. Recently, $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ has been shown to accommodate chemical substitutions, which can significantly tune its magnetic behavior. Here, we investigate the effects of substitution for non-magnetic $\mathrm{Zn}^{2+}$ impurities at the $\mathrm{Cu}^{2+}$ site, partitioning the spin ladders. Results from bulk thermodynamic and local muon magnetic characterization on the $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ solid solution ( $0 \leq x \leq 0.6$ ) indicate that $\mathrm{Zn}^{2+}$ partitions the $\mathrm{Cu}^{2+}$ spin ladders into clusters and can be considered using the  percolation theory. As the average cluster size decreases with increasing $\mathrm{Zn}^{2+}$ substitution, there is an evolving transition from longrange order to spin-freezing as the critical cluster size is reached between $x=0.1$ to $x=0.2$, beyond which the behavior became paramagnetic. This demonstrates well-controlled tuning of the magnetic disorder, which is highly topical across a range of lowdimensional $\mathrm{Cu}^{2+}$-based materials. However, in many of these cases, the chemical disorder is also relatively strong in contrast to $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ and its derivatives. Therefore, $\mathrm{Ba}_{2} \mathrm{Cu}_{1-}{ }_{2} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ provides an ideal model system for isolating the effect of defects and segmentation in low-dimensional quantum magnets.


## 1. INTRODUCTION

Copper oxides are excellent hosts for unusual magnetic phenomena. This is due to the quantum spin $S=1 / 2$ of $\mathrm{Cu}^{2+}$ cations combined with the strong Jahn-Teller effect, which leads to co-operative orbital ordering that effectively lowers the dimensionality of the interactions between the $\mathrm{Cu}^{2+}$ spins. The quantum spin and low dimensionality enhances quantum effects and can give rise to a range of exotic quantum magnetic phases and transitions, many of which are of technological value. ${ }^{1}$ As a result, copper-based transition metal oxides such as perovskites are desirable models to study existing and discover new low-dimensional quantum phenomena, e.g., high temperature superconductivity, frustrated magnetism, and quantum magnetic transitions. ${ }^{2-8}$
The two-leg spin ladder in $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ is an example of a low-dimensional copper perovskite. The 12R hexagonal perovskite structure of $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ has face-sharing $\mathrm{CuO}_{6}$ $\mathrm{TeO}_{6}-\mathrm{CuO}_{6}$ trimers linked by corner-sharing $\mathrm{TeO}_{6}$ units (Figure 1a). ${ }^{9}$ Through $\mathrm{Cu}-\mathrm{O}-\mathrm{Te}-\mathrm{O}-\mathrm{Cu}$ superexchange, this creates two-leg $\mathrm{Cu}^{2+}$ spin ladders along the $b$ axis of the monoclinic crystal structure, wherein the intra-ladder superexchange interactions are the $J_{\operatorname{leg}}$ and $J_{\text {rung }}$ interactions shown by the red arrows in Figure 1b. A weak inter-ladder exchange ( $\mathrm{J}_{\text {inter }}$ ) occurs through the face-sharing trimers, creating a highly quasi-two-dimensional system. ${ }^{90}$ This system has attracted interest as it lies very close to the quantum critical point (QCP) on the Nèel ordered side of the two-leg spin ladder phase diagram shown in Figure 1c. ${ }^{11-13}$ QCPs are


Figure 1. (a) Monoclinic structure of $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ showing the 12 R hexagonal stacking sequence. The intra-ladder ( $J_{\text {leg }}$ and $J_{\text {rung }}$ ) interactions between the $\mathrm{Cu}^{2+}$ cations (colored green) are indicated by the red arrows. The inter-ladder interaction $J_{\text {inter }}$ through the facesharing $\mathrm{CuO}_{6}-\mathrm{TeO}_{6}-\mathrm{CuO}_{6}$ trimer is indicated by the blue arrow. (b) Two-leg spin ladder structure of $\mathrm{Cu}^{2+}$ cations in $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ viewed along the $a$ axis. (c) Two-leg spin ladder phase diagram. The red arrow shows that $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ lies close to the quantum critical point (QCP) on the Nèel ordered side of the phase diagram.

[^0]
electronic phase transitions at absolute zero, and they occur in a range of technologically important materials (e.g., superconductors, insulators, and semiconductors). ${ }^{14-17}$

We have recently demonstrated that the intra-ladder interactions in $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ can be site-selectively tuned through $\mathrm{W}^{6+}$ substitution. ${ }^{18} \mathrm{~W}^{6+}$ is almost exclusively substituted for $\mathrm{Te}^{6+}$ at the corner-sharing site $\left(B^{\prime \prime}(c)\right)$ rather than the face-sharing trimer site $\left(B^{\prime \prime}(\mathrm{f})\right)$ indicated in Figure 1. Through the $d^{10} / d^{0}$ effect, the competing $d^{10} \mathrm{Te}^{6+}$ and $d^{0} \mathrm{~W}^{6+}$ interactions strongly suppress the $J_{\text {rung }}$, while the $J_{\text {leg }}$ is slightly strengthened. ${ }^{18,19}$ This tunes the system from a spin ladder toward a spin chain, further reducing the dimensionality of the $\mathrm{Cu}^{2+}$ interactions. It is possible that the dimensionality of the two-leg spin ladder could be modified from another perspective. Instead of between ladders, substitution could be performed directly within the ladder at the $\mathrm{Cu}^{2+}$ site. Nonmagnetic impurities, whether intrinsic or purposefully introduced, are an important consideration when synthesizing magnetic materials and have been considered using percolation theory in square lattices, spin chains, and spin ladders. ${ }^{20,21}$ In a two-leg spin ladder, any finite impurities will segment the ladder into clusters. This is due to the fact that a ladder is a one-dimensional system, and two neighboring non-magnetic impurities linked by a "rung" will create a break in the ladder interactions. The size of the clusters is controlled by the level of non-magnetic impurities. ${ }^{21}$

Non-magnetic $\mathrm{Zn}^{2+}$ impurities have been studied in two-leg spin ladders previously. Examples include $\operatorname{Sr}\left(\mathrm{Cu}_{1-x} \mathrm{Zn}_{x}\right)_{2} \mathrm{O}_{3}$, $\mathrm{Bi}\left(\mathrm{Cu}_{1-x} \mathrm{Zn}_{x}\right)_{2} \mathrm{PO}_{6}$, and $\left(\mathrm{C}_{7} \mathrm{H}_{10} \mathrm{~N}\right)_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x}(\mathrm{Br})_{4}{ }^{22-29}$ These two-leg spin ladders lie on the spin singlet side of the two-leg spin ladder phase diagram, where $J_{\text {inter }}$ is weak, creating near-isolated spin ladders. As expected, the introduction of $\mathrm{Zn}^{2+}$ creates "free" $\mathrm{Cu}^{2+}$ spins as singlet dimers are broken by the removal of $\mathrm{Cu}^{2+}$. ${ }^{3+}$ Unexpectedly, antiferromagnetic ordering has also been observed for $\operatorname{Sr}\left(\mathrm{Cu}_{1-x} \mathrm{Zn}_{x}\right)_{2} \mathrm{O}_{3}$ and $\mathrm{Bi}\left(\mathrm{Cu}_{1-x} \mathrm{Zn}_{x}\right)_{2} \mathrm{PO}_{6}$ with low $\mathrm{Zn}^{2+}$ concentrations ( $x=0.01-$ $0.02){ }^{24,25,29}$ It is proposed that antiferromagnetic order arises from $\mathrm{Cu}^{2+}$ moments generated in the vicinity of the $\mathrm{Zn}^{2+}$ impurity. ${ }^{28}$ The $\mathrm{Cu}^{2+}$ moments are independent of geometry and create antiferromagnetic correlations. ${ }^{29}$ Theoretical calculations suggest that the extended $\mathrm{Cu}^{2+}$ spin ladder interactions are not destroyed and only the local $\mathrm{Cu}^{2+}$ singlets are affected. ${ }^{30}$ The effect of $\mathrm{Zn}^{2+}$ impurities in Nèel ordered two-leg spin ladders with stronger $J_{\text {inter }}$ interactions remains experimentally unexplored. To investigate, a solid solution of $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}(0 \leq x \leq 0.6)$ was prepared and analyzed using a range of structural and magnetic characterization techniques.

## 2. EXPERIMENTAL SECTION

2.1. Synthesis. Polycrystalline powders of $\mathrm{Ba}_{2} \mathrm{Cu}_{1-}{ }_{x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}, 0$ $\leq x \leq 0.6$, were prepared by mixing high-purity $\mathrm{BaCO}_{3}$ ( $99.997 \%$ ), $\mathrm{CuO}(99.9995 \%), \mathrm{ZnO}$ (99.99\%), and $\mathrm{TeO}_{2}$ (99.995\%). The reactant mixture was pressed into a pellet and calcined in air for 12 h at 900 ${ }^{\circ} \mathrm{C}$. Calcined pellets were re-ground and pressed before heating at $1050-1100{ }^{\circ} \mathrm{C}$ under a flow of oxygen for 24 h . A total of $72 \mathrm{~h}(3 \times$ 24 h ) was required to achieve phase purity in all samples.
2.2. X-ray and Neutron Diffraction. A Rigaku Miniflex diffractometer $\left(\mathrm{Cu} \mathrm{K}_{\alpha 1} / \mathrm{K}_{\alpha 2}(\lambda=1.5405\right.$ and $\left.1.5443 \AA)\right)$ monitored the sample purity during the reaction. Neutron diffraction data were collected on the time-of-flight diffractometer HRPD at the ISIS Neutron and Muon Source. ${ }^{31,32}$ The data were collected at ambient temperature in a standard time-of-flight window of $30-130 \mathrm{~ms}$ with the sample contained in standard cylindrical vanadium cans. Data
were analyzed using Rietveld refinement as implemented in GSAS, TOPAS Academic v7, and PIEFACE for polyhedral distortions. ${ }^{33-35}$
2.3. Inductively Coupled Plasma-Optical Emission Spectroscopy. ICP-OES was performed on $x=0.1-0.6$ samples to determine the relative percentage of $\mathrm{Cu}^{2+}$ and $\mathrm{Zn}^{2+}$ in the samples. Powder samples were digested in an aqua regia mixture at $150^{\circ} \mathrm{C}$ before being analyzed by a Spectrogreen FMX46 ICP-OES where, upon ionization, the percentage $\mathrm{Cu}^{2+}$ and $\mathrm{Zn}^{2+}$ in each sample was determined from the light emitted at wavelengths of 324.754 nm $(\mathrm{Cu})$ and $213.856 \mathrm{~nm}(\mathrm{Zn})$ using an optical spectrometer.
2.4. Magnetic Susceptibility. Measurements were performed using a Quantum Design MPMS3 SQUID magnetometer. The DC susceptibility ( $\chi$ vs $T$ ) was measured between 2 and 300 K in both zero-field cooled (ZFC) and field-cooled (FC) modes using a 1000 Oe external field. AC susceptibility ( $\chi_{\mathrm{AC}}^{\prime}$ vs $T$ ) measurements were performed on the $x=0.1,0.2$, and 0.3 samples. Using a weak DC field of 25 Oe and an AC field of 5 Oe , the AC susceptibility was measured from 2 to 100 K in a frequency range of 10 to 467 Hz .
2.5. Heat Capacity. A Quantum design PPMS was used to perform heat capacity measurements. Shards of sintered pellets weighing $\sim 10 \mathrm{mg}$ were placed onto the sample puck using Apiezon N grease, and the heat capacity was measured using the thermal relaxation method between 2 and 100 K in the zero field. The contribution of the grease and puck was subtracted from the total measurement to give the heat capacity of the sample.
2.6. Muon Spin Relaxation. Muon experiments were performed at the Paul Scherrer Institut (PSI) using the GPS beamline. Approximately 1 g of polycrystalline powder ( $x=0,0.1,0.2$, and 0.3 ) was loaded into a silver foil packet and secured onto the sample fork. The sample fork was inserted into the muon beam and cooled to 1.5 K using a cryostat. Zero-field (ZF), transverse-field (TF), and longitudinal-field (LF) muon spin relaxation measurements were performed between 1.5 and 20 K . The data were analyzed using musfit. ${ }^{36}$

## 3. RESULTS

3.1. Crystal Structure. Our high resolution neutron diffraction data confirm that the same monoclinic $\mathrm{C} 2 / \mathrm{m}$ crystal structure is present across the series $0 \leq x \leq 0.6$ at $T=$ 300 K. Figure 2 shows an example of the Rietveld refinement


Figure 2. Rietveld refinement of the monoclinic $x=0.1$ model using the 300 K HRPD neutron diffraction data for $\mathrm{Ba}_{2} \mathrm{Cu}_{0.9} \mathrm{Zn}_{0.1} \mathrm{TeO}_{6}$ $\left(R_{\mathrm{wp}}(\%)=6.41\right.$ and $\left.\chi^{2}=2.924\right)$.
for $x=0.1$, wherein all the Bragg peaks are described well by the refined C2/m model shown in Table S1. It should be noted that the parent $x=0$ compound has a weak transition to a triclinic $P \overline{1}$ phase at $T=287 \mathrm{~K}$, and the $x=1$ composition is rhombohedral $(R \overline{3} m){ }^{9,10} C 2 / m$ to $P \overline{1}$ distortions could be observed for $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ upon cooling. This distortion will have a minor effect on the structure and interactions as the $C 2 / m$ and $P \overline{1}$ models are very similar. Consequently, the high-
temperature $\mathrm{C} 2 / m$ structure can be used to model the magnetic interactions in the low-temperature $P \overline{1}$ structure of $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6} .{ }^{12}$

Substitution of $\mathrm{Zn}^{2+}$ for $\mathrm{Cu}^{2+}$ leads to a systematic reduction in the $a$ lattice parameter, an increase in $b$ and $c$, and an increase in the monoclinic $\beta$ angle. This results from a weakening of the strength of the cooperative Jahn-Teller distortion with increasing $x$. Due to the significant irregularity in the shape and bond lengths of the $(\mathrm{Cu}, \mathrm{Zn}) \mathrm{O}_{6}$ octahedra, this may be better quantified through minimum bounding ellipsoid analysis ${ }^{34}$ than by the investigation of specific bond lengths and angles. The main parameter of interest from this calculation is the magnitude of the largest ellipsoidal principal axis (effectively the Jahn-Teller axis), $R_{1}$. As $x$ varies from 0 to 0.2 to 0.4 , for example, the magnitude of this parameter decreases from $2.371 \AA \AA$ to 2.356 and $2.343 \AA$. In addition, the variance of the principal axes indicates the overall strength of the distortion from an ideal polyhedron. As expected, this reduces monotonically with increasing $x$ with $\sigma(R)_{x=0}=0.192$, $\sigma(R)_{x=0.2}=0.179$, and $\sigma(R)_{x=0.4}=0.167$.
Given the similar $\mathrm{Cu}^{2+}$ and $\mathrm{Zn}^{2+}$ X-ray and neutron scattering lengths, ICP-OES was used to confirm the samples' stoichiometries. The ICP-OES results gave the percentages of $\mathrm{Cu}^{2+}$ and $\mathrm{Zn}^{2+}$ in each composition. The percentage of $\mathrm{Zn}^{2+}$ was divided by the total percentages of $\mathrm{Cu}^{2+}$ and $\mathrm{Zn}^{2+}$ in each sample. This gave the proportion of $\mathrm{Zn}^{2+}$ in the sample as a decimal where the total amount of $\mathrm{Cu}^{2+}+\mathrm{Zn}^{2+}=1$, and the $\mathrm{Cu}^{2+}$ portion was found by $\mathrm{Cu}^{2+}=1-\mathrm{Zn}^{2+}$. Table 1 shows that

Table 1. Results from ICP-OES Measurements of the $\mathrm{Ba}_{2} \mathrm{Cu}_{x} \mathrm{Zn}_{1-x} \mathrm{TeO}_{6} \boldsymbol{x}=\mathbf{0 . 1} \mathbf{- 0 . 6}$ Samples Showing the Amount of $\mathrm{Zn}^{2+}$ and $\mathrm{Cu}^{2+}$ in Each Sample as a Proportion of the Total Amount of $\mathrm{Cu}^{2+}$ and $\mathrm{Zn}^{2+}$, Where $\mathrm{Cu}^{2+}+\mathrm{Zn}^{2+}=1$

| $x$ | $\mathrm{Zn}^{2+}$ | $\mathrm{Cu}^{2+}$ |
| :---: | ---: | :---: |
| 0.1 | $0.1068(3)$ | $0.893(3)$ |
| 0.2 | $0.2116(6)$ | $0.788(2)$ |
| 0.3 | $0.312(3)$ | $0.688(6)$ |
| 0.4 | $0.417(4)$ | $0.583(6)$ |
| 0.5 | $0.516(3)$ | $0.484(2)$ |
| 0.6 | $0.621(2)$ | $0.388(1)$ |

the $\mathrm{Zn}^{2+}$ portion incrementally increases by $\sim 0.1$ for each $x=$ 0.1 increase in the $\mathrm{Zn}^{2+}$ concentration, while the $\mathrm{Cu}^{2+}$ portion decreases by $\sim 0.1$. This agrees with the sample stoichiometry of the $x=0.1-0.6$ samples, confirming no elemental losses.
3.2. DC Susceptibility. Figure 3 shows the $\chi$ vs $T$ data for $x=0,0.1,0.2,0.3,0.5$ and 0.6 . No ZFC and FC divergence was observed for any of the samples. There are clear changes in the features of the $\chi$ vs $T$ curve as $\mathrm{Zn}^{2+}$ is introduced to $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$. To most clearly show the effect of dilution of $\mathrm{Cu}^{2+}$ by $\mathrm{Zn}^{2+}$, the susceptibility has been scaled to $\mathrm{cm}^{3} \mathrm{~mol}^{-1}$ of $\mathrm{Cu}^{2+}$. Panel (a) shows the $\chi$ vs $T$ curve of $x=0$ has a broad maximum of about $T_{\max } \approx 74 \mathrm{~K}$, below which the susceptibility decreases leading to a low-temperature upturn of about $T_{\min } \approx$ $14 \mathrm{~K} . T_{\text {max }}$ represents the establishment of short-range ladder interactions. The low temperature upturn is thought to indicate entry to the Neel ordered state but is not a classical indication of antiferromagnetic order. ${ }^{10,18}$ Therefore, the upturn cannot be assumed to be the position of $T_{\mathrm{N}}$. Instead, magnetic ordering has been confirmed using other methods and places $T_{\mathrm{N}}$ at 14.1 K for $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}{ }^{5}$.

The introduction of $10 \% \mathrm{Zn}^{2+}$ (Figure 3b) causes a sharp rise in the low-temperature upturn feature and a shift in $T_{\max }$ toward lower temperatures. The decrease in $T_{\max }$ (Table 2) indicates weakening of the short-range interactions. The expansion in panel (b) shows a visible "kink" in the lowtemperature data at 10 K , close to the position of the $T_{\text {min }}$ upturn in $x=0$. Beyond $x=0.1$, there is no visible "kink" in the low-temperature data (see expansion in Figure 3c). The lowtemperature susceptibility continues to grow, and the $T_{\max }$ feature transitions into a large paramagnetic tail. The inverse $1 / \chi$ vs $T$ data between 150 and 300 K were fitted using the Curie-Weiss law (see Supplementary Figure S15). Table 2 shows the values of the Curie constant (C), Weiss constant $\left(\theta_{\mathrm{W}}\right)$, and effective magnetic moment $\left(\mu_{\text {eff }}\right)$. The linear change in $\theta_{\mathrm{W}}$ (plotted in Figure 3 g ) from $-89.3(4) \mathrm{K}$ for $x=0$ to a value of $-9.9(5) \mathrm{K}$ for $x=0.6$ shows a large weakening of the antiferromagnetic interactions. Table 2 shows that the $\mu_{\text {eff }}$ is close to the previously reported value for $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ and $\mathrm{Ba}_{2} \mathrm{CuTe}_{1-x} \mathrm{~W}_{x} \mathrm{O}_{6} .{ }^{10,18}$

The $\chi$ vs $T$ data for $0 \leq x \leq 0.3$ were modeled using the isolated two-leg spin ladder model between 35 and 300 K . The model is based on Quantum Monte Carlo (QMC) simulations of isolated two-leg spin ladders and has been employed to model $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ and $\mathrm{Ba}_{2} \mathrm{CuTe}_{1-x} \mathrm{~W}_{x} \mathrm{O}_{6}$ previously. ${ }^{10,18,22}$ The fitting parameters, $J_{\text {leg }} J_{\text {rung }} / J_{\text {leg }}$, and Landè $g$-factor, for $x=$ 0 were near identical to previous reports: $J_{\text {leg }}=89.2(3) \mathrm{K}, J_{\text {rung }} /$ $J_{\text {leg }}=0.972(6)$, and $g=2.231(2) .{ }^{10,18}$ The $x=0.1$ data could be described using the spin ladder model but, as shown in Figure 4, began to fail for $x=0.2$ as the $T_{\max }$ feature is suppressed. The model completely fails for $x=0.3$, indicating a change from spin ladder behavior. This can be seen by comparing the fits shown by the solid black lines in Figure 4. Hence, accurate fitting parameters could only be obtained for $x$ $=0.1$ and suggest slight strengthening of the $J_{\text {leg }}=99.5(2) \mathrm{K}$ interaction compared to $x=0$. The $J_{\text {rung }} / J_{\text {leg }}=0.17(2)$ ratio is significantly reduced from near unity in the $x=0$ compound, showing strong suppression of the $J_{\text {rung }}$ interaction. This agrees with the values of $\theta_{\mathrm{W}}$ and $\mu_{\mathrm{eff}}$, which suggest weakening of the overall intra-ladder interactions in $x=0.1$.
3.3. AC Susceptibility. The AC susceptibility data is shown in Figure 5. The $\chi_{\mathrm{AC}}^{\prime}$ vs $T$ curves in panels (a) $x=0.1$, (b) $x=0.2$, and (c) $x=0.3$ show no frequency-dependent shift. Neither were there any distinctive peaks in the imaginary component of the AC susceptibility ( $\chi_{\mathrm{AC}}^{\prime \prime}$ vs $T$ ) plotted in Supplementary Figure S16. As such, the expected AC signatures of a canonical spin glass are not observed in any of the samples.
3.4. Muon Spin Relaxation. Muon spin relaxation ( $\mu \mathrm{SR}$ ) experiments were performed on $x=0,0.1,0.2$, and 0.3 to learn more about the local magnetic behavior. Previous measurements of $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ on ARGUS at the RIKEN-RAL using a pulsed muon source have identified a single oscillation of frequency $f=4.3 \mathrm{MHz}$ in the ZF- $\mu \mathrm{SR}$ data at $2 \mathrm{~K} .{ }^{5}$ Continuous muon sources such as PSI offer improved time resolution and can detect higher frequency oscillations compared to at pulsed sources. In this work, $x=0$ was measured on GPS using a continuous PSI source, and the $1.5 \mathrm{~K} \mathrm{ZF}-\mu$ SR data in Figure 6a shows that the signal is actually composed of two oscillations. The presence of two oscillations shows there are two muon stopping sites in $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$. The two oscillations were described well using the polarization function in eq 1 . This sums a Gaussian cosine and an exponential cosine (to describe the two oscillations) with an exponential background term.


Figure 3. DC susceptibility data of the (a) $x=0$, (b) $x=0.1$, (c) $x=0.2$, (d) $x=0.3$, (e) $x=0.5$, and (f) $x=0.6 \mathrm{Ba}_{2} \mathrm{Cu}_{x} \mathrm{Zn}_{1-} \mathrm{TeO}_{6}$ samples. $\chi$ is scaled to $\mathrm{cm}^{3} \mathrm{~mol}^{-1}$ of $\mathrm{Cu}^{2+}$ to reflect dilution of the $\mathrm{Cu}^{2+}$ site. The $\chi$ vs $T$ data of $x=0.4$ was identical to other samples beyond $x \geq 0.3$. The position of $T_{\max }$ and/or $T_{\min }$ are indicated in the $\chi$ vs $T$ curves where appropriate. Expansions of the low-temperature data are shown in panels (b) and (c). There is a clear "kink" at $\sim 10 \mathrm{~K}$ in the $x=0.1$ curve that is not visible in the $x \geq 0.2$ curves. (g) Weiss constant ( $\theta_{\mathrm{W}}$ ) plotted as a function of $x$ in $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6} . \theta_{\mathrm{W}}$ increases linearly as $x$ increases, showing large weakening of the magnetic interactions.

Table 2. Results from DC $\chi$ vs $T$ Data for
$\mathrm{Ba}_{2} \mathrm{Cu}_{x} \mathrm{Zn}_{1-x} \mathrm{TeO}_{6}(0 \leq x \leq 0.6)$

| $x$ | $T_{\max }(\mathrm{K})$ | $\mathrm{C}\left(\mathrm{cm}^{3} \mathrm{~K} \mathrm{~mol}^{-1}\right)$ | $\theta_{\mathrm{W}}(\mathrm{K})$ | $\mu_{\text {eff }}\left(\mu_{\mathrm{B}}\right.$ per $\left.\mathrm{Cu}^{2+}\right)$ |
| :--- | :---: | :---: | :---: | :---: |
| 0 | 73.7 | $0.4450(7)$ | $-89.3(4)$ | $1.890(5)$ |
| 0.1 | 64 | $0.4688(8)$ | $-84.0(3)$ | $1.936(2)$ |
| 0.2 | $\sim 57$ | $0.4438(4)$ | $-71.70(9)$ | $1.8840(8)$ |
| 0.3 |  | $0.4365(7)$ | $-58.6(2)$ | $1.869(1)$ |
| 0.4 |  | $0.4291(7)$ | $-44.3(2)$ | $1.852(1)$ |
| 0.5 |  | $0.4214(6)$ | $-36.6(2)$ | $1.835(1)$ |
| 0.6 |  | $0.3751(8)$ | $-9.9(3)$ | $1.732(2)$ |



