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Key Points:

- We propose the utility of a cavity-expansion-based penetration model for interpreting snow mechanical properties from penetration data
- We perform a model comparison between a continuum cavity-expansion-based penetration model and a discrete microstructural model
- We determine sources of model discrepancy and discuss pros and cons from both models as well as propose the utility of combining the models

Supporting Information:

- Supporting Information S1

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Continuum cavity expansion and discrete micromechanical models for inferring macroscopic snow mechanical properties from cone penetration data

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Abstract Digital cone penetration measurements can be used to infer snow mechanical properties, for instance, to study snow avalanche formation. The standard interpretation of these measurements is based on statistically inferred micromechanical interactions between snow microstructural elements and a well-calibrated penetrating cone. We propose an alternative continuum model to derive the modulus of elasticity and yield strength of snow based on the widely used cavity expansion model in soils. We compare results from these approaches based on laboratory cone penetration measurements in snow samples of different densities and structural sizes. Results suggest that the micromechanical model underestimates the snow elastic modulus for dense samples by 2 orders of magnitude. By comparison with the cavity expansion-based model, some of the discrepancy is attributed to low sensitivity of the micromechanical model to the snow elastic modulus. Reasons and implications of this discrepancy are discussed, and possibilities to enhance both methodologies are proposed.

1. Introduction

Cone penetration tests (CPT) are often used to characterize the mechanical and structural properties of a wide range of media in several disciplines. For typical engineering applications, CPT are used to deduce mechanical properties of soil for trafficability [Whalley *et al.*, 2007] and structural stability. In agriculture, CPT measurements are used to assess suitability for root growth [Whalley *et al.*, 2007]. Recent studies have used penetrometer measurements analogously with the biophysical process of earthworm and plant roots growing or burrowing into soil [Bengough and Mullins, 1990; Ruiz *et al.*, 2015]. In snow science, CPT are used to derive snow properties, for instance, to investigate temporal and spatial changes in snow stratigraphy with regard to snow avalanche formation [Reuter *et al.*, 2015; Schneebeli *et al.*, 1999; Schweizer *et al.*, 2016; van Herwijnen *et al.*, 2009] and to derive fundamental structural properties [Proksch *et al.*, 2015].

The wide range of cone penetrometer applications relies on a variety of mechanical models for parameter estimation and other inferences from cone penetrometer measurements [Adamchuk *et al.*, 2004; Unger and Kaspar, 1994; Yu, 2006; Yu and Mitchell, 1998]. A class of continuum cavity expansion-penetration-based models has been used in the geotechnical literature [Yu, 2000; Yu and Mitchell, 1998; Yu and Carter, 2002; Yu, 1993, 2006] due to its ease of implementation and capability of describing elastic and plastic material deformation during cone penetration [Yu and Mitchell, 1998]. This methodology relies on macroscopic mechanical properties that could be determined independently from standard mechanical tests. The force penetration measurements are used to inversely determine mechanical properties and compare estimates to conventional mechanical tests under similar conditions [Ruiz *et al.*, 2016].

For snow applications, a micromechanical model has been developed to interpret snow mechanical properties from specialized (high-speed and highly resolved) cone penetrometer measurements [Johnson and Schneebeli, 1999]. Unlike conventional continuum models, the micromechanical model statistically characterizes snow microstructural parameters based on data from the high-resolution penetrometer SnowMicroPen (SMP) [Schneebeli *et al.*, 1999]. The microstructural parameters are upscaled to represent snow sample-scale mechanical properties. The original micromechanical model presented by Johnson and Schneebeli [1999] has since been updated to generalize the suggested peak counting method [Marshall and Johnson, 2009] and reformulated in terms of a Poisson shot noise process to reduce assumptions regarding spacing of snow microstructural elements [Löwe and Van Herwijnen, 2012] (details in supporting information section S1). The micromechanical method has the potential to quantify the highly variable behavior

during penetration through a layered snowpack. Nevertheless, there have only been a few attempts to assess the validity of the macroscopic mechanical properties obtained with this method [Capelli *et al.*, 2016; Reuter *et al.*, 2013; Sigrist, 2006]. Although, in general, a good correlation was found between macroscopic snow mechanical properties obtained from the micromechanical model and from other measurement techniques, the absolute values are often diverging. Indeed, the absolute value of the SMP-derived elastic modulus is generally underestimated with differences up to 2 orders of magnitude depending on the measurement method. [Capelli *et al.*, 2016; Reuter *et al.*, 2013; Sigrist, 2006]

Despite the benefit of the micromechanical model, it requires many underlying assumptions with regard to the distribution of rupture forces, inference of microstructural properties based on statistical parameters, and the scaling of these to bulk mechanical properties. We therefore investigate the utility of a continuum-based description of snow penetration. In this technical note, we explore the advantages, similarities, and differences between the microstructural model and the cavity-expansion model in order to assess relevant mechanical properties of snow. The specific goals of this study are to review the micromechanical model (MMM) in relation to macroscopic mechanical properties, introduce a continuum cavity expansion-penetration model (CEM), determine envelopes of compatibility amongst modeling approaches, and elucidate conditions for which the two models yield different interpretations.

We begin by introducing the current state of the art method for estimating essential microstructural parameters via the shot noise process [Löwe and Van Herwijnen, 2012]. Next, we determine the relationship between the micromechanical parameters and macroscopic mechanical properties [Johnson and Schneebeli, 1999; Marshall and Johnson, 2009]. We then introduce a continuum cavity expansion-penetration model, which uses macroscopic mechanical properties and compare the macroscopic properties determined by Johnson and Schneebeli [1999] with those inversely obtained from the continuum cavity expansion-penetration model [Ruiz *et al.*, 2016] and highlight instances of equivalence. Finally, we highlight scenarios where the macroscopic parameters determined in Johnson and Schneebeli [1999] are incompatible with the results obtained from continuum modeling [Ruiz *et al.*, 2016] and discuss reasons and implications of such discrepancies with regard to other mechanical tests that have been carried out on snow.

2. Theoretical Considerations

All symbols can be found in supporting information section S4. A more detailed discussion of the methodology development can be found in the supporting information [Ashby *et al.*, 1986; Fierz *et al.*, 2009; Johnson, 2003].

2.1. Statistical Determination of Microstructural Parameters (MMM)

The SMP measures the ensemble penetration force (F_p) which is interpreted as a superpositioning of microstructural forces resulting from snow microstructural elements deflecting to the point of rupture. These elements are assumed to be spaced at a specific characteristic length (Figures 1a–1c). The Poisson shot noise process is used to statistically infer the values of element rupture forces, rupture deflection, and characteristic element length [Löwe and Van Herwijnen, 2012]. Under the assumption that the rupture forces are equal in magnitude, the magnitude of a microstructural rupture force is defined as follows:

$$f_p = \frac{3}{2} \frac{\text{Var}(F_p(z))}{\text{Mean}(F_p(z))}, \quad (1)$$

where z (m) is the depth of penetration, $\text{Mean}(F_p(z))$ (N) is the mean value of the ensemble force, and $\text{Var}(F_p(z))$ (N^2) is the variance of the measured ensemble force (Figure 1b) over a moving window of 1.25 mm. The deflection at rupture according to Löwe and Van Herwijnen [2012] is estimated by the following expression:

$$\delta_{n,r} = -\frac{3}{2} \frac{\text{Cov}(F_p(z))}{\Delta \text{Cov}(F_p(z))} \Delta z, \quad (2)$$

where $\text{Cov}(F_p(z))$ (N^2) is the force covariance, $\Delta \text{Cov}(F_p(z))$ (N^2) is the incremental change in the force covariance, and Δz (m) is the resolution of the penetration depth (Figure 1d). The microstructural element size

Figure 1. Statistically inferring microstructural parameters from cone penetration measurements. (a) Idealized representation of the interaction between the penetrometer cone and snow microstructural elements (adapted from Johnson and Schneebeli [1999]). (b) The macroscopic penetration force profile determines the mean value and variance of the penetration forces (F_p) along a moving window. (c) F_p is interpreted as the super positioning of microstructural rupture forces (f_p) (arrow pointing to the ensemble of the individual ruptures). (d) From the individual failure events, the microstructural parameters are defined f_p as rupture force, $\delta_{n,r}$ as deflection at rupture, and L_n as element size.

according to Löwe and Van Herwijnen [2012] and Proksch et al. [2015] is the distance between given events and is defined as follows:

$$L_n = \left(\frac{A_s \delta_{n,r} f_p}{2 \text{Mean}(F_p(z))} \right)^{\frac{1}{3}}, \quad (3)$$

where A_s (m^2) is the surface area of the cone.

2.2. Derivation of Microyield and Macroyield Stress and Elastic Modulus

The micromechanical parameters can be used to infer macroscopic mechanical properties. We start by deriving an expression for the microstructural element stiffness [Johnson and Schneebeli, 1999; Marshall and Johnson, 2009]:

$$k_n = f_p / (\delta_{n,r} (1 + \mu \cot(\alpha)) \sin(\alpha)) \quad (4)$$

where μ (—) (assumed 0.25) is the interfacial friction between the penetrometer (steel) and the snow (ice) and α (30°) is the semiapex angle of the cone penetrometer. The micromechanical elastic modulus is then defined as [Johnson and Schneebeli, 1999; Marshall and Johnson, 2009] follows:

$$E_n = \frac{k_n}{L_n} \quad (5)$$

and the micromechanical yield stress is defined as [Johnson and Schneebeli, 1999] follows:

$$\sigma_n = \frac{k_n \delta_{n,r}}{L_n^2}. \quad (6)$$

The microscale properties are macroscopically scaled by taking the product of intact contact probability (defined as $\frac{\delta_{n,r}}{L_n}$) [Johnson and Schneebeli, 1999; Marshall and Johnson, 2009] and the respective microscopic parameters. The macroscopic elastic modulus is then [Johnson and Schneebeli, 1999]

$$E_{\text{mac}} = \frac{\delta_{n,r}}{L_n} E_n \quad (7)$$

and the macroscopic yield stress [Marshall and Johnson, 2009]

$$\sigma_{\text{mac}} = \frac{k_n}{V_e}, \quad (8)$$

where $V_e = \frac{4}{3}\pi\left(\frac{L_n}{2}\right)^3$ is the structural element volume [Marshall and Johnson, 2009]. Substituting (5) and (6) into (7) and (8), we see that the macroscopic yield stress is related to the macroscopic modulus of elasticity:

$$\sigma_{\text{mac}} = \frac{6}{\pi} \frac{\delta_{n,r}}{L_n} E_{\text{mac}}. \quad (9)$$

Furthermore, the ensemble penetration force can be described as a function of the macroscopic yield stress. By substituting (6), (4), and (3) into (8), we obtain the following expression for the ensemble penetration force as a function of macroscopic properties [Marshall and Johnson, 2009]:

$$F_p(z) = \pi \sigma_{\text{mac}} (1 + \mu \cot(\alpha)) A_c / 12, \quad (10)$$

where $A_c = A_s \sin(\alpha)$ (m^2) is the cross-sectional area of the cone. This formulation implies that mean penetration forces are explicitly related to the sample-scale yield stress only and only implicitly related to sample elasticity (in accordance with the second operating assumption [Johnson and Schneebeli, 1999] in supporting information section S1).

2.3. A Continuum Mechanical Model for Cone Penetration Forces

We propose an alternative method for estimating snow bulk mechanical properties. Independent estimates of soil macroscopic mechanical properties were obtained using a continuum-based cavity expansion-penetration model (CEM) [Bishop et al., 1945; Carter et al., 1986; Yu, 2006; Yu and Mitchell, 1998]. The model is derived from the static equilibrium equation that describes the decay of radial stress along the radius r from the center of the cavity into the surrounding medium:

$$\frac{\partial \sigma_r}{\partial r} = - \frac{\sigma_r - \sigma_\theta}{r}, \quad (11)$$

where σ_r (Pa) is the radial stress and σ_θ (Pa) is the circumferential stress. In this study, we neglect the effects of overburden stresses, as the strength of snow is primarily due to cohesion of snow elements. The transition from elastic to plastic deformation is expressed with the Von-Mises yield criteria:

$$\sigma_r - \sigma_\theta = \frac{2\sigma_{\text{mac}}}{\sqrt{3}} + \left(\frac{4}{3}\right) \eta \dot{\epsilon}_r, \quad (12)$$

where the full expression $\left(\frac{\sigma_{\text{mac}}}{\sqrt{3}}\right)$ (Pa) represents the materials macroscopic shear strength, η represents the material viscosity under yield, and $\dot{\epsilon}_r$ is the radial strain rate of the yielding material.

While snow is known to exhibit rate-dependent mechanical behavior [Mellor, 1974], for the first estimate we simplify the problem by only considering a quasi-static system ($\eta = 0$). Substituting (12) into (11) and integrating yields the following expression:

Figure 2. Relationship between cone penetration and cavity expansion. The (a) penetrating cone creates a radial stress that compresses a zone around the face of the cone. This is represented by the expansion of a (b) cavity, where the zone surrounding the cavity (r_c) is the plastic region, and it is bound by an elastoplastic interface at a radius R . The cavity expansion-penetration model (CEM) is used to interpret the (c) penetration force signal by changing the yield stress and elastic modulus to fit the characteristic penetration profile to the SMP signal using a least squares fit.

$$\sigma_r(r) = P_L - \frac{2\sigma_{\text{mac}}}{\sqrt{3}} \ln\left(\frac{r}{r_c}\right), \quad (13)$$

where P_L (Pa) is the radial pressure at the cavity walls as the cavity expands indefinitely and r_c (m) is the cavity radius. To solve for the limit pressure P_L , we assume that for large enough cavity radius r_c , there is a remote elastic zone beyond the radius R that contains the elastoplastic deformation. The ratio between the cavity zone and the plastic zone converges to the following expression [Bishop et al., 1945; Carter et al., 1986] (Figure 2):

$$\left(\frac{R}{r_c}\right)^2 \rightarrow \frac{\sqrt{3}E_{\text{mac}}}{2(1+\nu)\sigma_{\text{mac}}}, \quad (14)$$

where R (m) is the radius of the elastoplastic interface, r_c (m) is the cavity radius, $\nu = 0.2$ (—) is the Poisson's ratio [Mellor, 1974], E_{mac} (Pa) is the sample-scale elastic modulus, and σ_{mac} (Pa) is the sample-scale yield stress. At the elastoplastic interface, the radial stress equals the shear strength ($\sigma_r(R) = \frac{\sigma_{\text{mac}}}{\sqrt{3}}$), and thus, the limit pressure is of the form:

$$P_L = \frac{\sigma_{\text{mac}}}{\sqrt{3}} \left(\left(1 + \ln\left(\frac{\sqrt{3}E_{\text{mac}}}{2(1+\nu)\sigma_{\text{mac}}}\right) \right) \right). \quad (15)$$

The standard limit pressure theory of cavity expansion is independent of any length scale. However, scale plays a particularly important role when addressing expansion of small cavities (e.g., $r_c < 4.5$ mm) [Ruiz et al., 2016]. Johnson [2003] characterized scale effects as statistical deviations from the mean penetration resistance stress. Ladjal and Wu [2011] later derived the scale effect from a hardening parameter, which is a function of second-order strain gradient. The gradient-based approach was originally used by Vardoulakis and Aifantis [1991] to approximate the rate-dependent flow law of the yielding material. The scale effect can be managed without assuming higher-order gradient terms when considering rate dependency [Ruiz et al., 2017]. For the rate-independent penetration model considered in this study, we approximate of the scale dependency with the radial stress equation [Ruiz et al., 2016]. For radius r smaller than $r_c = 4.5$ mm, the radial stresses during penetration expansion can thus be expressed as follows:

$$\sigma_r(r) = \frac{\sigma_{\text{mac}}}{\sqrt{3}} \left(\left(1 + \ln\left(\frac{\sqrt{3}E_{\text{mac}}}{2(1+\nu)\sigma_{\text{mac}}}\right) \right) - 2 \ln\left(\frac{r}{r_c}\right) \right), \quad (16)$$

where r_c is the minimal cavity radius where the limit pressure is attained [Bishop et al., 1945; Ruiz et al., 2016] and r is the active cone radius. We determine the radial forces acting on the cone face [Ruiz et al., 2016]:

$$F_r = 2\pi \cot(\alpha) \int_0^{r_{\text{cone}}} \sigma_r(r) r dr, \quad (17)$$

where α is the semiapex angle of penetration (30° in this study) and r_{cone} is the cone base radius (2.5 mm). We determine the frictionless axial force required for cone penetration:

$$F_z = F_r \tan(\alpha) \quad (18)$$

and by factoring in the frictional effects, we determine the final measured axial force [Ruiz *et al.*, 2016]

$$F_P = F_z(1 + \mu \cot(\alpha)). \quad (19)$$

3. Materials and Methods

3.1. SMP Data and Signal Processing

We used SMP force signals obtained by *van Herwijnen and Miller* [2013], who performed laboratory experiments on homogeneous snow samples to a depth of 80 mm over time to determine the sintering rate of snow. Six snow samples were analyzed for the first 10 sintering times [*van Herwijnen and Miller*, 2013] (details in supporting information section S2 and Table S1).

