Supplementary Information for
“Asymmetry of collective excitations in electron and hole doped cuprate superconductors”

W. S. Lee1,*, J. J. Lee2, E. A. Nowadnick3,4, S. Gerber1, W. Tabis5,6, S. W. Huang7, V. N. Strocov7, E. M. Motoyama3, G. Yu5, B. Moritz1, H. Y. Huang8,9, R. P. Wang8, Y. B. Huang7,10, W. B. Wu8, C. T. Chen8, D. J. Huang8, M. Greven5, T. Schmitt7, Z. X. Shen1,2,3,*, and T. P. Devereaux1,*

1SIMES, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA
2Department of Applied Physics, Stanford University, Stanford, CA 94305, USA
3Department of Physics, Stanford, University, Stanford, CA 94305, USA
4Department of Physics, Columbia University, New York, NY 1002, USA
5School of Physics and Astronomy, University of Minnesota, Minneapolis, MN 55455, USA
6AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, 30-059 Krakow, Poland
7Paul Scherrer Institut, Swiss Light Source, CH-5232 Villigen PSI, Switzerland
8National Synchrotron Radiation Research Center, Hsinchu 30076, Taiwan, R. O. C.
9Program for Science and Technology of Synchrotron Light Source, College of Science, National Tsing Hua University, Hsinchu 30076, Taiwan, R. O. C.
10Beijing National Laboratory for Condensed Matter Physics, and Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

*Correspondence should be addressed to W.S.L., Z.X.S., and T.P.D
Supplementary Figure 1:

![XAS Cu-L3 spectra](image)

**Figure S1** a, X-ray absorption spectra (XAS) near the Cu L$_3$-edge for $x = 0.04$ and $0.147$ compounds, respectively. RIXS spectra were taken by tuning the photon energy to the peak of the XAS, as indicated by the arrow. b, The scattering geometry. $k_{\text{in}}$ and $k_{\text{out}}$ indicate the incident and scattered x-rays. The red arrows are the momentum transfer with the in-plane component $q_{//}$ as defined in the figures. The two orthogonal polarizations for the incident beam ($\sigma$ and $\pi$) are indicated in green.

Supplementary Note:

We emphasize that the observed low energy collective excitations do not originate from the $\sim 1\%$ (Nd,Ce)$_2$O$_3$ secondary phase generated during the annealing process which is required to make the as-grown crystal superconducting$^{32}$, because the spectral weight of the modes is significant and Cu L$_3$-edge RIXS is only sensitive to the Cu valence states.
Supplementary Method: Characterization of the doping levels of superconducting samples

Since the temperature dependence between the two measured superconducting samples are significant, it is important to characterize their actual doping levels more accurately. After our RIXS measurements, energy dispersive spectroscopy (EDS) were performed on the studied samples along with a scanning electron microscope (SEM) in order to obtain accurate knowledge about their doping levels. The EDS spectra were normalized by the Cu-$K\alpha$ peak, and the doping level was estimated by calculating the percentage of spectral weight of Ce (Fig. S2b). On each sample, four different locations were selected and the doping level is determined by taking their average (Fig. S2a). Our characterizations indicate that the doping levels of the studied superconducting samples are $0.147 \pm 0.001$ and $0.166 \pm 0.002$ for sample 1 and sample 2, respectively.

**Figure S2**

**a**, SEM image of the sample surfaces. Sample 1 was used for the RIXS measurements at the SLS, Switzerland. Sample 2 was used for measurements at NSRRC, Taiwan. The markers in the images indicate where the EDS/SEM measurements were performed. **b**, EDS spectra obtained by EDS/SEM. The spectra are normalized by the Cu $K\alpha$ peak. The inset highlights the normalized spectra near the Ce $L_{\alpha}$ peak for both samples. The fitted area of the Ce $L_{\alpha}$ peak was used to determine the doping level at each measured position. The doping level of sample 1 and 2 referred in the main text are the average of the doping level at the four selected locations (black circles in Fig. S2a) of each sample.
Supplementary Data: The width and spectral weight of the magnon and paramagnon in NCCO $x = 0.04$ and $x = 0.147$ compounds

Figure S3a summarizes the full-width at half-maximum (FWHM) and the spectral weight of the magnetic excitations found in $x = 0.04$ and 0.147 NCCO compounds. While the spectral width of the magnon excitation in the $x = 0.04$ compound is ~200 meV, the spectral width of the paramagnon is in the range of 300 ~ 600 meV, which is significantly broader. This indicates a significant shortening of the magnetic excitation lifetime, similar to those found in hole-doped compounds. In addition, as shown in Fig. S3b, we also find that the spectral weight of the paramagnon in the superconducting $x = 0.147$ compound remains significant. We note that the extracted spectral weight is an upper bound of the paramagnon, since the RIXS spectra also contain contributions from charge excitations that are yet to be determined.

**Figure S3 a,** Summary of the full-width half-maximum (FWHM) of the magnon in $x = 0.04$ (grey), and the paramagnon in $x = 0.147$ (red). The FWHM was determined by fitting with Gaussian functions as described in the main text. The error bars were estimated by the uncertainty of determining the zero energy loss. The spectral weights determined by the area of the Gaussian fit are summarized in **b.**
Supplementary Theory: Quantum Monte Carlo Simulation of single band Hubbard model

We consider the single-band Hubbard model:

\[ H = - \sum_{ij\sigma} t_{ij} (c_{i\sigma}^+ c_{j\sigma} + h.c.) - \mu \sum_{i\sigma} \hat{n}_{i\sigma} + U \sum_{i\sigma} \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} \]

with nearest and next-nearest-neighbor hopping parameters \( t_{ij} = t \) and \( t' \). Here \( c_{i\sigma}^+ \) creates an electron at site \( i \) with spin \( \sigma \), \( \hat{n}_{i\sigma} = c_{i\sigma}^+ c_{i\sigma} \), \( \mu \) is the chemical potential, which is adjusted to control the doping level, and \( U \) is the on-site electron-electron interaction strength.

We simulate this model using the determinant quantum Monte Carlo (DQMC) algorithm\(^{33,34}\), which is a numerically exact, non-perturbative approach for finite temperature calculations. The simulation yields the imaginary time spin susceptibility, defined as

\[ \chi(q, \tau) = \frac{1}{N} \sum_{ij} e^{i\mathbf{q} \cdot (\mathbf{R}_i - \mathbf{R}_j)} < T[\hat{n}_{i\uparrow}(\tau) - \hat{n}_{i\downarrow}(\tau)][\hat{n}_{j\uparrow} - \hat{n}_{j\downarrow}] > \]

In order to obtain the dynamic spin susceptibility in real frequency \( \chi(q, \omega) \), we perform analytic continuation using the maximum entropy method\(^{35}\). The dynamic spin structure factor \( S(q, \omega) \) is then calculated from the fluctuation-dissipation theorem.

The parameters for our simulation (in units of \( t \)) are: \( U = 8t \), \( t' = -0.3t \), and inverse temperature \( \beta = 3/t \). All calculations are performed on an 8 x 8 square lattice with periodic boundary conditions. This lattice gives access to \( S(q, \omega) \) at a discrete set of momentum points spaced by \( \Delta q_{x,y} = \pi/4 \).

Supplementary References:


