Negative magnetization of Li$_2$Ni$_2$Mo$_3$O$_{12}$: A spin system composed of distorted honeycomb lattices and linear chains

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We study the magnetism of a spin-1 substance Li$_2$Ni$_2$Mo$_3$O$_{12}$. The spin system consists of distorted honeycomb lattices and linear chains of Ni$^{3+}$ spins. Li$^+$ ions enter about 25% and 50% of the honeycomb and chain Ni sites, respectively, creating disorder in both spin subsystems. A magnetic phase transition occurs at $T_c$ = 8.0 K in the zero magnetic field. In low magnetic fields, the magnetization increases rapidly below $T_c$, decreases below 7 K, and finally becomes negative at low temperatures. We determine the magnetic structure using neutron-powder-diffraction results. The honeycomb lattices and linear chains show antiferromagnetic and ferromagnetic long-range order, respectively. We investigate static and dynamic magnetic properties using the local probe technique of muon spin relaxation. We discuss the origin of the negative magnetization.

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I. INTRODUCTION

Several antiferromagnets having plural magnetic ions or magnetic-ion sites show an interesting magnetic long-range order (LRO). In Cu$_2$Fe$_2$Ge$_4$M$_{12}$O$_{30}$ spin dimers (spin 1/2) are coupled to Fe$^{3+}$ spin chains (spin 5/2). An indirect Fe-Fe exchange coupling via Cu dimers is observed. This result reveals that the Cu dimers play the role of nonmagnetic media in the indirect magnetic interaction. The dynamics of dimer excitation in staggered and random fields generated by Fe spins are also identified experimentally. In Cu$_3$Mo$_2$O$_{10}$, three crystallographically independent Cu$^{2+}$ sites exist (Cu1, Cu2, and Cu3). Spins (1/2) on Cu1 sites form antiferromagnetic (AFM) dimers. Two spins on neighboring Cu2 and Cu3 sites form an AFM dimer. Each AFM chain is coupled to AFM dimers. Only a component parallel to the $b$ axis of magnetic moments on Cu1 sites is ordered below the AFM transition temperature $T_N$ = 7.9 K in the zero magnetic field. Perpendicular components are ordered in magnetic fields parallel to the $a$ or $c$ axis. A canted AFM LRO is stabilized only in finite magnetic fields. It is inferred that magnetic competition causes this unique LRO. In addition, the canted AFM LRO disappears in Cu$_3$Mo$_2$O$_{10}$ with a very small amount of doping (e.g., 0.2% Zn for Cu sites). Negative magnetization is an interesting phenomenon. It was observed in Li$_{0.5}$(FeCr)$_{2.5}$O$_4$ spinels and explained using the Néel model. In this model, two collinear sublattice magnetizations $M_A$ and $M_B$ with different temperature $T$ dependencies are considered. $|M_A|$ is larger than $|M_B|$ below $T_N$, while $|M_B|$ is larger than $|M_A|$ below a compensation temperature $T_{\text{comp}}$. If the directions of the sublattice magnetizations are fixed, the magnetization is observed as a negative value below $T_{\text{comp}}$. Several compounds showing a negative magnetization have been found. Some of them are considered to be explainable by the Néel model, while for the other compounds the origin is different or has not been determined explicitly. Details of the origins will be described later.

Here we turn our attention to Li$_2$Ni$_2$Mo$_3$O$_{12}$. Two crystallographically independent Ni$^{3+}$ (spin 1) sites (M1 and M2) exist as represented in Fig. 1. The M1 and M2 sites form distorted honeycomb lattices and linear chains, respectively. About 25% of the M1 sites and about 50% of the M2 sites are occupied by nonmagnetic Li$^+$ ions as shown by the formula Li(Li$_0$Ni$_{1.5}$)(Li$_0$Ni$_{1.5}$)Mo$_3$O$_{12}$. Unexpectedly, we found negative magnetization at low $T$. To investigate the origin of the negative magnetization, we determined the magnetic structure of Li$_2$Ni$_2$Mo$_3$O$_{12}$ using the neutron-powder-diffraction technique and its static and dynamic magnetic properties using the local probe technique of muon spin relaxation ($\mu$SR).

