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Evidence for current suppression in superconductor–superconductor bilayers

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Abstract

Superconducting radio frequency (SRF) cavities, which are critical components in many particle accelerators, need to be operated in the Meissner state to avoid strong dissipation from magnetic vortices. For a defect-free superconductor, the maximum attainable magnetic field for operation is set by the superheating field, B_{sh} , which directly depends on the surface current. In heterostructures composed of different superconductors, the current in each layer depends not only on the properties of the individual material, but also on the electromagnetic response of the adjacent layers through boundary conditions at the interfaces. Three prototypical bilayers $[Nb_{1-x}Ti_xN$ (50 nm)/Nb, $Nb_{1-x}Ti_xN$ (80 nm)/Nb, and, $Nb_{1-x}Ti_xN$ (160 nm)/Nb] are investigated here by depth-resolved measurements of their Meissner screening profiles using low energy muon spin rotation (LE- μ SR). From fits to a model based on London theory (with appropriate boundary and continuity conditions), a magnetic penetration depth for the thin $Nb_{1-x}Ti_xN$ layers of $\lambda_{Nb_{1-x}Ti_xN} = 182.5(31)$ nm is found, in good agreement with literature values for the bulk alloy. Using the measured $\lambda_{Nb_{1-x}Ti_xN}$, the maximum vortex-free field, B_{max} , of the superconductor-superconductor (SS) bilayer structure was estimated to be 610(40) mT. The strong suppression of the surface current in the $Nb_{1-x}Ti_xN$ layer suggests an optimal thickness of $\sim 1.4\lambda_{Nb_{1-x}Ti_xN} = 261(14)$ nm.

Keywords: penetration depth, Meissner effect, muon spin relaxation & rotation, type II superconductors, current suppression, superconductor-superconductor (SS), $Nb_{1-x}Ti_xNb/Nb$

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1. Introduction

A large accelerating gradient (E_{acc}) (energy gain per unit length) is required for high energy accelerators to limit their length and therefore their cost [1, 2]. Currently, the highest E_{acc} values are achieved using normal conducting radio frequency (RF) cavities, some exceeding 100 MV m^{-1} [3, 4]. In the case of field-emission-free superconducting radio frequency (SRF) cavities, the maximum E_{acc} is proportional to the highest sustainable vortex-free surface magnetic field, which is presently achieved by cavities made from niobium sheets. Some of these cavities have produced E_{acc} values as high as $\approx 49 \text{ MV m}^{-1}$ [5], corresponding to surface magnetic fields on the order of $\sim 210 \text{ mT}$, exceeding “clean” Nb’s lower critical field, $B_{\text{c1}} \approx 170 \text{ mT}$ at 2 K [6, 7]. While this achievement is commendable, it remains below the ultimate limit for bulk Nb, which is set by its superheating field, $B_{\text{sh}} \approx 240 \text{ mT}$ [8]. While Nb cavity operating conditions continue to approach this material limit, substantial advances in accelerator technology necessitate finding alternative materials.

1.1. SRF materials beyond niobium

One possibility to achieve surface magnetic fields beyond B_{sh} of Nb is to use a different superconducting material with a greater B_{sh} (e.g., Nb_3Sn or $\text{Nb}_{1-x}\text{Ti}_x\text{N}$) [9]; however, there is no viable replacement with a B_{c1} exceeding that of Nb. This is problematic, as all *real* SRF cavities possess both surface defects and topographic imperfections, facilitating vortex penetration below B_{sh} . Vortices that penetrate at these “weak points” often evolve into a thermomagnetic flux avalanche, quenching superconductivity at SRF cavity operating temperatures ($T \lesssim 4 \text{ K}$) [10–12].

To overcome this, a different approach has been proposed, wherein superconducting *multilayers* are used to push the field of first-flux penetration beyond Nb’s intrinsic B_{sh} (see e.g., [10, 12–14]). Gurevich [13] was the first to suggest the use of multilayer structures as a means of preventing thermomagnetic avalanches induced by vortex penetration at defects before they become predominant. The approach is to coat a conventional Nb cavity with several thin superconducting and insulating layers, the simplest version of which is one superconducting and one insulating layer on Nb, referred to as a superconductor-insulator-superconductor (SIS) structure. The insulating layer decouples the superconducting layers and if the layers are thinner than the London penetration depth (λ_{L}) of their material, nucleation of parallel vortices will only become energetically favorable at larger fields than B_{c1} of layer material. Kubo [10] suggested that a simpler structure containing only a single superconducting layer with a larger penetration depth on top of a Nb cavity can also increase the field of first vortex penetration (B_{vp}) due to the presence of an energy barrier at the superconductor–superconductor (SS) interface analogous to the vacuum-superconductor interface (i.e., the Bean–Livingston (BL) barrier [15]). Experimental evidence for this interface barrier has been reported in [16].

In summary, the maximum field in superconducting heterostructures that can be sustained while remaining in the Meissner state (B_{max}) depends on the thickness and superconducting properties of all individual layers in a correlated way. This is a direct consequence of Maxwell’s equations with continuity conditions enforced at interface boundaries [10].

1.2. Magnetic screening and current in superconducting heterostructures

Recall that, for a bulk superconductor in the “local” London limit (see e.g., [17]) with an ideal flat surface, the Meissner screening profile, $B(z)$, is given by [18]:

$$B(z) = B_0 \times \begin{cases} 1, & z < 0, \\ \exp\left(-\frac{z}{\lambda_{\text{L}}}\right), & z \geq 0, \end{cases} \quad (1)$$

where B_0 is the (effective) applied magnetic field, z is the depth below the superconductor’s surface, and λ_{L} is the London penetration depth. Equation (1) is well-known for its applicability to semi-infinite superconductors; however we are interested in SS bilayers comprised of dissimilar layers whose materials have different screening properties (i.e., $\lambda_{\text{L}s}$). Considering a naive exponential London decay in each component of the SS bilayer by treating the screening properties independently, the field screening profile is given by:

$$B(z) = B_0 \times \begin{cases} 1, & z < 0, \\ \exp\left(-\frac{z}{\lambda_{\text{s}}}\right), & 0 \leq z < d_{\text{s}}, \\ \exp\left(-\frac{d_{\text{s}}}{\lambda_{\text{s}}}\right) \exp\left(-\frac{z-d_{\text{s}}}{\lambda_{\text{sub}}}\right), & z \geq d_{\text{s}}, \end{cases} \quad (2)$$

where d_{s} is the thickness of the top superconducting layer, and the λ_i denote the penetration depth in the surface ($i = \text{s}$) and substrate ($i = \text{sub}$) layers, respectively. While equation (2) is both conceptually simple and qualitatively correct in its form, it does not consider any “coupling” between the adjacent layers. Notably, the substrate layer significantly influences the screening properties of the surface layer superconductor when their penetration depths differ. This occurs because an SS bilayer’s electromagnetic (EM) response depends on satisfying the boundary and continuity criteria for both the magnetic field and vector potential. Recently, it has been predicted that this coupling depends also on the surface layer’s thickness and is most effective when $d_{\text{s}} \sim \lambda_{\text{s}}$ [14]. For example, when the surface layer penetration depth is larger than the substrate’s (i.e., $\lambda_{\text{s}} > \lambda_{\text{sub}}$), the Meissner current in the surface layer is suppressed by the substrate layer’s counter-current (i.e., a counterflow current generated by the substrate in a multilayer superconductor [10–12, 19]) to satisfy the boundary and continuity condition at the interface. This results in a higher B -field for vortex entry in the outer layer with a correspondingly a reduced shielding of the substrate (higher field at

the substrate interface). This effect is expected for all superconducting heterostructures with and without insulating interlayers. Quantitatively, the field screening considering counter-current-flow induced by the substrate is derived by solving the relation between the applied field, B_0 and current density, J (or equivalently vector potential, A). For a SS structure this yields [10, 12, 14, 19]:

$$B(z) = B_0 \times \begin{cases} 1, & z \leq 0, \\ D_{S-S}^{-1} \left[\cosh\left(\frac{d_s - z}{\lambda_s}\right) + \left(\frac{\lambda_{\text{sub}}}{\lambda_s}\right) \sinh\left(\frac{d_s - z}{\lambda_s}\right) \right], & 0 < z \leq d_s, \\ D_{S-S}^{-1} \left[\exp\left(-\frac{z - d_s}{\lambda_{\text{sub}}}\right) \right], & z > d_s, \end{cases} \quad (3)$$

where the symbols have the same meaning as in equation (2), and, the common factor D_{S-S} is given by:

$$D_{S-S} = \cosh\left(\frac{d_s}{\lambda_s}\right) + \left(\frac{\lambda_{\text{sub}}}{\lambda_s}\right) \sinh\left(\frac{d_s}{\lambda_s}\right).$$