Figure 4. Modeling the $\chi$ vs $T$ data of $x=0,0.1$, and 0.2 using the QMC isolated two-leg spin ladder model. The fits are shown by the solid black lines. The data are offset along the $y$ axis. The isolated twoleg spin ladder model provides a good description of the $x=0$ and $x=$ 0.1 data, allowing extraction of the $J_{\text {leg }}, J_{\text {rung }} / J_{\text {leg }}$, and $g$ fitting parameters. The fit to the $x=0.2$ and $x=0.3$ data shows that the model increasingly fails to described the suppressing $T_{\max }$ feature.

$$
\begin{align*}
P(t)= & A_{0} e^{-\sigma^{2} t^{2}} \cos \left(2 \pi f_{1} t+ø_{1}\right)+A_{1} e^{-\lambda_{1} t} \cos \left(2 \pi f_{2} t+ø_{2}\right) \\
& +A_{2} e^{-\lambda_{2} t} \tag{1}
\end{align*}
$$



Figure 5. AC susceptibility data of the (a) $x=0.1$, (b) $x=0.2$, and (c) $x=0.3$ samples. None of the samples show any frequencydependent shift in their $\chi_{\mathrm{AC}}^{\prime}$ vs $T$ curve to suggest a canonical spin glass.
$A_{0}, A_{1}$, and $A_{2}$ are the initial asymmetries, $\sigma$ is the Gaussian decay rate, $\lambda_{1}$ is the exponential cosine decay rate, and $\lambda_{2}$ is the decay rate of the background term. $f_{1}$ and $f_{2}$ are the frequencies, and $\varrho_{1}$ and $\sigma_{2}$ are the phases of the respective oscillations. From the fit in Figure 6a, $f_{1}=3.81(1) \mathrm{MHz}, f_{2}=$ $6.67(9) \mathrm{MHz}$, and the phases were zero in zero-field. The values of $\sigma=1.4(1) \mu \mathrm{s}^{-1}$ and $\lambda_{1}=12.3(8) \mu \mathrm{s}^{-1}$ show that


Figure 6. GPS ZF- $\mu$ SR data of the (a) $x=0$, (b) $x=0.1$, (c) $x=0.2$, and (d) $x=0.3$ samples at 1.5 K . The black lines in the plots are the fits to the experimental data. Clear oscillations are observed for the $x=0$ and $x=0.1$ samples, demonstrating long-range order. Recovery of $1 / 3$ of the initial asymmetry implies static ordering for $x=0.2$, whereas exponential relaxation observed for $x=0.3$ reflects a mostly dynamic magnetic environment. The goodness of fit $\left(\chi^{2}\right)$ is shown in each panel.
muon relaxation is faster at the muon site described by the exponential cosine term.

Figure 6 compares the ZF- $\mu$ SR data of $x=0$ in panel (a) to the $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ (b) $x=0.1$, (c) $x=0.2$, and (d) $x=$ 0.3 data at 1.5 K . Clear oscillations are present in the $x=0.1$ data in panel (b), showing that long-range magnetic order is still present. The oscillations were described poorly using the $x$ $=0$ polarization function in eq 1 (see Supplementary Figure S17). Instead, the $x=0.1$ muon relaxation was better described using the polarization function involving Bessel functions in eq 2.

$$
\begin{align*}
P(t)= & A_{0} e^{-\lambda_{1} t} J_{0}\left(2 \pi f_{1} t+ø_{1}\right)+A_{1} e^{-\lambda_{2} t} J_{0}\left(2 \pi f_{2} t+ø_{2}\right) \\
& +A_{2} e^{-\lambda_{3} t} \tag{2}
\end{align*}
$$

Here, two exponential zeroth-order Bessel functions $J_{0}\left(2 \pi f_{1} t\right.$ $+\emptyset_{1}$ ) describe the two muon sites, and the exponential term describes the background. The fit in panel (b) of Figure 6 shows that eq 2 describes the muon relaxation of $x=0.1$ well when the phases of the Bessel functions were non-zero ( $\varnothing_{1}=$ $33.9(4.2)^{\circ}$ and $\left.\emptyset_{2}=-35.9(4.4)^{\circ}\right)$. The use of Bessel functions implies an incommensurate magnetic structure where the nonzero phase arises from the infinite number of magnetically inequivalent muon sites. ${ }^{37-39}$ However, this behavior has also been observed in materials with significant disorder whose magnetic structures are commensurate. ${ }^{8,40,41}$ The $x=0$ ZF$\mu$ SR data in Figure 6a was also fitted using eq 2 and is compared to the fit using eq 1 in Supplementary Figure S18. Both equations provided an adequate description of the muon polarization. The slight improvement in the fit using eq 2 implies that an incommensurate magnetic structure could also be plausible for $x=0$. The $\mathrm{ZF}-\mu \mathrm{SR}$ data for $x=0.1$ at above 1.5 K in Figure S19 shows that the magnetic oscillations decay on warming and are no longer visible above 8 K . This indicates that $10 \% \mathrm{Zn}^{2+}$ substitution lowered the transition temperature compared to $x=0\left(T_{\mathrm{N}}=14.1 \mathrm{~K}\right)$.
Panel (c) in Figure 6 shows the behavior of the $x=0.2$ sample differs to $x=0.1$. There are no oscillations to suggest long-range order. The muon polarization drops sharply at low
times but quickly retains $1 / 3$ of the initial asymmetry. Retention of $1 / 3$ of the initial asymmetry suggests a static component to the muon relaxation as well as a dynamic component that leads to the sharp drop in the initial asymmetry. The combination of static and dynamic behavior can be phenomenologically described using the sum of a Gaussian dynamic Kubo-Toyabe function and an exponential as in eq 3 , where $p_{\mathrm{z}}(t)$ is the static Kubo-Toyabe function (eq 4), $v$ is the muon hopping rate, $\delta$ is the width of the local field distribution, and $\lambda$ the exponential decay rate.

$$
\begin{align*}
& P(t)=\left(p_{\mathrm{z}}(t)+v \int_{0}^{t} g_{\mathrm{z}}\left(t_{1}\right) P_{\mathrm{z}}\left(t-t_{1}\right) \mathrm{d} t_{1}\right)+A_{1} e^{-\lambda t}  \tag{3}\\
& p_{\mathrm{z}}(t)=A_{0}\left(\frac{1}{3}+\frac{2}{3} e^{-1 / 2 \delta^{2} t^{2}}\left(1-\delta^{2} t^{2}\right)\right) \tag{4}
\end{align*}
$$

At $1.5 \mathrm{~K}, v$ is close to zero; therefore, the static KuboToyabe function mainly contributes to $P(t)$. This accounts for the $1 / 3$ retention of the initial asymmetry. This shows that the spins are frozen at 1.5 K . ZF- $\mu \mathrm{SR}$ measurements at higher temperatures show that the muon hopping rate increases on warming (see Supplementary Figure S20), causing gradual loss of the $1 / 3$ tail as the frozen static moments become dynamic. Note that the $x=0.2$ muon relaxation also resembles a spin glass. However, fits using stretch exponentials did not derive a meaningful stretching exponent (i.e., $\beta<0.5$ ) to support canonical spin glass behavior in agreement with the AC susceptibility data.