II. EXPERIMENT

Crystalline powder of Li$_2$Ni$_2$Mo$_3$O$_{12}$ was synthesized using a solid-state-reaction method at 973 K in air for 144 h with intermediate grindings. We used an isotope $^7$Li (purity: 99%) for neutron-diffraction samples to decrease the absorption of neutrons. We confirmed the formation of Li$_2$Ni$_2$Mo$_3$O$_{12}$ using an x-ray diffractometer (JDX-3500; JEOL). Li$_2$Ni$_2$Mo$_3$O$_{12}$ is insulating at room temperature.

We measured magnetization using a superconducting quantum interference device magnetometer (MPMS-5S; Quantum Design). We entered powder in paraffin molten by heating in the zero magnetic field and finally fixed the powder in solid paraffin. We measured the magnetization of the powder embedded in solid paraffin. It is likely that the powder directions are randomly distributed and powder reorientation does not occur in the magnetization measurements. The specific heat was measured using a relaxation technique with the Physical Property Measurement System (Quantum Design). We used a sintered pellet in the specific-heat measurements.

We determined the magnetic structure of $^7$Li$_2$Ni$_2$Mo$_3$O$_{12}$ from neutron-powder-diffraction data. The experiments were conducted using the high-resolution powder diffractometer...
FIG. 1. (Color online) Schematic drawing of M1 (8d) and M2 (4c) sites in Li$_2$Ni$_2$Mo$_3$O$_{12}$. The space group is orthorhombic Pnma (No. 62). Three sites are occupied by Ni$^{2+}$ (spin-1) or Li$^+$ (spin-0) ions. The occupation ratio of Ni$^{2+}$ ions is 0.757 at M1 and 0.566 at M2. The ratio between Ni numbers at M1 and M2 sites is 0.728:0.272. Three short M1-M1 bonds exist and form distorted honeycomb lattices. The M1-M1 distances are 3.03, 3.08, and 3.75 Å at 10 K for the first, second, and third shortest M1-M1 bonds, respectively. The respective exchange interaction parameters are defined as $J_1$, $J_2$, and $J_3$. The shortest M2-M2 bonds form linear chains. The M2-M2 distance is 2.53 Å. Distances in the other M1-M2 bonds ($i = 1$ or 2) are larger than 5.06 Å. Distances in M1-M2 bonds are larger than 5.19 Å.

for thermal neutrons$^{39}$ (HRPT) (wavelength $\lambda = 1.886$ Å) and the high-intensity cold neutron powder diffractometer DMC ($\lambda = 4.206$ Å) at the Swiss spallation neutron source (SINQ) at Paul Scherrer Institut (PSI). Powder was filled into a vanadium container 8 mm in diameter and 55 mm high. Rietveld refinements of diffraction data were performed using the FULLPROOF SUITE program package.$^{40}$ Symmetry analyses of possible magnetic configurations were conducted using the program BASIREP in the FULLPROOF SUITE program package.

The $\mu$SR measurements were performed using the Dolly spectrometer at the $\pi E1$ beamline at the Swiss Muon Source at PSI. In a $\mu$SR experiment nearly 100% spin-polarized muons are implanted into the sample one at a time. The positively charged $\mu^+$ thermalize at interstitial lattice sites, where they act as magnetic microprobes. In a magnetic material the muon spin precesses about the local magnetic field $B$ at the muon site with the Larmor frequency $f_\mu = \gamma_\mu/2\pi B$ (muon gyromagnetic ratio $\gamma_\mu/2\pi = 135.5$ MHz·T$^{-1}$). With a lifetime of $\tau_\mu = 2.2$ μs, the muon decays into two neutrinos and a positron, the latter being predominantly emitted in the direction of the muon spin at the moment of the decay. The measurement of both the direction of positron emission and the time between muon implantation and positron detection for an ensemble of hundreds of thousands of muons provides the time evolution of the muon spin polarization $P(t)$ in the initial muon spin direction. In a powder, two-thirds of the local magnetic-field components are perpendicular to the $\mu^+$ spin and cause a precession, while the one-third of longitudinal field components do not. The damping of the oscillation is a measure of the width of the static field distribution experienced by the muon ensemble. In a static magnetic environment, one-third of the muon spin polarization is conserved. In a dynamic magnetic material, however, this so-called one-third tail also relaxes and in certain limits the relaxation rate can be related to magnetic fluctuation rates. Since $\mu$SR is a local probe technique, the amplitudes of the different signals observed are a measure of the corresponding volume fractions. With this, it provides a direct measure of magnetic volumes, which is not easily possible with scattering techniques. For further details of the $\mu$SR technique, we refer the reader to the recent review in Ref. 41.