The current density distribution, $J(z)$ can be obtained from the field screening profiles using the expression $J(z) = -\frac{1}{\mu_0} \frac{dB(z)}{dz}$. Both equations (2) and (3) are essentially forms of exponential decay; however, the screening behavior is significantly modified in the surface layer. Figure 1 presents a comparison of the magnetic field profiles and current density distributions, with (a) showing normalized field screening behavior and (b) representing normalized current density distributions. In both figures, the solid red curve describes the London screening behavior in the absence of a “coupling” between the superconducting layers (equation (2)), whereas the blue dashed curve corresponds to screening according to Kubo’s counter-current-flow model (equation (3)). Here, the SS bilayer is $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb with assumed penetration depths of $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 200$ nm and $\lambda_{\text{Nb}} = 50$ nm for $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb, respectively. As alluded to above, the two models have qualitatively similar behavior; $B(z)$ ’s decay rate in the Nb substrate is identical, with the two curves differing only in their amplitudes at the SS boundary. This similarity is also observed in the decay rate of $J(z)$ in the Nb substrate. Conversely, a notable difference is apparent in the top $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer, where the decay rate is substantially reduced in Kubo’s model. This is the effect of the reduced current in the surface layer as seen in figure 1(b) due to the counter-current induced by the substrate.

In order to observe the effect of counter-current in SS bilayers, it is necessary to investigate the field screening profiles experimentally and quantify the penetration depths using an appropriate model. Perhaps the best way of achieving this through low-energy muon spin rotation (LE- μ SR) [20–24], which uses implanted positive muons to probe the sub-surface field distributions in the material under investigation. The key feature of this approach is the ability to control the energy at which the muons are implanted, conferring *depth-resolution* to

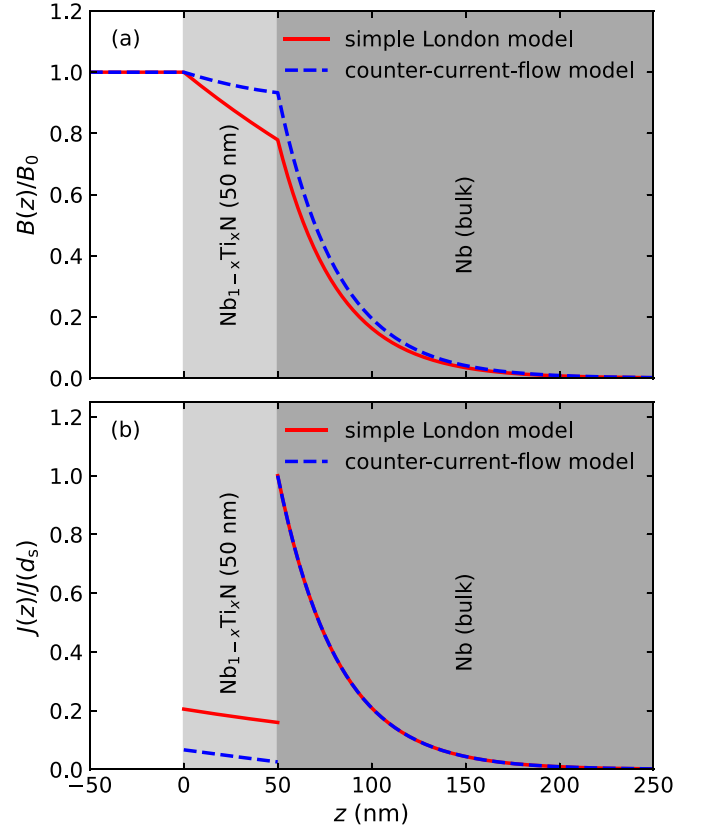


Figure 1. Magnetic field profiles given by equations (2) and (3) in (a) and the current density distributions of those equations normalized to the current density at interface in (b). The used magnetic penetration depths are $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 200$ nm and $\lambda_{\text{Nb}} = 50$ nm for the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb layers, respectively. The thickness of the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer is 50 nm. Comparing the two field profiles in (a), the strongest effect on field screening is observed in the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer due to the suppressed Meissner current in that as seen in (b).

the measurements. This is in contrast to other surface-sensitive techniques (e.g., magnetic force microscopy [25–29], scanning tunnelling microscopy [30, 31], etc), which provide *lateral* resolution across the surface, but lack the depth sensitivity to directly view the Meissner screening profile across a buried interface.

To this end, we measured the Meissner screening profile and observed suppression of screening current in the surface layer in $\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Nb}$ samples with different alloy thicknesses using the LE- μ SR technique. These measurements were conducted under applied fields ($15 \lesssim B_0 \lesssim 25$) mT. By globally fitting the field profiles of all the samples, we quantitatively determined the common magnetic penetration depths of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$, $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$, and Nb, λ_{Nb} . This quantitative assessment involves comparing Kubo’s counter-current-flow model (i.e., London theory with appropriate boundary and continuity conditions) with a simple London model without appropriate boundary conditions. The resultant comparison highlights the significant suppression of the Meissner current

in the surface $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer in $\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Nb}$ samples with film thicknesses shorter but close to the London penetration depth of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$.

2. Experiment

2.1. The LE- μSR technique

The LE- μSR experiments were performed at the Paul Scherrer Institute's (PSI) Swiss Muon Source located in Villigen, Switzerland, using the μE4 beamline [32]. In the beamline a thin film of condensed noble gas [33] is used to reduce the energy of a “surface” muon beam of ~ 4 MeV down to around 15 eV. Following that, the muons are accelerated to create a beam with an adjustable energy $E \leq 30$ keV which corresponds to an implantation depth of $\lesssim 150$ nm in Nb and Nb-based alloys. These low energy positive muons (μ^+) are $\sim 100\%$ spin-polarized. The μ^+ are implanted into a sample one at a time using a (quasi-)continuous beam [34], wherein they quickly thermalize in the target and their spins precess around the local magnetic field at the Larmor frequency, ω_μ permitting depth-resolved measurements of the field screening profile in surface-parallel applied fields up to ~ 30 mT [35].

When a μ^+ decays, it emits a positron preferentially along its spin direction at the moment of decay. The emitted positrons are detected as a function of time in a set of positron detectors symmetrically placed surrounding the sample. This allows for the temporal evolution of the muon's spin orientation to be deduced, and consequently, the properties of the magnetic fields it experiences.

In this experiment, the *asymmetry* of μ^+ decay is determined in a transverse field arrangement wherein a magnetic field is applied perpendicular to the initial direction of muon spin-polarization and parallel to the sample surface. The positron event rate in one (or more) “counters” i , is given by:

$$N_i(t) = N_{0,i} \exp\left(-\frac{t}{\tau_\mu}\right) [1 + A_i(t)] + b_i, \quad (4)$$

where $\tau_\mu = 2.2 \mu\text{s}$ is the muon lifetime, $N_{0,i}$ represents the total number of “good” decay events (i.e., decays from muons stopped in the sample), b_i is the time-independent rate from uncorrelated “background” events, and $A_i(t)$ represents the time-evolution of the muon ensemble asymmetry:

$$A_i(t) = A_{0,i} P(t), \quad (5)$$

where $A_{0,i}$ is the experimental decay asymmetry and $P(t)$ is the polarization of the muon ensemble.

In a transverse-field experiment, the time-evolution of $P(t)$ is given by:

$$P(t) = \int_0^\infty p(B) \cos(\gamma_\mu B t + \phi) dB, \quad (6)$$

where $p(B)$ is the internal magnetic field distribution sensed by the muons, $\gamma_\mu = 2\pi \times 135.54 \text{ MHz T}^{-1}$ is the gyromagnetic ratio of the muon, B is the magnitude of the local magnetic field at the muon site, t is the time after implantation,

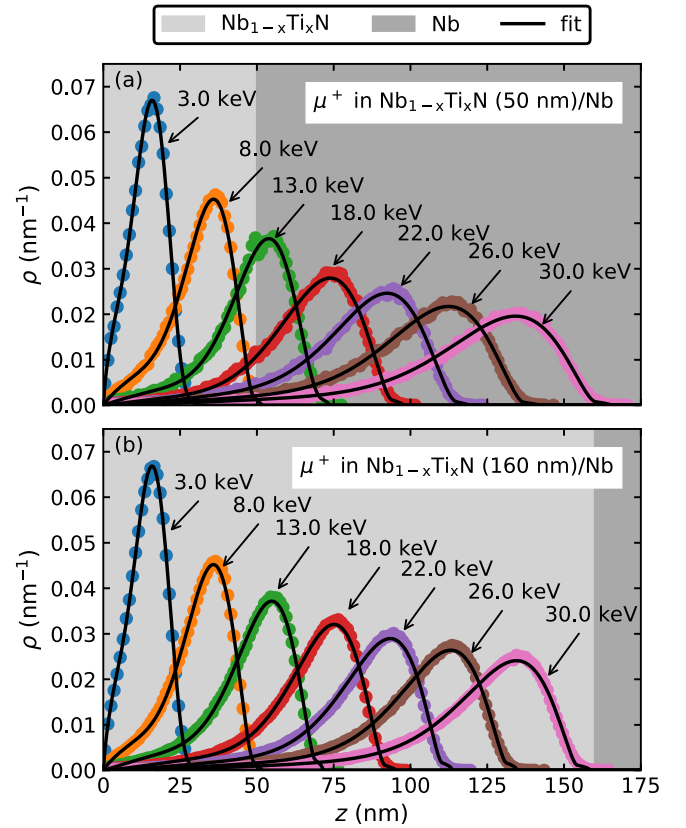


Figure 2. Typical stopping profiles for μ^+ implanted in (a) $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb, and (b) $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (160 nm)/Nb SS bilayer, simulated using the Monte Carlo code TRIM.SP [36]. The densities of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb are 6.6223 g cm^{-3} , and 8.57 g cm^{-3} , respectively. The light gray color in the first 50 nm of figure (a) and 160 nm of figure (b) refers to the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ film thickness on bulk Nb substrate (i.e., dark gray color). The normalized stopping distribution ρ of μ^+ is plotted against the depth z below the surface. The black solid curves are fits to the stopping profile (represented as a histogram) using equations (7) and (8). These fits clearly capture all features of the stopping profiles.

and ϕ is the phase factor (i.e., angle between the initial muon spin-polarization and the effective symmetry axis of a positron detector).

2.2. Muon stopping profiles

As mentioned in section 2.1, LE- μSR has the ability to explore the local field in a depth resolved manner. Muons of a particular energy stop over a specific range distribution when implanted into a sample. In this experiment, a range of implantation energies (~ 2 keV to ~ 30 keV) were used (see figure 2), providing depth-resolution on the nm scale (i.e., ~ 10 nm to ~ 150 nm).

The stopping profile of muons can be accurately simulated [20, 37, 38] using the TRIM.SP code (a Monte Carlo code) [36], which treats all collisions within the target using the binary collision approximation. Simulation results for μ^+ implanted in a $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb, and a

Table 1. Superconducting properties of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ films from several literatures [9, 39–44]. Here, T_c is the critical temperature, B_c is the thermodynamic critical field, B_{c1} is the lower critical field, B_{sh} is the superheating field, B_{c2} is the upper critical field, λ is the penetration depth, and ξ is the BCS [45] coherence length.

Sample	T_c (K)	B_c (mT)	B_{c1} (mT)	B_{sh} (mT)	B_{c2} (mT)	λ (nm)	ξ (nm)	Reference
$\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Nb}$	15.97		35					[39]
$\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Al}_2\text{O}_3$	17.3		30		15 000	150–200		[9]
$\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Al}_2\text{O}_3$	15.8		25	186		208		[40]
$\text{Nb}_{0.62}\text{Ti}_{0.38}\text{N}/\text{Si}$	~ 15.0						2.4(3)	[41]
$\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{MgO}$	~ 15.0							[42]
$\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Al}_2\text{O}_3$	~ 13.1							[43]
$\text{Nb}_{0.62}\text{Ti}_{0.38}\text{N}/\text{Si}$	~ 16.0					200(20)		[44]

$\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (160 nm)/Nb SS bilayer are shown in figure 2, illustrating LE- μSR 's typical range of spatial sensitivity. For our analysis (see section 3), it was convenient to have the ability to describe these profiles at arbitrary E , which can be accomplished by fitting the simulated profiles and interpolating their “shape” parameters [20]. Empirically, we found the μ^+ stopping probability, $\rho(z)$, at a given E can be described by:

$$\rho(z) = \sum_i^m f_i p_i(z), \quad (7)$$

where $p_i(z)$ is a probability density function, $f_i \in [0, 1]$ is the i^{th} stopping fraction, constrained such that

$$\sum_i^m f_i \equiv 1,$$

and z is the depth below the surface. For our SS bilayers, the stopping data are well-described using $m = 2$ and a $p(z)$ is given by a modified beta distribution. Explicitly,

$$p(z) = \begin{cases} 0, & \text{for } z < 0, \\ \frac{(z/z_0)^{\alpha-1} (1 - z/z_0)^{\beta-1}}{z_0 B(\alpha, \beta)}, & \text{for } 0 \leq z \leq z_0, \\ 0, & \text{for } z > z_0, \end{cases} \quad (8)$$

where $z \in [0, z_0]$ is the depth below the surface and $B(\alpha, \beta)$ is the beta function:

$$B(\alpha, \beta) \equiv \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)},$$

with $\Gamma(s)$ denoting the gamma function. Further details of the stopping profile simulation can be found elsewhere [20, 37].

2.3. Sample preparation

In this study, $\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Nb}$ SS bilayer samples were prepared by growing thin films of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ on “bulk” Nb substrates using direct current (DC) magnetron reactive sputtering (R-DCMS) in a vacuum chamber with a base pressure of low 1×10^{-10} mbar. The sputtering target consisted of 80/20 (wt %) Nb/Ti alloy, was used within an Ar and N_2 ($P_{\text{N}_2}/P_{\text{Ar}}$) gas mixture at a pressure of 2×10^{-3} mbar.

Films with nominal thicknesses of 50 nm, 80 nm, and 160 nm were deposited at 450 °C on 3 mm thick bulk Nb substrates, with respective T_c values of 15.8 K, 16.3 K and 16.3 K [46]. Following standard practice for preparing the surface of a Nb SRF cavity, the substrates were prepared by mechanical polishing (MP) followed by 5 μm cold electropolishing (EP) or by 50 μm buffered chemical polishing (BCP) (see e.g., [47]). Specifically, the 50 nm sample was prepared using EP and the others using BCP. Prior to film growth, the substrates were baked at 600 °C for 24 h under vacuum and the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ films were annealed at 450 °C after deposition. The typical surface roughness of the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer is similar to the original substrate roughness (1 nm for MP+ EP substrates and microns for BCP substrates [48]). All film depositions were performed at Thomas Jefferson National Accelerator Facility (JLab) and further details on deposition technique can be found in [39]. Note that, unlike the elemental superconductors, the magnitude of superconducting properties (such as the penetration depth and the coherence length) of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ are not robust. This is due to the fact that $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ is not a “natural” compound [39]. Therefore, the superconducting properties of some $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ films prepared using different target stoichiometries, deposition techniques, and preparation methods have been reviewed from the literature and are listed in table 1 for reference. Although the tabulated values for various samples show considerable variation, the attributes derived from all reviewed research are in fair agreement with one another. These will be used to compare our measured penetration depths in section 3 as well as for the prediction of critical fields in section 4.1.

3. Results

Typical muon spin-precession signals are shown in figure 3(a) for the normal conducting state (20 K) and in figures 3(b)–(f) for the Meissner state (2.7 K) in $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb. In the normal state, there is no substantial energy dependence to the time evolution of the muon ensemble polarization. This means muons implanted at different depths experience almost the same local field. By contrast, in the Meissner state the temporal evolution of $A(t)$ varies as the implantation energy increases, wherein the μ^+ spin-precession rate is greatly reduced, and the signal is more strongly damped at high implantation energies.

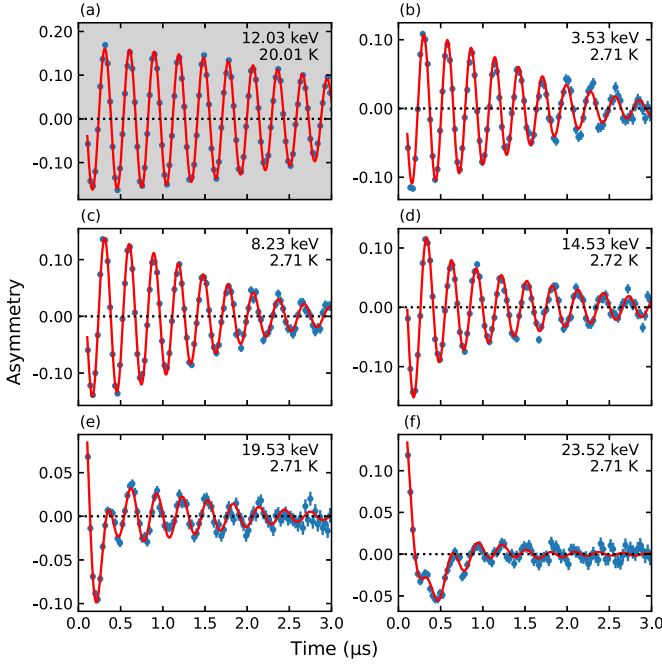


Figure 3. Asymmetry as a function of time for different implantation energies (given in the panel's inset) in $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb in both the normal (20 K) and Meissner state (2.7 K) at an applied magnetic field of ~ 25 mT parallel to the sample surface. In the normal state (gray shaded background panel), there is no substantial energy dependence to the time evolution of the muon ensemble polarization, meaning the implanted muons experience the same local field. By contrast, it is evident that the temporal evolution of $A(t)$ varies in the Meissner state (plain white background panels). As the implantation energy increases, the μ^+ spin-precession frequency is reduced, and the signal is more strongly damped. The solid red lines denote fits to *all* of the data (i.e., a global fit) using equations (11)–(13). Clearly, the model captures all the data's main features.

Figure 4 shows the Fourier amplitude (i.e., $\sqrt{\text{Fourier power}}$ [49]) of the LE- μ SR time spectra depicted in figure 3 in the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb sample as a function of field (note $\omega_\mu = \gamma_\mu B$), in the normal (20 K) and Meissner (2.7 K) state. In the Fourier transform of the data, it is evidenced that a large damping rate in the time domain signal corresponds to a wider distribution of frequencies (i.e., local fields) (see figures 4(b)–(f)). For energies above ~ 14.5 keV, the Fourier spectra show two distinct peaks, implying at least two unique field regions are sensed, consistent with the different materials in the SS bilayer.

The measured internal field distribution, $p(B)$, in the Meissner state depends on energy via the muon implantation depth profile and the magnetic screening due to the Meissner current. We will now consider how to approximate $p(B)$ in equation (6) for our analysis. In the Meissner state, the applied field decays to zero monotonically below the sample surface and the field screening is expected to be intrinsically asymmetric. For the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ /Nb samples, it is found that a sum of two skewed Gaussian (SKG) components (i.e., one

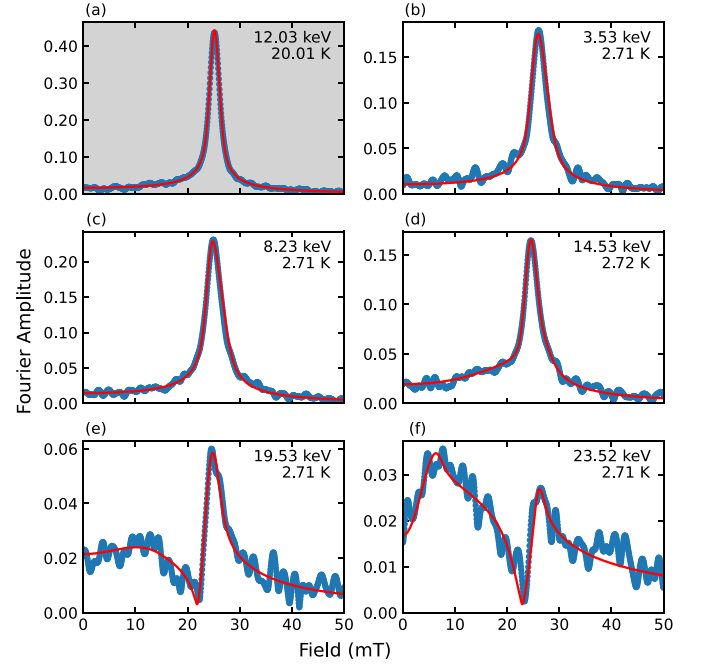


Figure 4. Fourier amplitude of the LE- μ SR data (shown in figure 3) in $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb as a function of field (note $\omega_\mu = \gamma_\mu B$), in the normal (20 K) and Meissner (2.7 K) state with an applied magnetic field of ~ 25 mT. The red lines are skewed Gaussian fits corresponding to the field distribution described by equations (11)–(13). Above ~ 14.5 keV two distinct peaks are observed indicating that muons of a single implantation energy sense the field in both layers of the SS bilayer.

for each material) gives a good fit describing the data in all measurement conditions. Because each layer in the SS has a different screening properties, the SKG distribution function is defined as [50]:

$$P_{\text{SKG}}(B) = \sqrt{\frac{2}{\pi}} \frac{\gamma_\mu}{(\sigma_+ + \sigma_-)} \times \begin{cases} \exp\left[-\frac{1}{2} \frac{(B - B_p)^2}{(\sigma_+/\gamma_\mu)^2}\right], & B \geq B_p, \\ \exp\left[-\frac{1}{2} \frac{(B - B_p)^2}{(\sigma_-/\gamma_\mu)^2}\right], & B < B_p, \end{cases} \quad (9)$$

where B_p is the “peak” field (i.e., *not* the mean) and σ_\pm denotes the distribution's “width” on either side of B_p .