The $1.5 \mathrm{~K} \mathrm{ZF}-\mu \mathrm{SR}$ of $x=0.3$ in panel (c) is exponential with no recovery of $1 / 3$ of the asymmetry. The muon relaxation was described using two exponentials to reflect the two muon sites:

$$
\begin{equation*}
P(t)=A_{1} e^{-\lambda_{1} t}+A_{2} e^{-\lambda_{2} t} \tag{5}
\end{equation*}
$$

ZF- $\mu$ SR measurements on warming show that the hightemperature muon relaxation is quickly recovered as the dynamic fluctuations increase with temperature (see Supplementary Figure S21). Transverse field (TF)- $\mu$ SR measurements were also performed on warming. Dampening of the


Figure 7. TF- $\mu$ SR data for the (a) $x=0.1$, (b) $x=0.2$, and (c) $x=0.3$ samples. TF- $\mu$ SR data were collected at various temperatures between 1.5 and 20 K using a TF field of 30 G . Clear dampening is observed for the $x=0.1$ and $x=0.2$ samples, whereas the TF oscillations for $x=0.3$ are only slightly damped at 1.5 K . Panel (d) plots the normalized TF asymmetry $A_{\mathrm{T}}(T) / A_{\mathrm{T}}(20 \mathrm{~K})$ for $x=0.1,0.2$, and 0.3 as a function of temperature, $T$. $A_{\mathrm{T}}(T) / A_{\mathrm{T}}(20 \mathrm{~K})$ was determined by fitting the TF oscillations using eq 6.


Figure 8. LF- $\mu$ SR measurements of the (a) $x=0.1$, (b) $x=0.2$, and (c) $x=0.3$ samples. LF measurements were performed at 1.5 K using LF fields of $50-1000 \mathrm{G}$.

TF- $\mu$ SR oscillations indicates static magnetism as the muon spins begin to feel the effects of the internal fields and decouple from the TF field. While the majority of the $\mathrm{Cu}^{2+}$ moments in $x=0.3$ are dynamic, slight dampening of the $\mathrm{TF}-\mu \mathrm{SR}$ oscillations upon cooling in Figure 7c implies a small fraction of frozen spins. The TF- $\mu$ SR asymmetry $A_{\mathrm{T}}(T)$ was determined by fitting the TF oscillations using eq 6. The normalized $A_{T}(T) / A_{T}(20 \mathrm{~K})$ for $x=0.3$ plotted in red in Figure 7d noticeably decreases below 10 K and suggests that $\sim 14 \%$ of the spins are frozen at 1.5 K .

$$
\begin{equation*}
P(t)=A_{\mathrm{L}} e^{-\lambda_{\mathrm{L}} t}+A_{\mathrm{T}} e^{-\lambda_{\mathrm{T}} t} \cos (2 \pi f t+\emptyset)+A_{\mathrm{bkgd}} e^{-\lambda_{\mathrm{L}} t} \tag{6}
\end{equation*}
$$

Figure 7 also shows the TF- $\mu$ SR data of (a) $x=0.1$ and (b) $x=0.2$. Figure 7a shows complete dampening of the TF oscillations for $x=0.1$. The $A_{\mathrm{T}}(T) / A_{\mathrm{T}}(20 \mathrm{~K})$ plot for $x=0.1$ (green) in Figure 7d shows that long-range ordering is complete below 8 K , showing that $\mathrm{Zn}^{2+}$ lowered the ordering temperature of $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}\left(T_{\mathrm{N}}=14.1 \mathrm{~K}\right)$. The transition was gradual and occurred over a wider temperature range than
might be expected for long-range ordering. The transition temperature was chosen as the point at which $A_{T}(T) / A_{T}(20$ K) plateaued to a constant value. Strong dampening was observed for $x=0.2$ (Figure 7b). The value of $A_{T}(T) / A_{T}(20$ K) in Figure 7d plateaued to a constant value, indicating that freezing of the magnetic spins was complete below 4 K for $x=$ 0.2 (shown in pink). It was noted that the TF oscillations were not completely damped at 1.5 K for $x=0.2$ in Figure 7 b , whereas they were in the $x=0.1$ data in Figure 7a. This supports the existence of a small dynamic fraction ( $\sim 6 \%$ ) at 1.5 K. It is also noted that the transitions in the $x=0.1$ and $x=$ 0.2 samples are also gradual. This is clearly shown in the plot in Figure 7 d comparing the $A_{\mathrm{T}}(T) / A_{\mathrm{T}}(20 \mathrm{~K})$ data of $x=0.1,0.2$, and 0.3 .

Longitudinal field (LF) $-\mu \mathrm{SR}$ measurements of $x=0.1,0.2$, and 0.3 are compared in Figure 8. LF- $\mu$ SR measurements indicate the field strength required to repolarize the muon spin in the direction of the LF field. Figure 8 shows the LF data of (a) $x=0.1$ and (b) $x=0.2$. For $x=0.1$, suppression of the muon relaxation occurs above 100 G and complete


