Two-dimensional ultra-small angle X-ray scattering with grating interferometry

P. Modregger,1,2,a) S. Rutishauser,3 J. Meiser,4 C. David,3 and M. Stampanoni1,5
1Swiss Light Source, Paul Scherrer Institut, Villigen, Switzerland
2Centre d’Imagerie BioMédicale, École Polytechnique Fédérale de Lausanne, Lausanne, Switzerland
3Laboratory for Micro- and Nanotechnology, Paul Scherrer Institut, Villigen, Switzerland
4Institute of Micro Structure Technology, Karlsruhe Institute of Technology, Karlsruhe, Germany
5Institute for Biomedical Engineering, UZH/ETH Zürich, Zürich, Switzerland

(Received 30 May 2014; accepted 24 June 2014; published online 16 July 2014)

It was recently established that the pixel-wise ultra-small angle x-ray distribution can be retrieved with grating interferometry. However, in these one dimensional approaches the contrast was limited to the direction orthogonal to the structure of the line gratings. Here, we demonstrate that sensitivity in two contrast directions can be achieved by using two pairs of crossed line gratings and by adapting scan procedures and data analysis accordingly. We demonstrate the retrieval of two-dimensional scattering distributions with grating interferometry, thus overcoming the previously reported limit of seven observable, complementary contrasts. In addition, we give further evidence for the superiority of the signal-to-noise ratio for the dark-field contrast, if a deconvolution-based instead of the standard analysis is utilized. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4890090]

Phase contrast imaging with hard x-ray grating interferometry (GI) (Refs. 1 and 2) has been established as an important tool for biomedical applications3–5 and wave front sensing.6,7 Typically, the setup consists of at least two line gratings (beam splitter and analyzer grating) and provides differential phase contrast (DPC) (Ref. 8) and dark-field contrast9 in the direction orthogonal to the grating structure in addition to absorption contrast. Sensitivity in two directions can be achieved by either rotating the gratings10,11 or the sample12 and scanning multiple times. Alternatively, gratings with a two-dimensional (2D) structure can be utilized to obtain 2D sensitivity, which was experimentally demonstrated without an analyzer grating13–15 and with an analyzer grating present.16 Alternative x-ray phase-contrast techniques have also been shown to provide 2D contrast, e.g., edge illumination,17 analyzer-based imaging,18 or phase propagation imaging.19

In addition to the standard contrasts provided by one-dimensional (1D) GI, it was further established that the angular resolved ultra-small angle x-ray scattering distribution (USAXS) can be retrieved by 1D GI. Two distinct approaches to obtain the USAXS distributions were proposed. In Ref. 20, multiple scans acquired with different inter-grating distances, model-based data fitting, and consecutive Fourier transform were utilized. Later, this approach was combined with tomography.21

Recently, we have demonstrated that the need for multiple scans and the assumption of Gaussian-shaped scattering distributions can be avoided if the appropriate data analysis (GI-USAXS) is used.22 We also proposed the combination of GI-USAXS with tomography that preserves these beneficial characteristics.23 The data analysis of GI-USAXS involves the deconvolution of noisy, periodic data, which was achieved by a careful selection of the parameters for Lucy-Richardson deconvolution.24 The three standard contrasts provided by 1D-GI (i.e., absorption, differential phase, and dark-field) can be regarded as parameters describing the pixel-wise USAXS distribution and the following relations were found:22 absorption corresponds to the 0th moment of the USAXS distribution, DPC to the 1st moment, and dark-field to the 2nd moment. Thus, GI-USAXS can also be used as an alternative data analysis for obtaining the standard contrasts and it was shown that GI-USAXS provides a superior signal-to-noise ratio (SNR) for the dark-field contrast.25

In this Letter, we combine the benefits of GI-USAXS with 2D-GI by utilizing two pairs of crossed line gratings (Fig. 1). The experiment was carried out at the beamline for TOMographic Microscopy and Coherent rAdiology experimenTs (TOMCAT) of the Swiss Light Source of the Paul Scherrer Institute (Villigen, Switzerland). A photon energy of 25 keV was selected by a double multilayer monochromator, and the sample, a knot in a nylon thread, was placed at 25 m distance from the source. The beam splitter unit was mounted on a 2D Piezo actuator to allow for lateral positioning with nm precision. Each beam splitter grating

---

a)Electronic mail: peter.modregger@psi.ch
(material: Si, pitch: \( g_1 = 4.767 \mu m \), and structure height: 32 \( \mu m \)) introduced a phase shift of \( \pi \) into the beam, which realized a checkerboard-type 2D phase grating and provided a mesh-like interference pattern at an inter-grating distance of \( z = 174 \text{mm} \).\(^{26}\) At this position, the analyzer grating (material: Au, pitch: \( g_2 = 2.4 \mu m \), and structure height: 50 \( \mu m \)) was placed directly in front of the detector unit, which had a pixel size of 7.4 \( \mu m \) (pco.2000, PCO AG, Kelheim, Germany). The beam splitter gratings were manufactured by the Laboratory for Micro- and Nanotechnology\(^{27}\) of the Paul Scherrer Institute (Villigen, Switzerland), while the analyzer gratings were produced by the Institute of Micro Structure Technology of the Karlsruhe Institute of Technology (Karlsruhe, Germany).\(^{28}\)