By substituting equation (9) into equation (6) for $p(B)$, the polarization formula can be written as:

$$P_{\text{SKG}}(t) = P_{\text{SKG}}^+(t) + P_{\text{SKG}}^-(t), \quad (10)$$

where

$$P_{\text{SKG}}^\pm(t) = \left(\frac{\sigma_\pm}{\sigma_+ + \sigma_-} \right) \exp\left(-\frac{\sigma_\pm^2 t^2}{2}\right) \left[\cos(\gamma_\mu B_p t + \phi) \mp \sin(\gamma_\mu B_p t + \phi) \text{Erfi}\left(\frac{\sigma_\pm t}{\sqrt{2}}\right) \right], \quad (11)$$

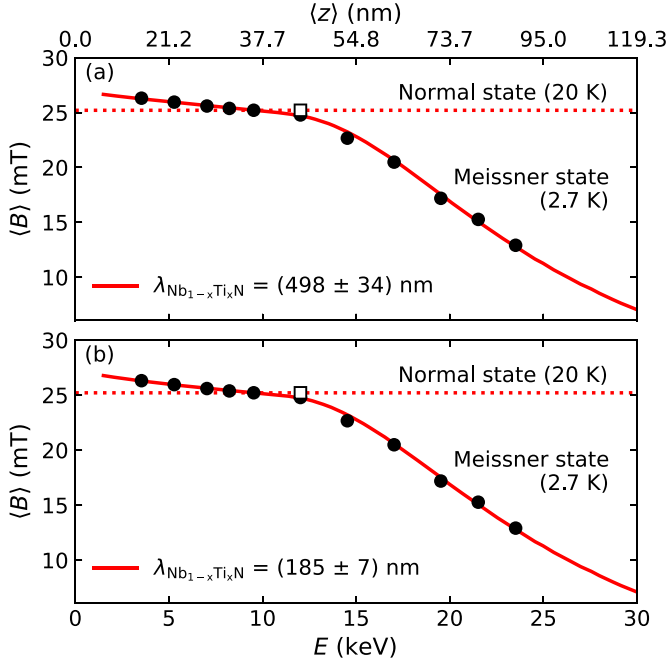


Figure 5. $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb field profile: Plot of the mean magnetic field, $\langle B \rangle$, sensed by μ^+ at different implantation energies, E , in a $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb sample at an applied field ($B_0 \sim 25$ mT) parallel to the sample surface in the Meissner ($T = 2.7$ K) and normal state ($T = 20$ K). The closed circles and open squares are the data points in the Meissner state and normal state, respectively. The implantation energy E is related to the mean implantation depth $\langle z \rangle$ as shown in the top x -axis. The solid red lines are fits to the data in the Meissner state and the dashed red lines are fits to the normal state data. Both figures represent the same data points fitted to different models. In the Meissner state $\langle B \rangle$ decays with increasing E as expected. The fit to figure 5(a) represents the field screening using equation (2) i.e., a simple London model with fit parameters $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 498(34)$ nm and $\lambda_{\text{Nb}} = 42.9(30)$ nm. Figure 5(b) is fitted with the equation (3) which considers counter-current-flow induced by the substrate layer and the extracted fit parameters are $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 185(7)$ nm and $\lambda_{\text{Nb}} = 43.6(29)$ nm.

where $\text{Erfi}(x)$ is the imaginary error function. Therefore, the total asymmetry signal $A(t)$ yields:

$$A(t) = A_0 \sum_{i=1}^n k_i P_{\text{SKG},i}(t), \quad (12)$$

where k reflects the fraction of muons stopping in each component of the SS bilayer, constrained such that

$$\sum_{i=1}^n k_i \equiv 1. \quad (13)$$

To fit the data, the program *musrfit* was used [51]. The red lines in figure 3 are fits to *all* the data (i.e., a global fit) of the 50 nm sample using equations (9)–(13), where the phase, ϕ is shared as a common parameter. The imposition of this restriction is necessary because in situations where $A(t)$ is significantly damped at high implantation energies in the Meissner state, the phase becomes poorly defined, and only a few complete precession periods can be resolved. The fit was con-

strained such that for $E \leq 14.5$ keV (i.e., mean stopping depths ≤ 50 nm) we assumed $n = 1$ in equation (12) and used $n = 2$ at higher implantation energies. This choice gave the best fit to the data at all measurement conditions, as evidenced by the goodness of fit criterion (i.e., reduced- $\chi^2 = 1.06$).

In order to construct the Meissner screening profile in the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb sample, the mean field, $\langle B \rangle$, needs to be derived from $p(B)$ for each implantation energy E . The $\langle B \rangle$ is a convenient means of encapsulating the $p(B)$'s shift to lower fields as the E increases. The $\langle B \rangle$ is derived using the fit parameters $B_{p,i}$, $\sigma_{+,i}$, and $\sigma_{-,i}$ (see appendix) of equation (12):

$$\langle B \rangle = \sum_{i=1}^n k_i \left[B_{p,i} + \sqrt{\frac{2}{\pi}} \left(\frac{\sigma_{+,i} - \sigma_{-,i}}{\gamma_\mu} \right) \right]. \quad (14)$$

The field screening profile of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb at an applied field of $B_0 \sim 25$ mT as a function of energy E (bottom scale) and corresponding mean implantation depth $\langle z \rangle$ (top scale) in the Meissner ($T = 2.7$ K) and normal state ($T = 20$ K) is shown in figure 5. The closed circles and open squares in figure 5(a) and (b) represent the mean field $\langle B \rangle$ of the same data. In the normal state the $\langle B \rangle$ is not screened and in the Meissner state $\langle B \rangle$ decays with increasing E as expected.

In order to fit $\langle B \rangle$, we shall consider a model that describes all essential features of the data. In equations (6) and (9)–(14), $\langle B \rangle$ is derived by fitting a field distribution $p(B)$ at a given energy E . At specific E , muons sample over a range of depths (i.e., distribution) which is simulated and quantified by $\rho(z)$ as discussed in section 2.2. The quantities $\rho(z)$ and $p(B)$ are both energy dependent. Hence, $\langle B \rangle$ depends on the Meissner screening and the μ^+ implantation distribution $\rho(z)$. The mean field $\langle B \rangle$ as a function of E is therefore:

$$\langle B \rangle(E) = \int_0^\infty B(z) \rho(E, z) dz, \quad (15)$$

where the dependence on E is accounted for *implicitly* by $\rho(E, z)$ which is predetermined from fits to simulated implantation profiles (see figure 2). The screening profile $B(z)$ is derived from equations (2) and (3). Note that the applied magnetic field, B_{applied} in both equations (2) and (3) is enhanced in the Meissner state due to the sample geometry, which needs to be accounted. This is done by using:

$$B_0 = B_{\text{applied}} \times \frac{1}{(1 - N)}, \quad (16)$$

where the demagnetization factor N depends on the geometry of the sample [52–54]. To compare our measured penetration depths with literature values (see table 1), the T dependence of λ was assumed to follow the phenomenological power law [55]:

$$\lambda(T) = \frac{\lambda(0)}{\left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{1/2}}, \quad (17)$$

where $\lambda(0)$ is the magnetic penetration depth at 0 K.

Table 2. Fit results of the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb bilayer with a counter-current-flow (i.e., equation (3)) and a naive bi-exponential model (i.e., equation (2)). Here, B_{applied} is the applied magnetic field, N is the demagnetization factor, $d_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ is the thickness of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer, and $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$, λ_{Nb} are the penetration depths of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb at 0 K.

Model	B_{applied} (mT)	N	$d_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ (nm)	$\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ (nm)	λ_{Nb} (nm)
Counter-current-flow	25.209(31)	0.079(5)	58.5(13)	185(7)	43.6(29)
Simple London	25.209(31)	0.070(4)	59.0(15)	498(34)	42.9(30)

The fits to the normal state data in figure 5 are represented by dashed red curves. The solid red curves denote fits to the Meissner state data using equation (15) and one of the screening models introduced in section 1.2 (i.e., counter-current-flow or simple London model). Figure 5(a) is fitted with a simple London model (equation (2)), and the counter-current-flow model field distribution (equation (3)) is used to fit the data in figure 5(b). It can be seen that both models capture all physically meaningful details of the data and give excellent fits. The fit parameters for both models are tabulated in table 2. The values of the extracted parameters N , λ_{Nb} , and $d_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ are almost identical in the two models. However, a large discrepancy exists between the determined values of $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$. The simple London model gives $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 498(34)$ nm, while Kubo's counter-current-flow model gives $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 185(7)$ nm. Interestingly, the value determined using Kubo's counter-current-flow model (equation (3)) is in good agreement with literature estimates (see table 1), whereas the expression in equation (2) overestimates $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ by a factor of ~ 2.5 . This observation strongly supports the predictions of the counter-current-flow theory [10] and suggests that equation (2) is not appropriate for quantifying $B(z)$ in superconducting heterostructures.

To be more conclusive about this observation, we measured the field screening profile in three samples with different $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ thicknesses (50 nm, 80 nm, and 160 nm) deposited on Nb substrates, see figure 6. Using the counter-current-flow model, the field screening profiles were fitted simultaneously (i.e., global fit) with the penetration depth values at 0 K of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb as shared fit parameters, this is justified by the fact that when the profiles for each sample were fit separately, identical λ values were obtained. Other fit parameters were the thickness of each film and individual demagnetization factors. The thickness of the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (160 nm)/Nb sample cannot be determined from the fit as all muons are stopped in the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer, see figure 2(b). This parameter was therefore directly measured using transmission electron microscopy (TEM) and found to be 168 nm [56].

The best fit parameters were determined to be: $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}(0 \text{ K}) = 182.5(31)$ nm (using T_c of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ mentioned in section 2.3) and $\lambda_{\text{Nb}}(0 \text{ K}) = 43.3(19)$ nm (using $T_c = 9.25$ K for Nb [7]). All fit parameters can be found in table 3. Although the magnetic screening is very different for each sample, the fact that the global fit gives excellent agreement with the entire data, with the penetration depths of the layer and the substrate as common fit parameters, further confirms the applicability of the counter-current-flow model to the data.

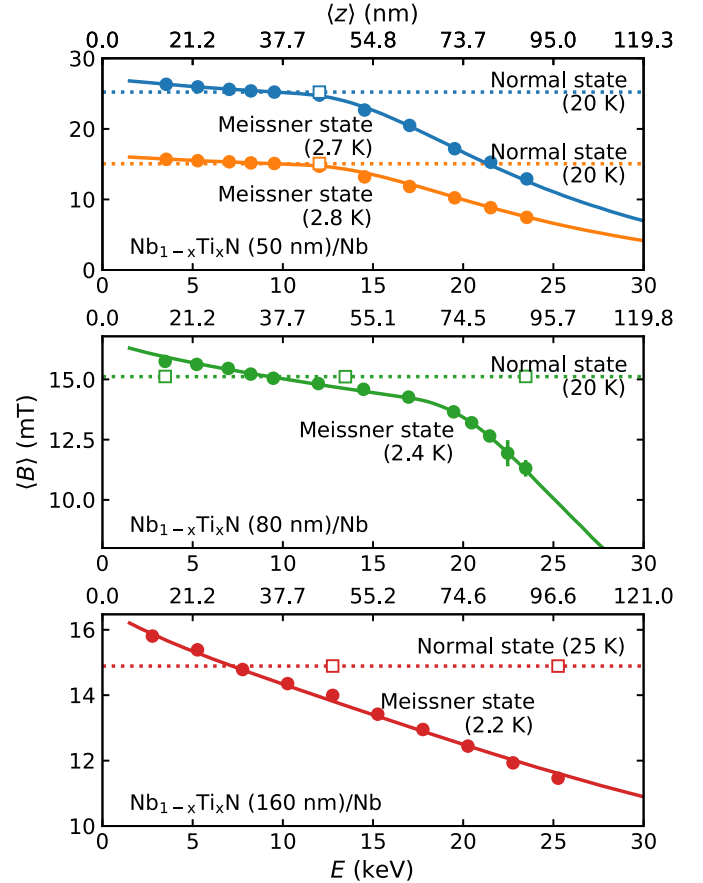


Figure 6. Plot of the mean magnetic field, $\langle B \rangle$, sensed by μ^+ at different implantation energies, E in $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ /Nb samples with different $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ thicknesses (i.e., 50 nm, 80 nm, and 160 nm) at applied fields of $(15.0 \lesssim B_0 \lesssim 25.0)$ mT, parallel to the sample surface in the Meissner state ($T \lesssim 2.8$ K) and normal state ($T \gtrsim 20$ K). The mean implantation depth $\langle z \rangle$ corresponding to E of each sample is shown in the top x-axis on each panel. The colored closed circles and open squares are the data derived from the LE- μ SR measurements. The solid and dashed lines represent a (global) fit to the data using equation (15) where $B(z)$ is the field screening formula, i.e., equation (3). In the normal state, there is no energy or depth dependence to $\langle B \rangle$, which represents the strength of the applied magnetic field. However, in the Meissner state, $\langle B \rangle$ decays with increasing E . The apparent difference in $\langle B \rangle$ at $E \sim 0$ keV between the Meissner and normal state is due to the field “enhancement” in the Meissner state. The fit parameters are shown in the table 3.

4. Discussion

From figure 6, it is obvious that in both the 50 nm and 80 nm samples the decay of $B(z)$ is weaker in the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layers, whereafter it is attenuated strongly in the Nb substrate.

Table 3. Individual parameters derived from a global fit to the counter-current-flow model of three $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ samples. The magnetic penetration depths at 0 K of the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer and the Nb substrate were derived as global fit parameters, using the analysis approach described in section 3. Here, B_{applied} is the strength of the magnetic field applied parallel to the sample surface, N is the demagnetization factor, and $d_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ is the thickness of the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer.

Sample	B_{applied} (mT)	N	$d_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ (nm)	$\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ (nm)	λ_{Nb} (nm)
$\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb	15.058(29)	0.0801(22)	57.5(9)	182.5(31)	43.3(19)
	25.214(29)				
$\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (80 nm)/Nb	15.115(20)	0.0977(35)	84.0(9)		
$\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (160 nm)/Nb	14.89(5)	0.115(7)	168 (fixed)		

This bipartite screening represents the presence of two distinct penetration depths (i.e., $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ and λ_{Nb}), each associated with a distinct region in the SS bilayer. This spatially segregated response is directly resolved by the raw LE- μ SR data, as evidenced by the Fourier spectra in figure 4. Note that a low-temperature baked [57] Nb was considered an “effective” bilayer due to the anomalous Meissner screening [58] near the surface, however, more recent analysis evidenced that this bipartite screening profiles is absent (i.e., there is no evidence for an effective SS bilayer) [20]. The analysis shown in figure 6, gives λ values that are independent of the particular sample used and measurement conditions, implying that the measured quantities are intrinsic to the individual materials (originating from this batch of “stocks” and the coating procedure). The experimentally obtained $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ in table 3 agree with the literature values shown in table 1, highlighting the suppression of the Meissner current in the surface layer. The obtained $\lambda_{\text{Nb}} = 43.3(19)$ nm exceeds the average literature estimate of Nb’s penetration depth in the “clean” limit $\lambda = 28.0(15)$ nm [20, 24]. We propose that this increased λ_{Nb} is due to the suppression of the electron mean free path, ℓ . There might be some impurity added to the Nb substrate due to the material “doping” while exposing its surface to the Ar/N₂ mixture during sputtering at 450 °C. Commonly, low temperature baking of Nb in N₂ reduce ℓ [59–61] and consequently increase λ based on the Pippard’s approximation [62].