Figure 9. Heat capacity data for the $x=0,0.1,0.2$, and 0.3 samples. Panel (a) shows the $C_{P} / T$ vs $T$ data between 1.8 and 120 K. Panel (b) shows the low-temperature $C_{\mathrm{P}} / T$ vs $T^{2}$ data. Debye-Einstein fits for the $x=0$ and $x=0.1$ data between 1.8 and 109 K are shown by the black lines. The horizontal dotted lines at 8 and 4 K indicate the $T_{\mathrm{N}}$ of $x=0.1$ and spin-freezing temperature of $x=0.2$, respectively, determined from the TF- $\mu$ SR measurements.
repolarization occurs by 1000 G . The small suppression observed between 0 and 50 G represents decoupling from weak static nuclear spins. Larger LF fields are required to decouple electronic spins compared to nuclear spins. Repolarization requires weaker LF fields than might be expected for a long-range ordered sample owing to the weak $\mathrm{Cu}^{2+}$ moment and quantum fluctuations. The effects of the LF field can be seen at 100 G in the $x=0.2$ dataset in Figure 8b. The stronger suppression between 50 and 100 G might represent decoupling from dynamic or static electronic spins as well as static nuclear spins. Similar to the $x=0.1$ data, the largest changes occur above 100 G and the muon polarization is nearly completely recovered at 1000 G . The LF- $\mu$ SR data for $x=0.3$ in Figure 8c behaves differently. The muon polarization is gradually recovered as the LF field increases and is nearly complete at 1000 G . Suppression at 50 G likely represents decoupling from static nuclear spins, while the gradual recovery above 50 G resembles decoupling from dynamic electronic spins.
3.5. Heat Capacity. The zero-field heat capacity $\left(C_{P} / T\right.$ vs $T$ ) data of the $x=0,0.1,0.2$, and 0.3 samples is plotted in Figure 9a. As in previous reports, no clear Nèel transition can be observed for $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ at $\sim 14 \mathrm{~K} .{ }^{10,18}$ The close proximity to the QCP creates quantum fluctuations that smear out the lambda ( $\lambda$ ) ordering peak. The $C_{\mathrm{P}} / T$ vs $T$ curve of $x=0.1$ is similar, with no evidence of a $\lambda$-peak about the $T_{\mathrm{N}}=8 \mathrm{~K}$ indicated by the dotted line in the $C_{\mathrm{P}} / T$ vs $T^{2}$ plot in Figure 9 b . This indicates that strong quantum fluctuations persist upon $\mathrm{Zn}^{2+}$ substitution. The $C_{\mathrm{P}} / T$ vs $T^{2}$ data of $x=0$ and $x=$ 0.1 between 1.8 and 109 K was linear and could be fitted well using the Debye-Einstein equation to determine the electronic $(\gamma)$ and phonon ( $\beta_{\mathrm{D}}$ ) contribution to the heat capacity.

$$
\begin{equation*}
C_{\mathrm{P}}=\gamma T+\beta_{\mathrm{D}} T^{3} \tag{7}
\end{equation*}
$$

The $\gamma$-contribution was almost zero for $x=0(\gamma=1.4(1) \mathrm{mJ}$ $\mathrm{mol}^{-1} \mathrm{~K}^{-2}$ ), in agreement with previous reports, where $\gamma=$ $3.5(4) \mathrm{mJ} \mathrm{mol}^{-1} \mathrm{~K}^{-2}$. ${ }^{18}$ The value of $\gamma$ was also close to zero for $x=0.1\left(\gamma=8.6(1) \mathrm{mJ} \mathrm{mol}^{-1} \mathrm{~K}^{-2}\right)$. The low-temperature $C_{\mathrm{P}} / T$ vs $T^{2}$ data for $x=0.2$ and $x=0.3$ could not be fitted using eq 7. The $x=0.2$ data shown in red in Figure 9b deviates slightly from linear behavior and has a slight bump at 4 K (dotted line). This is close to the spin-freezing transition identified in the $\mathrm{TF}-\mu \mathrm{SR}$ measurements, so an association may be formed with this. The $x=0.3$ data in black deviates from linear behavior as the temperature decreases, leading to an
upturn below 4 K . The upturn indicates a clear change in the behavior between $x=0.2$ and $x=0.3$.

## 4. DISCUSSION

$\mathrm{Zn}^{2+}$ was successfully substituted for $\mathrm{Cu}^{2+}$ within the spin ladder, forming a monoclinic $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ solid solution. In-depth magnetic characterization using both bulk and local techniques showed that the spin ladder behavior changes as the $\mathrm{Zn}^{2+}$ concentration increases. Replacing magnetic $\mathrm{Cu}^{2+}$ with non-magnetic $\mathrm{Zn}^{2+}$ breaks local magnetic interactions in the system. Percolation theory can be used to explain the effects of such non-magnetic impurities on different spin systems. ${ }^{20,21}$ In this approach, below a critical impurity level known as the percolation threshold, the system contains one infinitely large cluster and smaller isolated clusters. Above this level, only isolated clusters remain. Therefore, the properties of these systems are different below and above the percolation threshold. For example, on a square, this threshold is $40.7 \%{ }^{20}$ The situation is different in one-dimensional systems such as spin chains and spin ladders. The percolation threshold is essentially zero as any finite level of impurities will break the system into isolated clusters. In a two-leg spin ladder, this occurs when two non-magnetic impurities neighbor each other, cutting the local ladder interactions.

The size of the clusters in spin ladders with non-magnetic impurities is still determined by percolation theory. We can understand the observed changes in the properties of $\mathrm{Ba}_{2} \mathrm{CuTe}_{1-x} \mathrm{Zn}_{x} \mathrm{O}_{6}$ by considering how the cluster size changes with increasing $x$. The cluster size distribution intuitively depends on the impurity concentration $x$ and is approximated as a geometric distribution:

$$
\begin{equation*}
\rho(l)=\zeta(1-\zeta)^{l-1} \tag{8}
\end{equation*}
$$

$\rho(l)$ is the probability of finding a cluster of $l$ sites, which are mainly comprised of $\mathrm{Cu}^{2+} S=1 / 2$ spins as well as potential isolated $\mathrm{Zn}^{2+}$ impurities that do not break the ladder interactions. $\zeta$ is the probability of breaking the ladder and is given by eq 9 , where $x$ is the impurity concentration that has a value between 0 and 1 .

$$
\begin{equation*}
\zeta=\frac{1}{2}[1+x-(1-x) \sqrt{1+4 x(1-x)}] \tag{9}
\end{equation*}
$$

The average cluster size $(\bar{l})$ is the expected value of the geometric distribution

$$
\begin{equation*}
\bar{l}=\frac{1}{\zeta} \tag{10}
\end{equation*}
$$

Any value of $x$ leads to segmentation of the ladder into clusters; however, below a certain critical value $\left(x_{c}\right)$, the clusters are large enough to form long-range magnetic order. This agrees well with the result for $x=0.1$. Using eqs 9 and 10 , the average cluster size for $x=0.1$ is calculated as $\bar{l}=40$ sites. There are clear oscillations in the $\mu$ SR data below $T_{\mathrm{N}}=8 \mathrm{~K}$, showing that long-range order is retained. TF- $\mu \mathrm{SR}$ measurements in Figure 7 show that the transition is gradual. This can be explained by the distribution of cluster sizes, which are ordered at slightly different temperatures. The Weiss constant and position of $T_{\max }$ indicate slight weakening of the magnetic interactions. However, the susceptibility data could still be described using the isolated two-leg spin ladder model, where the reduced $J_{\text {rung }} / J_{\text {leg }}$ ratio also supports the weakening of the ladder interactions. There are remnants of the $T_{\min }$ feature from the "kink" in the low-temperature data. Also, like $x=0$, the electronic contribution to the heat capacity was found to be near-zero, reflecting insulating behavior. There is some indication that the magnetic structure of $x=0.1$ might be incommensurate. Low-temperature neutron diffraction would determine this, although it would require a high flux instrument given the very weak $\mathrm{Cu}^{2+}$ magnetic scattering. In any case, it is clear that the behavior of $x=0.1$ closely resembles that of $\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$.