III. RESULTS

Figure 2 portrays the $T$ dependence of the specific heat in the zero magnetic field. A peak is apparent at 8.0 K. As will be shown later, magnetization increases rapidly on cooling below 8.0 K. Therefore, the peak indicates the occurrence of a magnetic phase transition at $T_c = 8.0$ K.

Figure 3(a) shows the $T$ dependence of the magnetization $M$ of Li$_2$Ni$_2$Mo$_3$O$_{12}$ in a very weak residual magnetic field. The value of the magnetic field was estimated as $6.3 \times 10^{-5}$ T (0.63 Oe) from paramagnetic magnetization values in this field and 0.1 T. The magnetization increases rapidly below 8.2 K, shows a maximum around 7.2 K, and decreases below 7.2 K. It should be noted that the magnetization is negative below 4.2 K. We did not observe any difference between magnetizations measured for increasing and decreasing $T$. Figure 4 represents the magnetic-field $H$ dependence of $M$ at 1.7 K after the residual field cooling process. We can see a small residual magnetization and hysteresis. The rapid increase below 8.2 K and small residual magnetization indicate the appearance of a canted AFM LRO or ferromagnetic (FM) LRO with small magnetic moments. Figure 3(b) shows the $T$ dependence of $M$ in the field of $1 \times 10^{-3}$ T. We can see a hysteresis between magnetizations measured in the residual field cooling (RFC) and field cooling (FC) processes. The absolute value of $M$ in the FC process is larger than that in the RFC process. In 0.1 T [Fig. 3(d)], only the RFC magnetization becomes negative. In 0.2 T [Fig. 3(e)], both the RFC and FC magnetizations are positive. In 1 T [Fig. 3(f)], the hysteresis is small. Figure 3(g) shows the $T$ dependence of paramagnetic susceptibility in 0.1 T. The susceptibility above 20 K obeys the Curie-Weiss law. We determined the value of the Curie constant as 1.12 emu K/mol Ni from the data above 200 K. The $g$ value

FIG. 2. (Color online) Temperature dependence of the specific heat of Li$_2$Ni$_2$Mo$_3$O$_{12}$ in the zero magnetic field.
was calculated as 2.11 on the assumption that the spin value is 1. This $g$ value is reasonable for Ni$^{2+}$ spins.

Figure 5 depicts the neutron-powder-diffraction pattern of paramagnetic $^{7}$Li$_2$Ni$_2$Mo$_3$O$_{12}$ recorded using the HRPT diffractometer with $\lambda = 1.886$ Å at 10 K, which is slightly higher than $T_c = 8.0$ K. The refinement based on the crystal structure of Li$_2$Ni$_2$Mo$_3$O$_{12}$ as determined by room-temperature single-crystal x-ray diffraction$^{38}$ fits the experimental neutron-diffraction pattern at 10 K well. Structural parameters are presented in Table I.

Figure 6(a) shows the difference between two neutron-powder-diffraction patterns of $^{7}$Li$_2$Ni$_2$Mo$_3$O$_{12}$ at 2.0 and 10 K collected using the DMC diffractometer ($\lambda = 4.206$ Å). The reflections of the difference pattern can be indexed in the chemical cell with the propagation vector $\mathbf{k} = \mathbf{0}$. The profile matching Le Bail fit, in which the integral intensities are the refined parameters, shows that all the peaks are well described with $\mathbf{k} = \mathbf{0}$ with the reliability factors $R_{wp} = 2.34$, $R_{exp} = 1.72$, and $\chi^2 = 1.86$. The new reflections appear below $T_c$; therefore, they must be magnetic reflections.