Note that, Pippard’s nonlocal electrodynamics [62] were not considered in describing the superconducting properties of Nb. Our previous LE- μ SR investigation on “bare” and “N₂ doped” Nb samples [20] have shown that the London model sufficiently describes these properties, suggesting that the effects of nonlocal electrodynamics could be even more prominent in higher purity samples [63].

Regarding the field screening in the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (160 nm)/Nb sample the field decays far more rapidly in the first few nanometers than in the other samples but the whole data can be fitted with a single $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$ value. Agreement of the film thicknesses extracted as fit parameters with the nominal thicknesses of the films for different measurement conditions further confirm that the counter-current-flow model can very well describe the material properties. Also, the different magnitude of B_{applied} for the 50 nm sample does not have any effect on the other fit parameters.

An (apparent) difference in applied fields, B_{applied} for measurements in the normal and Meissner state is observed

in figure 6. B_{applied} is used as a shared fit parameter between Meissner and normal state data for all the samples, while for the Meissner state the magnetic field enhancement is accounted for by the demagnetization factor, N as an individual fit parameter (see equation (16)). From the fit, the extracted value of B_{applied} agrees with the nominal applied fields of the samples.

We also find that a non-superconducting layer (i.e., “dead layer”) at the surface is absent in our model. While such feature is often found in “real” superconductors, its absence here is not unexpected, given the surface roughness of our samples and the chemical stability of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$. The 50 nm sample is mirrored surface finished and others are prepared by BCP however, we did not observe any effect of surface roughness in the field screening profile. The surface of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ oxidizes on exposure to the ambient atmosphere (forming NbO_x and TiO_x), with the thickness of the oxide layer saturating quickly to ~ 1.3 nm [64]. This layer is too thin for observation by LE- μ SR at the implantation energies used here. Thus, while we can not completely rule out the existence of a thin ~ 1 nm non-superconducting region at the surface of our samples, we assert that such a feature is too small to meaningfully impact the material quantities reported here.

4.1. Predictions of critical fields

As discussed in sections 1 and 3, the counter-current-flow model predicts that multilayer superconductors can maximize the field of first-flux entry beyond the individual superheating field of its layers and substrate. For a *crude* estimate of this quantity, the superheating field, B_{sh} and Ginzburg–Landau (GL) parameter, $\kappa \equiv \lambda/\xi_{\text{GL}}$ (i.e., the ratio between the magnetic penetration depth λ and the GL coherence length ξ_{GL}), of each layer are required, which we consider below.

Through linear stability analysis using GL theory (strictly valid at $T \simeq T_c$) B_{sh} for $\kappa > 1.1495$ was derived to be [8]:

$$B_{\text{sh}} \approx B_c \left(\frac{\sqrt{20}}{6} - \frac{0.55}{\sqrt{\kappa}} \right), \quad (18)$$

where B_c is the thermodynamic critical field.

Following that, κ is calculated for each material from experimentally measured penetration depth with the literature value of the London penetration depth λ_L and Bardeen–Cooper–Schrieffer (BCS) [45] coherence length ξ_0 :

Table 4. Superconducting parameters GL parameter κ , thermodynamic critical field B_c , lower critical field B_{c1} , and superheating field B_{sh} were calculated from the measured penetration depths of $\lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 182.5(31)$ nm and $\lambda_{\text{Nb}} = 43.3(19)$ nm. B_c for Nb and BCS coherence length ξ_0 for both materials are taken from literature.

Material	λ_L (nm)	ξ_0 (nm)	κ	B_{c1} (mT)	B_c (mT)	B_{sh} (mT)
$\text{Nb}_{1-x}\text{Ti}_x\text{N}$	150 [9]	2.4(3) [41]	102(17)	22.9(11)	710(40)	570(40)
Nb	28.0(15) [20, 24]	40.3(35) [20]	1.83(25)	74(11)	199(1) [7]	229(6)

$$\kappa = \frac{\lambda}{\xi_{\text{GL}}} = \frac{2\sqrt{3}}{\pi} \frac{\lambda^2}{\xi_0 \lambda_L}, \quad (19)$$

using the fact that ξ_0 and ξ_{GL} both are correlated to the magnetic flux quantum Φ_0 [65]. Here, λ_L and ξ_0 are the fundamental properties of the metal defined by the clean stoichiometric material.

The next quantity is the lower critical field B_{c1} , which is derived for both materials from [66]:

$$B_{c1} = \frac{\Phi_0}{4\pi \lambda^2} \ln(\kappa + 0.497). \quad (20)$$

Now, B_c of equation (18) needs to be determined, which is well-defined for Nb [7]. However, since $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ is not a natural compound B_c is not readily available from the literature, which we can be self-consistently evaluated when B_{c1} is known [65]:

$$B_c = \frac{\sqrt{2}\kappa B_{c1}}{\ln \kappa}, \quad (21)$$

To summarize the results of these calculations, the values of κ , B_{c1} , B_c , and B_{sh} for $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb are presented in table 4.

Finally, the maximum field for which the SS bilayer can remain in the Meissner state B_{max} is derived by solving the relation between applied field and screening current density in the London model, with appropriate boundary and continuity conditions [10, 12, 19]:

$$B_{\text{max}} = \min \left\{ \gamma_1^{-1} B_{\text{sh}}^{(s)}, \gamma_2^{-1} B_{\text{sh}}^{(\text{sub})} \right\}, \quad (22)$$

where $B_{\text{sh}}^{(s)}$ and $B_{\text{sh}}^{(\text{sub})}$ are the superheating fields of the surface and substrate layers, respectively, and the terms γ_1 and γ_2 arise as coefficients while solving the relation for B_{max} (see [10] for details). Explicitly, the γ_i s are:

$$\gamma_1 = \frac{\sinh \frac{d_s}{\lambda_s} + \frac{\lambda_{\text{sub}}}{\lambda_s} \cosh \frac{d_s}{\lambda_s}}{\cosh \frac{d_s}{\lambda_s} + \frac{\lambda_{\text{sub}}}{\lambda_s} \sinh \frac{d_s}{\lambda_s}}, \quad (23)$$

and

$$\gamma_2 = \frac{1}{\cosh \frac{d_s}{\lambda_s} + \frac{\lambda_{\text{sub}}}{\lambda_s} \sinh \frac{d_s}{\lambda_s}}. \quad (24)$$

In equation (22), the term $\gamma_1^{-1} B_{\text{sh}}^{(s)}$ is related to the maximum applied field for the surface layer, whereas the term