Curie-Weiss fitting shows further weakening of the interactions as the $\mathrm{Zn}^{2+}$ content increases. The $T_{\max }$ feature is suppressed, and the $T_{\min }$ upturn transitions into a large paramagnetic tail, suggesting the generation of "free" spins as $\mathrm{Zn}^{2+}$ breaks the $\mathrm{Cu}^{2+}$ interactions. Above $x>0.1$, the two-leg spin ladder model began to deviate as the $T_{\max }$ was suppressed. This indicates that the $\mathrm{Zn}^{2+}$ concentration has exceeded the critical value beyond which the cluster size is too small to facilitate long-range magnetic order. For $x=0.2$, the expected cluster size is only $\bar{l}=11$. There was no evidence of long-range order in the $\mu \mathrm{SR}$ data. Instead, the ZF- $\mu \mathrm{SR}$ measurements indicate frozen spins from the $1 / 3$ recovery of the initial muon polarization below 4 K . This likely represents the formation of a long-range disordered static state, wherein the spins within clusters are statically ordered, but between clusters, the $\mathrm{Cu}^{2+}$ spins are long-range disordered. Similar to $x=0.1$, the transition is gradual, reflecting the freezing of the different cluster sizes. However, the relaxation curve is not typical of static order, with a strong relaxing component at short times indicating that there is also a dynamic component to the muon relaxation. TF- $\mu$ SR measurements suggest a small $\sim 6 \%$ fraction of dynamic electronic spins at 1.5 K . Decoupling of the ZF muon relaxation occurred at slightly weaker LF fields above 50 G compared to the $>100 \mathrm{G}$ required for $x=0.1$, implying that dynamic electronic spins are present. The dynamic fraction arises from the portion of small clusters in the distribution in which there is too few spins to freeze.

The behavior further changes between $x=0.2$ and $x=0.3$. The heat capacity data of $x=0.3$ shows an upturn that is not present in the $x \leq 0.2$ data. There are a variety of plausible explanations for the low-temperature upturn, e.g., magnetic defects, spin fluctuations, or weak ferromagnetism. ${ }^{42,43}$ The $T_{\max }$ feature is suppressed in the $\chi$ vs $T$ curve and can no longer be described by the two-leg spin ladder model. At 1.5 K , the ZF muon relaxation is mostly dynamic with only a small frozen fraction of spins $(\sim 14 \%)$. The expected cluster size is $\bar{l}=6$ for $x=0.3$; therefore, the small frozen fraction is likely to represent freezing of the small portion of large clusters in the distribution. Helium dilution fridge experiments would reveal
whether this frozen fraction increases below 1.5 K . The LF$\mu \mathrm{SR}$ data supports dynamic behavior, showing a gradual repolarization of the muon spins as the LF field increased. Therefore, as the average cluster size further decreases from $\bar{l}=$ 6 to $\bar{l}=4$ between $x=0.3$ and $x=0.4$, the system approaches a purely paramagnetic state. This leads to a Curie-like magnetic susceptibility for $x \geq 0.3$, in which there is no $T_{\max }$ feature.

## 5. CONCLUSIONS

$\mathrm{Ba}_{2} \mathrm{CuTeO}_{6}$ has been shown to be a versatile structure, accommodating chemical substitution at both the magnetic $\mathrm{Cu}^{2+}$ site and non-magnetic $B^{\prime \prime}$ sites. Non-magnetic $\mathrm{Zn}^{2+}$ substitution at the $\mathrm{Cu}^{2+}$ site produced a $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ solid solution ( $0 \leq x \leq 0.6$ ). The results can be understood from the viewpoint of the percolation theory, whereby the $\mathrm{Zn}^{2+}$ impurities segmented the two-leg spin ladder into clusters. We observe three distinct types of behavior depending on the cluster size. For $x=0.1$, the cluster size was large enough that long-range magnetic order was retained and the magnetic properties were similar to $x=0$. As the cluster size is further reduced, the critical cluster size for long-range order is exceeded and a long-range disordered static state is proposed for $x=0.2$. The behavior changes further between $x=0.2$ and $x=0.3$. Dynamic muon behavior was observed for $x=0.3$, indicating a mostly paramagnetic state as the cluster size is too small to facilitate ordering or spin-freezing. This makes $\mathrm{Ba}_{2} \mathrm{Cu}_{1-x} \mathrm{Zn}_{x} \mathrm{TeO}_{6}$ an excellent model for studying nonmagnetic impurities in two-leg spin ladders as the structural disorder (apart from that introduced by $\mathrm{Zn}^{2+}$ ) is low and the changes in magnetic behavior closely follow that expected for the percolation of a two-leg spin ladder.

## ASSOCIATED CONTENT

## (s) Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.chemmater.2c02939.

Curie-Weiss fitting, imaginary AC susceptibility, analysis of the muon spin relaxation data, QMC isolated two-leg spin ladder model, and average cluster size for nonmagnetic dilution of a spin ladder (PDF)
CIF file for $\mathrm{Ba}_{2} \mathrm{Cu}_{0.9} \mathrm{Zn}_{0.1} \mathrm{TeO}_{6}$ (CIF)

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## Notes

The authors declare no competing financial interest.

## - ACKNOWLEDGMENTS

E.J.C., O.M., and C.P. acknowledge financial support from the Leverhulme Trust Research Project Grant No. RPG-2017-109. O.M. is grateful for funding via the Leverhulme Trust Early Career Fellowship ECF-2021-170. A.S.G. acknowledges funding through an EPSRC Early Career Fellowship EP/ T011130/1. A.S.G. and H.T. acknowledge funding through the Humboldt Foundation and the Max Planck Institute for Solid State Research. Sabine Prill-Diemer is gratefully acknowledged for sample preparation and characterization. The authors thank the Science and Technology Facilities Council for beamtime allocated at ISIS through proposal RB1990046 (DOI: 10. 5286/ISIS.E.RB1990046) and the Swiss Muon Source at the Paul Scherrer Institute through proposal numbers 20150959 and 20211440. The authors are grateful for access to the MPMS3 instrument at The Royce Discovery Centre at the University of Sheffield (EPSRC grant no. EP/R00661X/1) and the PPMS instrument at the University of St. Andrews (EPSRC grant no. EP/T031441/1).

## ■ ABBREVIATIONS

QCP, quantum critical point; $\mu \mathrm{SR}$, muon spin relaxation; QMC, Quantum Monte Carlo.

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[^0]:    Received: September 26, 2022
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