The typical scan procedure for 1D GI involves a lateral scan of one of the line gratings by equally distant fractions of the grating’s pitch.\(^6\) This phase stepping method yields an oscillatory, periodic intensity pattern for each detector pixel, which is called a phase stepping curve (PSC). The extension to two dimensions is straightforward: 2D raster-like scans of the beam splitter unit in both lateral beam directions\(^{16}\) (indicated by the arrows in Fig. 1). Performing this scan twice, once with a sample present in the beam and once without, provides 2D intensity patterns \( s(x, y, \phi_x, \phi_y) \) with the sample and \( f(x, y, \phi_x, \phi_y) \) without the sample for each pixel \( x, y \). The lateral offsets of the beam splitter unit in horizontal \( \Delta_x \) and in vertical \( \Delta_y \) direction are expressed as \( \phi_x = 2\pi \Delta_x / g_2 \) and \( \phi_y = 2\pi \Delta_y / g_2 \). Here, we acquired 16 \times 16 phase steps over one period with four accumulations for noise reduction and 150 ms exposure time for each frame. The resulting 2D PSCs outside and inside the sample are exemplified in Figs. 2(a) and 2(b).

For the 1D case, the scattering distribution \( g(\phi) \) was implicitly defined by the convolution \( s(\phi) = g(\phi) \otimes f(\phi) \).\(^{22}\) Again, the extension to two dimensions is straightforward yielding

\[
s(x, y, \phi_x, \phi_y) = g(x, y, \phi_x, \phi_y) \otimes f(x, y, \phi_x, \phi_y)
\]

with a 2D convolution with respect to \( \phi_x \) and \( \phi_y \). Thus, the 2D USAXS distribution \( g(x, y, \phi_x, \phi_y) \) can be obtained by a pixel-wise 2D deconvolution of the sample and the flat PSCs. We utilized Lucy-Richardson deconvolution with 250 iterations, which took approximately 4 h on a modern desktop PC with 4 cores to process the entire data set consisting of 1800 \( \times \) 500 \( \times \) 16 \( \times \) 16 data points. Examples for the retrieved 2D USAXS distributions are shown in Fig. 2(c) in an area outside the sample and in Fig. 2(d) at the edge of the sample (corresponding pixel positions are indicated by black circles in Fig. 3). Ideally, \( g(\phi_x, \phi_y) \) outside of the sample should have the form of Dirac’s \( \delta \)-distribution and deviation from this ideal (Fig. 2(c)) are due to the angular response function of the utilized data analysis procedure. The increased width of \( g(\phi_x, \phi_y) \) at the edge of the sample (Fig. 2(d)) accounts for the smoothed PSC in Fig. 2(b), which demonstrates the consistency of the proposed approach.

Fig. 3 shows a montage of the retrieved scatter images of the nylon sample. Individual scatter images present \( g(x, y, \phi_x, \phi_y) \) as it varies over the field of view for a constant angular vector \( (x_l, y_l) \) with the refraction angles \( x_{l, x} = \phi_{l, x} g_2 / (2\pi) \).\(^8\) The angular step from image to image is 0.9 \( \mu \text{rad} \) in both directions.

\[
\begin{align*}
&\begin{array}{c}
\text{FIG. 2. Deconvolution procedure for 2D GI-USAXS. (a) and (b) are the 2D PSCs without, } f(\phi_x, \phi_y), \text{ and with, } s(\phi_x, \phi_y), \text{ the sample present in the beam. (c) and (d) are the retrieved scattering distributions, } g(\phi_x, \phi_y), \text{ at the locations outside and at the edge of the sample as indicated by the circles in Fig. 3. In (c), } g \text{ outside the sample shows a } \delta_y \text{-like shape, which meets the expectation of basically deconvolving a function with itself. In (d), the broadening of } g \text{ accounts for the increased scattering at the edge of the sample as expected.}
\end{array}
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
\text{FIG. 3. Montage of scatter images of a knotted nylon thread as retrieved by 2D GI-USAXS. Each scatter image is positioned according to its angular vector } (x_l, y_l) \text{ as illustrated by the coordinate system. Out of the } 16 \times 16 \text{ available complementary contrasts the central } 5 \times 5 \text{ are shown. Triangles mark an inversion of contrast at the edge of the thread in both horizontal and vertical directions, which demonstrates 2D sensitivity as expected. The circles in the central image indicate the spatial positions (inside and outside of the sample) of the data used in Fig. 2.}
\end{array}
\end{align*}
\]
In combination with the two diago-
comparison, we define the 0th moment
comparing the results of GI-USAXS to the standard contrasts.