Using the determined propagation vector, we performed a symmetry analysis according to Izyumov et al.$^{42}$ to derive possible magnetic configurations for the space group $Pnma$. The space group has eight one-dimensional irreducible representations (IRs). The observed magnetic patterns were compared with calculated patterns using the structural parameters determined from the structural refinement. After sorting out the basis functions of all eight IRs, we found that only $\tau_5$ fits the experimental pattern well. The IR $\tau_5$ has the characters 1, $-1$, 1, $-1$, 1, $-1$, 1, $-1$ for the symmetry elements listed in the caption of Fig. 7. The magnetic moments of Ni$^2+$ atoms are restricted by $\tau_5$ symmetry to be parallel to the $b$ axis, thus forming a ferromagnetic sublattice. The $y$ components of Ni$^2+$ spins are also forced to be ferromagnetically aligned, whereas the spins in $xz$ plane have an antiferromagnetic configuration with the basis vectors listed in the caption of Fig. 7.

In the refinement of the magnetic structure, all the atom structure parameters were fixed by the values obtained from HRPT data (Table I). The magnetic structure is depicted in Fig. 7. The magnetic moment at the NiI($1$) position is $[0.27(4), 0.04(12), -1.58(3)]\mu_B/\text{Ni}$ and its magnitude is $1.60(3)\mu_B/\text{Ni}$ at 2.0 K. The ordered NiI moment mainly points in the $c$ direction with a small component in the $a$ direction. The size of the ferromagnetic $b$ component is refined to a small value close to zero within experimental error bars. The arrangement of the moments indicates that the $J_1$, $J_2$, and $J_3$ interactions are FM, FM, and AFM, respectively. The magnetic moment at the NiI($1$) position is $[0, -0.6(1), 0]\mu_B/\text{Ni}$. NiI$^2+$ moments form the FM LRO. The Ni$^2+$ moment is reduced in comparison with ordinary ordered Ni$^{2+}$ moments. The reduction may be
caused by the one dimensionality of the Ni2 spin system and large disorder due to about 50% substitution of Li ions.

The IR $\tau_5$ fits the experimental data well for all the measured temperatures. Figure 8 shows the $T$ dependence of the magnetic moments. The absolute value of the FM component $m_{2b}$ increases monotonically with decreasing $T$. The AFM components $m_{1a}$ and $m_{1b}$ have a significantly weaker dependence of temperature with step-like change at $T_c$. The FM component $m_{1b}$ stays close to zero with a slight tendency to increase.

To further elucidate the static and dynamic magnetic properties of Li$_2$Ni$_2$Mo$_3$O$_{12}$, we performed zero-field $\mu$SR experiments for various temperatures. In Fig. 9 zero-field $\mu$SR spectra are shown for characteristic temperatures above and below $T_c = 8.0$ K. Figures 9(a) and 9(b) show the same data but plotted on different time scales to highlight the observed changes for the fast and slow relaxing signals. At high temperatures a slowly relaxing spectrum of Gauss-Kubo-Toyabe form is observed. This is the distinctive identifying characteristic of a nonmagnetic material. The small Gaussian relaxation $\sigma_{nm}$ is due to static random nuclear moments only. Below $T_c = 8.0$ K an overdamped spontaneous muon spin precession is observed, indicating a very broad field distribution at the muon site(s), i.e., a complicated or disordered magnetic structure or many muon sites in the crystallographic lattice. This result is consistent with structural disorder caused by Li ions.

The $\mu$SR spectra have been fitted using the following functional form for the polarization function:

$$P(t) = (1 - f_{\text{mag}}) G_{\text{nn}}(t) + f_{\text{mag}} G_{\text{mag}}(t)$$

with

$$G_{\text{nn}}(t) = \left( \frac{3}{2} \right) \left[ 1 - \sigma_{\text{nn}} t^2 \right] e^{-\sigma_{\text{nn}} t^2/2} + \frac{1}{3} e^{-\lambda_{\text{dyn}} t}$$

and

$$G_{\text{mag}}(t) = \left( \frac{2}{3} \right) \cos(2\pi f_\mu t) e^{-\sigma_{\text{mag}} t^2/2} + \frac{1}{3} e^{-\lambda_{\text{dyn}} t}.$$  (3)

Here $G_{\text{mag}}$ and $G_{\text{nn}}$ represent the relaxation functions for the magnetic $f_{\text{mag}}$ and nonmagnetic $1 - f_{\text{mag}}$ volume fractions with their corresponding relaxation rates $\sigma_{\text{mag}}$ and $\sigma_{\text{nn}}$, respectively, and $\lambda_{\text{dyn}}$ is the dynamic relaxation rate.

Figure 10 shows the parameter values obtained from the fits using the preceding equations. A sharp magnetic transition at $T_c = 8.0$ K is observed in all the parameters. The magnetic fraction $f_{\text{mag}}$ reaches $\approx 100\%$ shortly below $T_c$. The magnetic order parameter measured via the $\mu$SR frequency $f_\mu$ and the static field width $\sigma_{\text{mag}}$ continuously increase below $T_c$. The values of $\sigma_{\text{mag}}$ are larger than the values of $f_\mu$, indicating the very broad width of the internal field distribution. Approaching the transition from above, an additional exponential damping that increases toward $T_c$ is observed. This is typical for the slowing down of electronic moments approaching $T_c$.

The corresponding dynamic relaxation rate $\lambda_{\text{dyn}}$ is shown in Fig. 10(d). Below the transition $\lambda_{\text{dyn}}$ is nearly zero, indicating a static magnetic state. Decreasing the temperature further, a dynamic relaxation of the one-third tail is observed again. This is already clearly visible from the comparison of the $\mu$SR spectra at 7.6 and 1.9 K in Fig. 9(b). The dynamic relaxation at low temperature had to be fitted with a root-exponential time dependence [Eq. (3)], which is indicative of a broad distribution of fluctuation times. Interestingly, the dynamic relaxation rate starts to increase below $\approx 4$ K where the magnetization becomes negative.

IV. DISCUSSION

Let us consider the origin of the negative magnetization in Li$_2$Ni$_2$Mo$_3$O$_{12}$. The Néel model requires two FM sublattices, which in our case are formed by the $m_{1b}$ and $m_{2b}$ components.

### TABLE I. Structural parameters of Li$_2$Ni$_2$Mo$_3$O$_{12}$ derived from Rietveld refinement of the HRPT neutron-powder-diffraction pattern at 10 K. The space group is orthorhombic $Pnma$ (No. 62). The lattice constants at 10 K are $a = 5.0639(5)$ Å, $b = 10.397(1)$ Å, and $c = 17.467(2)$ Å. Estimated standard deviations are shown in parentheses. The atomic displacement parameters $B_{\text{iso}}$ were constrained to be the same for the same atom type and for the atoms occupying the same site. The reliability factors of the refinement amounted to $R_{\exp} = 3.21$, $R_{\text{mag}} = 1.77$, $\chi^2 = 3.31$, and $R_{\text{Bragg}} = 2.76$.

<table>
<thead>
<tr>
<th>Atom</th>
<th>Site</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$B_{\text{iso}}$ (Å$^2$)</th>
<th>Occupancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni1</td>
<td>8d</td>
<td>0.7473(3)</td>
<td>0.4302(1)</td>
<td>0.0258(1)</td>
<td>0.27(3)</td>
<td>0.757(2)</td>
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<tr>
<td>Li1</td>
<td>8d</td>
<td>0.7473(3)</td>
<td>0.4302(1)</td>
<td>0.0258(1)</td>
<td>0.27(3)</td>
<td>0.243(2)</td>
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<tr>
<td>Ni2</td>
<td>4c</td>
<td>0.3881(6)</td>
<td>0.75</td>
<td>0.2497(2)</td>
<td>0.27(3)</td>
<td>0.566(2)</td>
</tr>
<tr>
<td>Li2</td>
<td>4c</td>
<td>0.3881(6)</td>
<td>0.75</td>
<td>0.2497(2)</td>
<td>0.27(3)</td>
<td>0.434(2)</td>
</tr>
<tr>
<td>Li3</td>
<td>4c</td>
<td>0.7562(13)</td>
<td>0.25</td>
<td>0.1967(3)</td>
<td>0.27(3)</td>
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</tr>
<tr>
<td>Mo1</td>
<td>4c</td>
<td>0.7149(4)</td>
<td>0.75</td>
<td>0.0562(1)</td>
<td>0.06(2)</td>
<td></td>
</tr>
<tr>
<td>Mo2</td>
<td>8d</td>
<td>0.2225(3)</td>
<td>0.4739(1)</td>
<td>0.1562(1)</td>
<td>0.06(2)</td>
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<tr>
<td>O1</td>
<td>4c</td>
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<td>0.25</td>
<td>0.0064(1)</td>
<td>0.29(2)</td>
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<tr>
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<td>0.75</td>
<td>0.1544(1)</td>
<td>0.29(2)</td>
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<tr>
<td>O3</td>
<td>8d</td>
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<td>0.6184(2)</td>
<td>0.2050(1)</td>
<td>0.29(2)</td>
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<tr>
<td>O4</td>
<td>8d</td>
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<td>0.3797(2)</td>
<td>0.2137(1)</td>
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<tr>
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<td>8d</td>
<td>0.9411(4)</td>
<td>0.3864(2)</td>
<td>0.1256(1)</td>
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<td>8d</td>
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<td>0.5122(2)</td>
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<td>O7</td>
<td>8d</td>
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<td>0.6146(2)</td>
<td>0.0363(1)</td>
<td>0.29(2)</td>
<td></td>
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</table>
The values of \( m_{2b} \) are negative. The values of \( m_{1b} \) are very small and show both signs. At 2.0 K, the error bar of \( m_{1b} \) is the smallest and the value of \( m_{1b} \) is positive. Therefore, if the \( T \) dependence of \( m_{1b} \) is similar to that of \( -m_{1c} \) or \( m_{1a} \), \( m_{1b} \) may be intrinsically positive. Compensation of \( m_{1b} \) and \( m_{2b} \) is possible. The sublattice magnetizations \( M_A \) and \( M_B \) defined in Sec. I correspond to \( m_{1b} \) and \( m_{2b} \), respectively. \( M_A \) has a weaker \( T \) dependence than \( M_B \) in the Néel model. If \( m_{1b} \) is proportional to \( -m_{1c} \) or \( m_{1a} \), \( m_{1b} \) has a weaker \( T \) dependence than \( m_{2b} \), as in the Néel model. We calculated the average spontaneous magnetization per Ni \( m_s \) using \( p_1 m_{1b} + (1 - p_1) m_{2b} \) and plotted it together with the RFC magnetization in Fig. 8(c). Here the proportion coefficient \( p_1 = 0.728 \) is calculated from the occupancies listed in Table I. The contribution of \( m_{1b} \) is large in \( m_s \) because of the large number of Ni1 atoms. Although the component \( m_{2b} \) shows a well-defined decrease with decreasing temperature [Fig. 8(b)], \( m_s \) is practically constant [Fig. 8(c)]. The difference in the \( T \) dependence of \( m_{2b} \) and \( m_s \) suggests compensation of the FM components. A reversal of \( m_s \), however, was not obtained.

The absolute values of \( m_s \) seem much larger than the absolute values of the RFC magnetization represented by the solid line in Fig. 8(c).

The negative magnetization is too small to be detected using neutron-powder-diffraction techniques. Therefore, we could not verify the validity of the Néel model. The magnetic scattering length was calculated as \( 2.7gSF(Q) \) fm using a Born approximation. Here \( g \) and \( S \) are the \( g \) value and magnitude of ordered spins, respectively, and \( f(Q) \) is the magnetic form factor whose magnitude is on the order of unity or less. The nuclear coherent scattering length of Ni was obtained as 10.3 fm (natural average). The intensity of the Bragg peaks is proportional to the square of the scattering lengths. The lower limit of magnetic moments that can be evaluated in neutron-powder-diffraction experiments is about 0.1\( \mu_B \). As shown in Fig. 8, the error bars are on the order of 0.1\( \mu_B \).

The following four alternative models may be considered as possible explanations of the negative magnetization effect. In model 1 applicable to GdCrO\(_3\) or (La\(_{1-x}\)RE\(_x\))CrO\(_3\) (\( X_{RE} = \) Pr or Nd),\(^{18-24}\) magnetic moments of Cr ions form a canted AFM LRO, while rare-earth (RE) ions are paramagnetic. A Dzyaloshinskii-Moriya (DM) interaction exists between Cr and \( X_{RE} \) spins. The main component \( x \) of the Cr moment produces an internal magnetic field parallel to the \( z \) direction on RE ions. The polarization of paramagnetic moments on RE ions induced by the internal magnetic field is aligned opposite the canted components parallel to the \( z \) direction.

**FIG. 6.** (Color online) (a) Fragment of the difference between two neutron-powder-diffraction patterns of \( ^7\)Li\(_2\)Ni\(_2\)Mo\(_3\)O\(_{12}\) at 2.0 and 10 K (magnetic neutron-diffraction pattern) measured using the DMC diffractometer (\( \lambda = 4.206 \) Å). Lines on the observed pattern and at the bottom show a Rietveld refined pattern in the Le Bail profile matching mode and the difference between the observed and the Rietveld refined patterns. Hash marks represent positions of magnetic reflections. Several indices of the reflections are written. (b) Neutron-powder-diffraction pattern at 2.0 K. Lines on the observed pattern and at the bottom show a Rietveld refined pattern for the magnetic structure model \( J_1 \) and the difference between the observed and the Rietveld refined patterns. Upper and lower hash marks represent the positions of nuclear and magnetic reflections, respectively.

**FIG. 7.** (Color online) Magnetic structure of Li\(_2\)Ni\(_2\)Mo\(_3\)O\(_{12}\). Ni2 moments form a FM LRO. To show the arrangement of Ni1 moments clearly, the symmetry operators of M1 \((8d)\) sites are given: \( x,y,z[u,v,w]\); \( \bar{x}+1/2,\bar{y},\bar{z}+1/2[u,v,\bar{w}]\); \( \bar{x},\bar{y}+1/2,\bar{z}[u,v,\bar{w}]\); \( x+1/2,\bar{y}+1/2,\bar{z}+1/2[u,v,\bar{w}]\); \( \bar{x},\bar{y},\bar{z}[u,v,w]\); \( x+1/2,\bar{y},\bar{z}+1/2[u,v,\bar{w}]\); \( \bar{x}+1/2,\bar{y},\bar{z}+1/2[u,v,\bar{w}]\); and \( \bar{x}+1/2,\bar{y},\bar{z}+1/2[u,v,\bar{w}]\). Here \( u, v, \) and \( w \) indicate components of the magnetic moment.
Li$_2$Ni$_2$Mo$_3$O$_{12}$ cannot be explained by model 1. Effects cannot account for the negative magnetization in Li$_2$Ni$_2$Mo$_3$O$_{12}$.

The magnitude of the sum of the canted components is larger than the magnitude of the sum of the polarization of the paramagnetic magnetization below $T_N$, while the latter becomes larger than the former below $T_{\text{comp}}$. The direction of the main component of the Cr moments is fixed. Therefore, the direction of the canted component is also fixed. As a result, the negative magnetization appears in GdCrO$_3$ or (LaRE)CrO$_3$ ($X_{\text{RE}} = $ Pr or Nd). In Li$_2$Ni$_2$Mo$_3$O$_{12}$, the distance between Ni1 and Ni2 sites is larger than 5.19 Å. Exchange interactions between Ni1 and Ni2 spins are probably weak. Internal magnetic fields produced by Ni1 (Ni2) moments on Ni2 (Ni1) sites are small. Accordingly, the negative magnetization in Li$_2$Ni$_2$Mo$_3$O$_{12}$ cannot be explained by model 1.

In model 2, which is applicable to LaVO$_3$ (Refs. 26–28) or YVO$_3$, only the canting component of the magnetic moment on V ions reverses and becomes antiparallel to external fields. Therefore, negative magnetization was observed. In LaVO$_3$, a structural phase transition occurs just below $T_N$ due to the Jahn-Teller effect. As a result, the DM vector reverses and therefore the canting component of the magnetic moment reverses. In YVO$_3$, the canting direction is determined mainly by the single-ion anisotropy at high $T$ and by the DM interaction at low $T$. As a result, the canting direction reverses at low $T$. In Li$_2$Ni$_2$Mo$_3$O$_{12}$, the LRO of Ni2 spins is the FM order (not the canted AFM order). Consequently, model 2 cannot account for the negative magnetization in Li$_2$Ni$_2$Mo$_3$O$_{12}$.

In model 3, which is applicable to (SmGd)Al$_2$, compensation between the spin and the orbital parts of the ordered moments occurs. We observe a sum of spin and orbital magnetic moments. Therefore, the results in Fig. 8 indicate directly that compensation between the spin and the orbital parts of the ordered moments is not the origin of the negative magnetization. In addition, in 3$d$ shells, only spin magnetic moments exist because of quenching of the orbital angular momentum.

In model 4, which is possibly applicable to Sr$_2$X$_{\text{RE}}$RuO$_6$ ($X_{\text{RE}} = $ Yb, Y, or Lu), exchange bias (EB) effects may cause negative magnetization. The effects can appear in composites having both FM and AFM LROs. Ordered magnetic moments of the two LROs should be collinear. The FM transition temperature should be higher than the AFM transition temperature. In Li$_2$Ni$_2$Mo$_3$O$_{12}$, however, Ni1 and Ni2 moments are nearly perpendicular to each other. Only one transition was observed within experimental accuracy. In addition, the distances between Ni1-Ni2 bonds are larger than 5.19 Å, suggesting that interactions between Ni1 and Ni2 spins are weak. Consequently, EB effects cannot account for the negative magnetization in Li$_2$Ni$_2$Mo$_3$O$_{12}$.

At present, we think that the Néel model may be valid for the negative magnetization in Li$_2$Ni$_2$Mo$_3$O$_{12}$, although we cannot prove the validity of the Néel model. We cannot absolutely deny the possibility of another (unknown) origin. As described by the μSR results, the dynamic relaxation rate starts to increase below $\approx 4$ K where the magnetization becomes negative. It is tempting to conclude that the
magnetization inversion mechanism involves a dynamic process. An alternative idea is that an undetected structural change may cause reversal of \(m_{zb}\). Experimental investigations into \(\text{Li}_2\text{Ni}_2\text{Mo}_3\text{O}_{12}\) using other techniques and further theoretical considerations are necessary.

V. CONCLUSION

We studied the magnetism of the spin-1 substance \(\text{Li}_2\text{Ni}_2\text{Mo}_3\text{O}_{12}\). The spin system is composed of the distorted honeycomb lattices and linear chains formed by two distinct \(\text{Ni}^{2+}\) sites, \(\text{Ni}1\) and \(\text{Ni}2\), respectively. \(\text{Li}^{+}\) ions substitute about 25% and 50% of the honeycomb and chain Ni sites, respectively, creating the disorder in both spin subsystems. A magnetic phase transition occurs at \(T_c = 8.0\ \text{K}\). In low magnetic fields, the magnetization increases rapidly just below \(T_c\), decreases below 7 K, and finally becomes negative at low temperatures on cooling. We determined the magnetic structure using neutron-powder-diffraction results. The magnetic order on both Ni sites develops according to a single irreducible representation \(\tau_5\) of the space group \(\text{Pnma}\) and propagation vector \(\mathbf{k} = 0\). The honeycomb lattices show antiferromagnetic long-range order. The ordered Ni1 moment mainly points in the \(c\) direction with a small AFM component in the \(a\) direction. The size of the ferromagnetic \(b\) component is very small. The ordered Ni2 moment on linear chains has only the \(b\) component and shows FM long-range order. The magnetic moment sizes amounted to 1.60(3)\(\mu_B\) and 0.6(1)\(\mu_B\) per Ni atom for the honeycomb lattice and chain Ni sites, respectively. The sharp magnetic transition at \(T_c\) was also observed in the \(\mu\text{SR}\) results. The overdamped spontaneous muon spin precession below \(T_c\) indicates a very broad field distribution at the muon site(s). The dynamic relaxation rate starts to increase below \(\approx 4\ \text{K}\) where the magnetization becomes negative. We have examined several models known from literature that can account for the negative magnetization. The Néel model that is based on compensation of the ferromagnetic sublattices may be valid in our case, but our experimental data do not allow us to prove this model quantitatively.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure10.png}
\caption{Parameter values obtained from the fits to the zero-field muon spin relaxation measurements. The lines are guides to the eye.}
\end{figure}

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