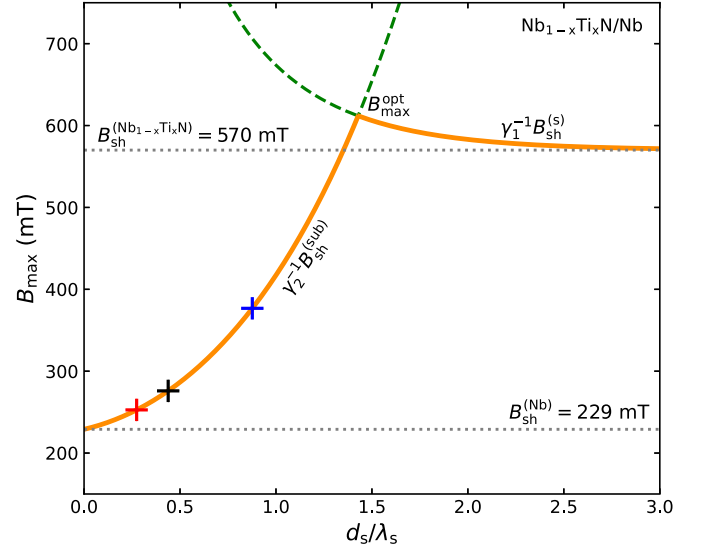


Figure 7. Prediction of the maximum applied field B_{max} where the Meissner state can be sustained for an SS bilayer as a function of thickness of the top $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ superconducting layer, d_s (i.e., $d_{\text{Nb}_{1-x}\text{Ti}_x\text{N}}$) in $\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Nb}$. The orange curve starting from the left represents B_{max} of the substrate Nb layer and the curve starting at right corresponds to the surface $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer. Here the measured penetration depths of $\lambda_s = \lambda_{\text{Nb}_{1-x}\text{Ti}_x\text{N}} = 182.5(31)$ nm and $\lambda_{\text{sub}} = \lambda_{\text{Nb}} = 43.3(19)$ nm were used to find the magnitude of γ_1 and γ_2 using equations (23) and (24). The predicted values of the superheating field of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb are $B_{\text{sh}}^{(s)} = B_{\text{sh}}^{(\text{Nb}_{1-x}\text{Ti}_x\text{N})} = 570(40)$ mT and $B_{\text{sh}}^{(\text{sub})} = B_{\text{sh}}^{(\text{Nb})} = 229(6)$ mT, respectively. The +, + and + are the position of maximum fields for each of the 50 nm, 80 nm, and 160 nm samples.

$\gamma_2^{-1} B_{\text{sh}}^{(\text{sub})}$ corresponds to the substrate. As B_{max} is a function of the surface layer thickness, d_s , there exists an optimum where its value is maximized [10, 12, 19]

$$B_{\text{max}}^{\text{opt}} = \sqrt{\left(B_{\text{sh}}^{(s)}\right)^2 + \left[1 - \frac{\lambda_{\text{sub}}^2}{\lambda_s^2}\right] \left(B_{\text{sh}}^{(\text{sub})}\right)^2}. \quad (25)$$

B_{max} is plotted in figure 7 as a function of d_s , wherein the entire $\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Nb}$ SS structure remains in the Meissner state. The predicted maximum applied fields for our different film thicknesses (50 nm, 80 nm, and 160 nm) were found to be 253(5) mT, 276(5) mT and, 377(5) mT, indicated in figure 7 by “plus” (+, +, and +) symbols, respectively. Clearly, these values exceed the intrinsic field limit of the Nb substrate. The orange curve in figure 7 represents the criteria for the surface and substrate layer to remain in the Meissner state. For zero film thickness, the substrate can sustain its Meissner state up to the superheating field of the substrate B_{sh}

(229(6) mT for our Nb substrates). Upon increasing the d_s , B_{\max} is initially increased, as the applied field is shielded by the surface superconductor before it reaches the SS interface. B_{\max} reaches its optimum (i.e., $B_{\max}^{\text{opt}} = 610(40)$ mT) for a surface layer thickness, $d_m \sim 1.4\lambda_s = 261(14)$ nm according to equation (25).

Note that a surface layer thicker than λ_s can only remain in the Meissner state above B_{c1} in the presence of a BL barrier [15] just like a bulk superconductor of same material. The strong suppression of the screening current by the counter-current-flow between substrate and surface layers therefore suggests that multilayer structures with several interlayers to stop vortices are necessary in order to achieve largest B_{\max}^{opt} .

5. Summary

In conclusion, the depth-dependent field screening profile in SS bilayers composed of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ films (50 nm, 80 nm, and 160 nm) deposited on Nb substrates were measured using LE- μ SR. A fit of the magnetic screening profile to a counter-current-flow model yielded a penetration depth for $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ of 182.5(31) nm in agreement with literature values. This is contrasted by fits to a naive biexponential model, which was found to overestimate λ by a factor of ~ 2.5 . For the Nb substrates, a common λ of 43.3(19) nm was found. This comparison highlights the pronounced suppression of the Meissner current within the surface layer and serves as an experimental validation of the counter-current-flow model. Using these quantities, the optimum maximum field that can be sustained before first-flux entry by a $\text{Nb}_{1-x}\text{Ti}_x\text{N}/\text{Nb}$ heterostructure with these material properties was predicted to be 610(40) mT. This study emphasizes that the samples tested can collectively be well described by a London model with appropriate boundary conditions.

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Data availability statement

The raw data that support the findings of this study are openly available at the following URL/DOI: www.psi.ch/en/smus/data-archiving.

Conflict of interest

The authors have no conflicts to disclose.

Appendix. Fit parameters of LE- μ SR time spectra data of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb sample

Figure 8 shows fit parameters of the Meissner state (2.7 K) data of the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb sample presented in figure 3. The size of the error bars in the fit parameters signifies the robustness of the skewed Gaussian approach to present the field distribution.

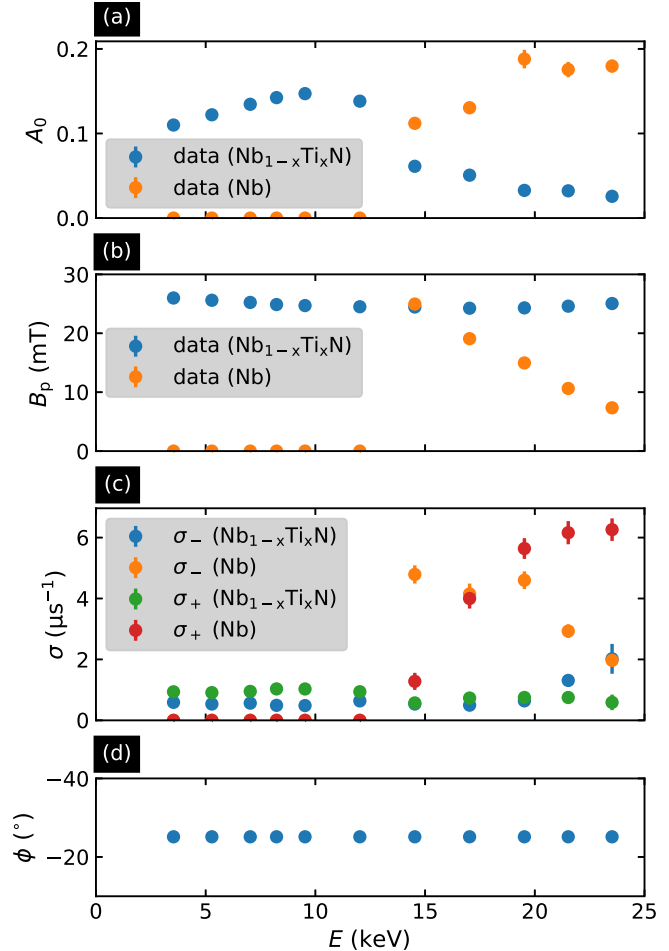


Figure 8. Plot of the fit parameters A_0 , B_p , σ_{\pm} , and ϕ of equations (9)–(13) as a function of E in $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ (50 nm)/Nb in the Meissner state (2.7 K) at an applied magnetic field of ~ 25 mT. For $E \leq 14.5$ keV the fit is constrained such that $n = 1$ in equation (12) indicating the μ^+ sample is only implanted in the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ layer. (a) The blue and orange closed circles are the asymmetry, A_0 data points corresponding to the $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb layer, respectively. (b) the peak field, B_p of $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb layer are denoted by the blue and orange closed circles, (c) the distribution's "width" on either side of B_p , σ_{\pm} is plotted for both $\text{Nb}_{1-x}\text{Ti}_x\text{N}$ and Nb layers indicated by colored closed circles shown in the figure inset, and (d) represents the shared parameter, phase ϕ .

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