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Systematic effects in the search for a muon electric dipole moment using the frozen-spin technique

Chavdar Dutsov,[a,*] **Massimo Giovanozzi,**[b] **Timothy Hume**[a,c]
and Philipp Schmidt-Wellenburg[a] **on behalf of the muonEDM collaboration**

[a]*Paul Scherrer Institute,*
 Forschungsstrasse 111, 5232 Villigen, Switzerland

[b]*CERN Beams Department,*
 Esplanade des Particules 1, 1211 Meyrin, Switzerland

[c]*ETH Zürich,*
 8092 Zürich, Switzerland

 E-mail: chavdar.dutsov@psi.ch

ABSTRACT: At the Paul Scherrer Institute we are developing a high precision instrument to measure the electric dipole moment (EDM) of the muon. The salient feature of the experiment is the use of the frozen-spin technique to suppress the anomalous precession of the muon spin, allowing for a sensitivity that cannot be achieved with conventional g-2 muon storage rings. With this technique, the expected statistical sensitivity for the EDM after one year of data taking is $6 \times 10^{-23}\,\mathrm{e \cdot cm}$ with a $p = 125\,\mathrm{MeV/c}$ muon beam available at PSI.

Reaching this goal necessitates a comprehensive analysis on spurious effects that mimic the EDM signal. This work discusses a quantitative approach to study systematic effects for the frozen-spin technique when searching for the muon EDM with a focus on the kinematics of decay positrons in the context of this experiment, which operates at muon momenta well below the ultrarelativistic limit.

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*Corresponding author.

Contents

1 Introduction

In an EDM experiment the signal of interest is the precession of the particle's spin around the electric-field vector. For muons, this can be observed from the direction of the emitted decay positrons, which is correlated to the spin phase. Systematic effects leading to a false EDM signal can be categorized into real and apparent spin-precession effects: the former from actual precession due to the magnetic dipole moment coupling to the electromagnetic field, and the latter from artifacts of positron detection causing misinterpretation of the spin phase. Here we show the initial steps in the study of apparent spin-precession systematic effects by exploring the kinematics of decay positrons. The study is rooted in the context of ongoing experimental efforts, preparing a search for the muon EDM at the Paul Scherrer Institute (PSI) [1].

Similarly to the anomalous magnetic moment $a = (g - 2)/2$ one can define a parameter η that characterizes the EDM. The sensitivity of an oscillating signal to η is [2]:

$$\sigma_\eta = \frac{\sqrt{2}}{\gamma\tau(e/m)EAP\sqrt{N}}, \tag{1.1}$$

where P is the initial polarization of the muon beam ($P \geq 95\%$ at the πE1 beamline at PSI). E is the electric field in the muon rest frame, where, denoting the field intensities in the laboratory reference frame with primes, $E = E' + \gamma c|\vec{\beta} \times \vec{B}'|$. The total number of detected positrons is N, and τ is the muon lifetime at rest. The parameter A is the longitudinal asymmetry along $\vec{B}'$ of the detected positrons for 100% muon beam polarization. Due to parity violation in the weak interaction, decay positrons with high energies are more likely to be emitted in the direction of the spin, while at low energies, they tend to be emitted opposite to the spin. This allows the spin phase to be inferred at the time of the muon decay, the key observable in $g - 2$ and EDM experiments. The average asymmetry over the whole range of decay positron energies is $1/3$.

Our work explores how an optimal energy cut-off could be defined to enhance the average asymmetry A and at the same time keeping the number of detected positrons N high. We accomplish this by studying the decay positron kinematics. The insights and derivations from this analysis can further our understanding of potential systematic effects associated with apparent spin-precession.

2 Michel decay positron kinematics in a boosted reference frame

In the rest frame of the muon the angular distribution of Michel decay positrons is given by [3]:

$$W(x, \cos\alpha)dx\, d\cos\alpha = x^2\left((3-2x)+(2x-1)\cos\alpha\right)dx\, d\cos\alpha, \tag{2.1}$$

where $x = E/E_{\max}$, and α is the angle between the momenta of the muon and decay positron. In the boosted (laboratory) reference frame the energy of the positron is given by:

$$E'_y = \gamma(\cos\alpha + \beta)E, \quad E'_z = E\sin\alpha \quad \text{and} \quad E' = \sqrt{(E'_y)^2 + (E'_z)^2}.$$

This is valid under the assumption that the rest mass of the positron is much smaller than its kinetic energy. Here, the boost and the direction of the spin are aligned along the x-axis. The direction of the spin is also along the x-axis. To allow for arbitrary spin directions we can substitute $\cos\alpha \to \cos\alpha\sin\phi\sin\theta + \sin\alpha\cos\theta$, where ϕ is the azimuthal angle and θ the polar angle (indicated in figure 1) of the spin with respect to the momentum. The reference frame is chosen such that the z-axis coincides with the magnetic field.

We define the Lorentz boosted variables u and v:

$$u = \frac{E'}{E'_{\max}} = x\frac{\sqrt{\gamma^2(y+\beta)^2 + 1 - y^2}}{\gamma(1+\beta)}, \quad v = \cos\alpha' = \cos\left(\arctan\frac{E'_x}{E'_z}\right) = \frac{1}{\sqrt{(E'_x/E'_z)^2 + 1}}. \tag{2.2}$$

We then express the non-boosted variables in terms of the boosted ones:

$$x(u,v) = u\frac{v\beta - 1}{\beta - 1}, \quad y(v) = \cos\alpha = \frac{\beta - v}{v\beta - 1} \quad \text{and the Jacobian} \quad \frac{\delta x}{\delta u}\frac{\delta y}{\delta v} = \frac{1+\beta}{v\beta - 1}. \tag{2.3}$$

The boosted angular and energy distribution is then:

$$\int_0^1 \int_{-v_{\lim}}^{v_{\lim}} W'(u,v)\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}dv\, du = \int_0^1 \int_{-1}^1 W(x,y)dy\, dx, \quad \text{where } v_{\lim} = \frac{u+\beta - 1}{u\beta}. \tag{2.4}$$

The energy spectrum N can be obtained by integrating over all kinematically allowed angles:

$$N(u) = \int_{-v_{\lim}}^{v_{\lim}} W'(u,v)\frac{\partial x}{\partial u}\frac{\partial y}{\partial v}dv, \quad \text{and its mean over } \phi \text{ as } \tilde{N}(u) = \frac{1}{2\pi}\int_{-\pi}^{\pi} N(u,\phi,\theta)d\phi. \tag{2.5}$$

The energy spectrum in the case of the Phase I muonEDM experiment ($\beta = 0.256$) is shown in figure 1 for spin parallel to the momentum $N(\theta = \frac{\pi}{2}, \phi = \frac{\pi}{2})$ and anti-parallel $N(\theta = \frac{\pi}{2}, \phi = -\frac{\pi}{2})$.

In the quest to improve the sensitivity of the search for a muon EDM, we delve into an analysis of asymmetries in muon decay. Two types of asymmetries are particularly interesting in this context: directional and intensity asymmetries. Directional asymmetry A_d refers to the asymmetry in positrons emitted on either side with respect to the boost, i.e., between positrons emitted in positive or negative z (or x), and is:

$$A_d(u)du = \frac{1}{\tilde{N}(u)}\int_{-\pi/2}^{\pi/2}\left(N(u,\phi,\theta) - N(u,\phi+\pi,\theta)\right)du\, d\theta, \quad \text{where } \phi = \frac{\pi}{2}. \tag{2.6}$$
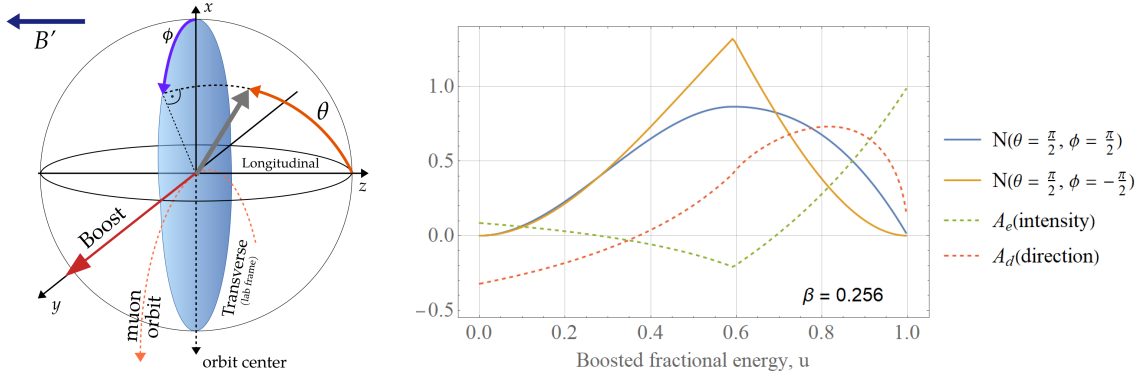
Figure 1. Left: polar angle θ and azimuthal angle ϕ of the spin (grey arrow) with respect to B'. Right: directional and intensity asymmetries in the case of $p = 28\,\text{MeV/c}$ muons.

On the other hand, the intensity asymmetry is the asymmetry with respect to the longitudinal axis, i.e., between positrons emitted along or opposite the boost:

$$A_{\text{e}}(u)du = \frac{1}{\tilde{N}(u)} \int_0^\pi \Big(N(u, \phi, \theta) - N(u, \phi + \pi, \theta) \Big) du\, d\phi, \text{ where } \theta = \frac{\pi}{2}. \tag{2.7}$$

The intensity asymmetry effectively gives the change of the energy spectrum as a function of ϕ, with the two extreme cases being where the spin is parallel or anti-parallel to the momentum, or $\phi = \pm \pi/2$ at $\theta = \pi/2$ (see figure 1, right). In the limiting case of $\beta \to 1$, eq. (2.7) gives the same results for the asymmetry as used in the FNAL $g - 2$ experiment $A_e(u)du = (-8u^2 + u + 1)/(4u^2 - 5u - 5)$ [4].

The distinction between these two types of asymmetries, (2.6) and (2.7), is of significant consequence not only in experiments involving moderately boosted muons, such as the PSI experiment, but also for those with highly boosted muons such as the FNAL $g - 2$ experiment or the JPARC $g - 2$/EDM. In the rest frame, the only observable asymmetry is directional, with respect to the spin phase and momentum. For highly boosted muons the positrons are emitted in a narrow cone around the momentum and in practice only the intensity asymmetry remains relevant. Nevertheless, the directional asymmetry remains the EDM signal also in the FNAL and JPARC experiments.

The PSI muon EDM experiment operates in a region between these two extremes. This allows the simultaneous examination of both types of asymmetries, giving complementary information about the muon spin at the time of decay, and potentially restoring the theoretical sensitivity in cases where one method would be insensitive due to detector acceptance.

The zero-crossing point of the directional asymmetry,[1] denoted as u_{thr}, is defined by the equation $u_{\text{thr}} = (1 - \beta)/2$. For the Phase I experiment, where $E_{\text{max}} = 68.9\,\text{MeV/c}^2$, the zero-crossing is $u_{\text{thr}} = 0.372$. Above this energy threshold, the average asymmetry, $\tilde{A}_{\text{d}} = \frac{1}{1 - u_{\text{thr}}} \int_{u_{\text{thr}}}^1 A_{\text{d}}(u)du$, is found to be 0.462, with the fraction of positrons above this energy being 0.82, i.e., looking only at positrons above the threshold energy results in 54% sensitivity gain due to the decay asymmetry and only 9.4% sensitivity loss due to decrease in the number of detections.

Taken together, the careful analysis of these asymmetries can further our sensitivity to the muon EDM and significantly contribute to the ongoing efforts in the study of apparent spin-precession related systematic effects.

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[1]That would be the energy of the decay positrons where directional information cannot be extracted.

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