For this, we define the 0th moment
retrieved scattering distributions
make up the seven complementary contrasts of the standard

tion, ter strength, which are related to the dark-field contrast

The last two entries refer to the horizontal and vertical scat-
ner strength, which are related to the dark-field contrast $B_{x,y}$
via $S_{x,y} = -2 \log B_{x,y}^{29}$ In combination with the two dia-
gonal scatter contrasts reported in Ref. 16, the listed quantities
make up the seven complementary contrasts of the standard
analysis (i.e., 2D FCA). A 2D moment analysis of the
retrieved scattering distributions $g(\phi_x, \phi_y)$ will be used to
compare the results of GI-USAXS to the standard contrasts.
For this, we define the 0th moment $M_0$ and the first moments
in both directions $\mu_x$ and $\mu_y$ according to

$$M_0 = \int d\phi_x d\phi_y g(\phi_x, \phi_y)$$

$$\mu_{x,y} = \int d\phi_x d\phi_y \phi_{x,y} g(\phi_x, \phi_y)/M_0,$$

and the higher order, centralized moments as

$$M_{nm} = \int d\phi_x d\phi_y (\phi_x - \mu_x)^n (\phi_y - \mu_y)^m g(\phi_x, \phi_y)/M_0$$

with the natural numbers $m > 1$ and $n > 1$. The PSCs were
acquired in the interval $\phi_{x,y} \in [-\pi, \pi]$ and in order to account
for this asymmetric interval, we symmetrized the scattering
distributions according to $g(\phi_x + \pi, \phi_y + \pi) = g(\phi_x, \phi_y)$.

Fig. 4 compares the standard contrasts modalities to the
corresponding moments of the scattering distributions for the
knot sample. An excellent visual agreement is found for all
contrasts. Thus, 2D GI-USAXS can be regarded as an alterna-
tive to 2D FCA for data analysis. We used the standard devia-
tion, $std$, within a $50 \times 50$ pixel wide region of interest located
in the background of the images to determine the performance
of 2D FCA and 2D GI-USAXS. While we found negligible
difference for the absorption ($std(A) / std(M_0) = 1.00$) and the
DPC contrasts ($std(P_x)/std(\mu_x) = 1.03$, $std(P_y)/std(\mu_y)$
$= 1.08$), 2D GI-USAXS outperformed 2D FCA for the scattering
strengths ($std(S_x)/std(\sqrt{M_{20}}) = 4.25$, $std(S_y)/std(\sqrt{M_{20}})$
$= 4.38$). Thus, we conclude that the superiority of GI-USAXS
as used for the retrieval of the scatter strength over FCA that
was previously reported on for the 1D case,25 holds also true in
the 2D case.

In addition, the horizontal third $M_{30}$ and fourth moment
$M_{40}$ of the scattering distributions of the sample are dis-
played in Fig. 5. The third moment relates to the skew of the
scattering distributions, and positive values mean that the
positive tail is longer than the negative (vice versa for negative
values). The fourth moment corresponds to the kurtosis,
which quantifies the weight of the tails compared to the cen-
tral peak. These higher order moments are not accessible by
the standard data analysis approach, which shows the addi-
tional information provided by 2D GI-USAXS.

In conclusion, we extended GI-USAXS to two contrast
dimensions by utilizing two pairs of crossed line gratings
and by adapting scan procedures as well as data analysis.
This provided the 2D ultra-small x-ray angle scattering
distributions for each pixel (i.e., 4D data sets) and increased
the number of obtainable contrasts from previously seven to 16 × 16 (and more). A consecutive moment analysis of scattering distributions established that GI-USAXS can be used as an alternative approach to data analysis for 2D GI and it was demonstrated that 2D GI-USAXS delivers superior noise characteristics over the established data analysis for the scatter strength images (i.e., dark-field contrast). Further, we showed that higher order moments of the scattering distributions are provided by 2D GI-USAXS, which are not accessible by the previously established data analysis.

We would like to acknowledge Gordan Mikuljan (Paul Scherrer Institut, Villigen, Switzerland) for his support on the mechanical design and Irene Zanette (Technische Universität München, Garching, Germany) for the fruitful discussions. This study was supported by Centre d’Imagerie BioMédicale (CIBM) of the UNIL, UNIGE, HUG, CHUV, and EPFL and the Leenaards and Jeantet Foundations. This work was partly carried out with the support of the Karlsruhe Nano Micro Facility (KNMF), a Helmholtz Research Infrastructure at Karlsruhe Institute of Technology (KIT).


FIG. 5. Horizontal third and fourth moment of the scattering distributions demonstrating previously inaccessible information.