3.2. Estimating Mechanical Properties With MMM

For the different snow types, the micromechanical parameters were determined with the Poisson shot noise process [*Löwe and Van Herwijnen*, 2012] using a moving window of 1.25 mm size and averaging the value for the entire signal length (mean of values of all windows up to 80 mm) with no overlap (illustrated in Figure 1b). Microstructural parameters were used to determine the bulk mechanical properties (E_{mac} and σ_{mac}).

3.3. Estimating Mechanical Properties With CEM

For each experiment, the curve characteristic to an ideal penetration profile (Figure 2c) was fit to the entire data using a least squares algorithm and adjusting the macroscopic yield stress and elastic modulus [Ruiz *et al.*, 2016; *Wraith and Or*, 1998]. Initial values for the yield stress and elastic modulus (E_{mac} and σ_{mac}) were chosen based on average values from the range presented in *Mellor* [1974] for a given density of snow (Details in supporting information S2).

3.4. Comparison to Snow Mechanical Properties in Mellor [1974]

In the absence of independent and standard measurements of the snow mechanical properties, we opted for comparison of the mechanical properties deduced from the CEM and the MMM with the expected mechanical properties for different snow densities compiled by *Mellor* [1974]. Values of macroscopic yield stress and macroscopic elastic modulus obtained from the CEM and the MMM were plotted alongside the ranges of elastic moduli and yield stress values as a function of snow density as summarized in Figures 2 and 17 of *Mellor* [1974]. Snow properties from *Mellor* were estimated for snow at temperatures ranging from -6 to -25°C [*Mellor*, 1974].

4. Results

4.1. Comparing Mechanical Properties Obtained Using MMM and CEM

We assess the bulk snow macroscopic elastic modulus and yield stress for all samples deduced from the MMM and CEM models as depicted in Figure 3. While both estimates for yield stress (Figure 3a) correlate well ($R^2 = 0.96$), there is a bias between the two approximated values, with yield stress determined by the MMM being consistently an order of magnitude greater than the yield stress determined by the CEM. The comparison between the macroscopic elastic modulus was not as well correlated for all of the density values (Figure 3b). Focusing on the low-density samples, the estimates for the elastic modulus from the two models are similar to one another (graphically near the 1:1 line); however, the macroscopic elastic modulus for the dense snow is estimated many orders of magnitude higher for CEM than with the MMM.

4.2. Comparison to Snow Mechanical Properties in Mellor [1974]

For the macroscopic yield stress (Figure 4a) the estimates from the MMM perform very consistently with the yield stress expected from literature for the higher-density snow samples. For the lower-density snow samples, the MMM is consistently out of range by nearly an order of magnitude from the expected strength

Figure 3. Comparison between (a) bulk snow yield stress and (b) the bulk snow effective elastic modulus obtained from the micromechanical model (MMM) and the cavity expansion-penetration model (CEM) penetration model for the different snow types for the first cavity expansion estimates. Snow types are plotted in order of descending bulk densities (see Table S1 for details). The 1:1 line is marked red. For each sample, 10 penetration tests were considered for different sintering times; thus, each sample has 10 colored symbols.

values. The CEM predicts lower yield stress values for higher-density snow samples, with few of the predicted values overlapping the lower bound of the expected magnitude. However, the CEM predicts values for the yield stress within the expected range for the lower-density samples. Looking at the expected values for elastic modulus (Figure 4b), the MMM is several orders of magnitude less than the expected range for the higher-density snow. For the low-density samples the MMM appears to be reasonably within the expected range. However, for the higher-density samples, the MMM is 2 orders of magnitude below the expected range of elastic properties. The CEM-estimated elastic modulus values fall within the expected range for the samples considered in this study and are still consistent with the average penetration forces.

5. Discussion and Conclusion

In this study we presented a new approach to derive the yield stress and the elastic modulus of snow samples from SMP data using a method based on the continuum cavity expansion model. The macroscopic yield stress (Figure 3a) estimated by the MMM was an order of magnitude larger than that of the CEM. The macroscopic interpretation of the force signal by the MMM neglects the zone of compression around the cone tip, and thus, the influence of a remote elastic zone plays no explicit role in the MMM estimation of mechanical properties. Thus, the burden of the penetration force has to be entirely compensated by the macroscopic yield stress (and a factor of friction). This may be a reasonable estimate for lower-density snow samples, where the SMP could be pulling down a net of loose snow elements during penetration, thus failing in tension. However, for the higher-density snow samples, there will be compaction around the cone [van Herwijnen, 2013]. This is considered in the CEM; thus, the penetration force is partitioned between the macroscopic yield stress and the remote influence of elasticity.

The CEM and MMM estimated different values for the macroscopic elastic modulus (Figure 3b). While the two models produced similar values for the low-density elastic modulus, the estimates for the elastic modulus by the CEM were 2 orders of magnitude greater than the MMM for the high-density snow samples. For the dense samples, the compression zone around the cone tip will have a nonnegligible influence on the force measurement. The failure zone is remotely bounded by the elasticity of the snow. This remote rebound contributes to the force measurement; however, the effect is dissipated in the proximity of the cone. The MMM would thus interpret this dissipated local effect as the elastic modulus and underestimates the elastic modulus.

These parameters were plotted in context with expected values [Mellor, 1974] of macroscopic snow elastic modulus and yield stress (Figure 4). Observing the comparison to the yield stress (Figure 4a), there are no clear drastic deviations from the expected values. The MMM estimates yield stress values that are an order

Figure 4. (a) Bulk snow compressive yield stress and (b) bulk effective elastic modulus obtained from the micromechanical model (MMM) and the cavity expansion-penetration model (CEM) penetration model as a function of snow density taken from Mellor 1974 [Mellor, 1974] (orange shaded region in the plots). Snow types are plotted in order of descending bulk densities (see Table S1 for details). Open circles were the results from the CEM, and closed diamonds are from the fit of the MMM.

of magnitude greater than the expected range for the low snow density. However, it predicts the yield stresses well for the high-density samples. The CEM is within the expected range of yield stress values for the low-density snow and below the lower bound for the higher-density snow. The estimated influence of scale in this study resulted in compressive strength 30% lower than the values that would be obtained with a scale-free model. Considering the span of the data in Figure 4, scale dependency presented little consequence on the trends of the CEM predictions. Despite falling out of the range presented by Mellor [1974], they are consistent with cohesion values obtained for similar snow densities [Shapiro *et al.*, 1997].

The elastic modulus values obtained by the MMM (Figure 4b) were in good agreement for the loose snow samples. However, the MMM-predicted values for elastic modulus do not seem to vary with density. Previous studies have shown that the elastic modulus of snow varies as a function of density over several orders of magnitude [Mellor, 1974]; thus, the MMM-predicted elastic modulus does not seem sufficient for characterizing the elasticity of dense snow. The values for elastic modulus estimated by the CEM for high-density snow were consistent with other studies that used independent methods to measure the elastic modulus and compared the values with the penetration forces obtained with the SMP ([Capelli *et al.*, 2016; Benjamin Reuter *et al.*, 2013; Sigrist, 2006]).

While the determination of snow microstructural parameters is sufficient (in particular, for loose snow), the macroscopic mechanical parameterization has difficulties estimating the elastic modulus in dense snow. Since it has been observed that there is a considerable zone of compaction local to the cone surface during penetration, neglecting the remote influence of elasticity is insufficient for properly inferring the macroscopic elastic modulus. While there have been attempts to incorporate the compaction zone in a version of the MMM [Johnson, 2003], there have been no rigorous attempts to upscale microstructural parameters to macroscale mechanical properties. Therefore, a new parameterization is required that can account for the nonnegligible influence of this compacted zone.

Alternatively, the continuum mechanical model (CEM) infers snow bulk mechanical properties considering a remote elastic influence that dense snow would have during cone penetration and also allows the model to predict the higher elasticity of snow while characterizing the penetration force profile (details in supporting information section S3). Furthermore, the CEM is capable of considering effects of different cone penetration rates and potential of snow viscous deformation [Mellor, 1974] by considering different rheological formulations [Ruiz *et al.*, 2017], which also manages scale dependency. However, unlike the MMM, estimates of

mechanical properties require an initial value. For a given snow density, we tested the upper and lower bounds of the expected mechanical properties (E_{mac} and σ_{mac}) [Mellor, 1974]. The estimated properties were in the same range as presented in Figure 4, with a consistent under estimation of σ_{mac} for dense snow samples. The CEM also fails to resolve changes in properties over individual layers of stratified snow cover, which is a key function implicit in the MMM.

The importance for inference of snow bulk mechanical properties warrants further investigation and independent validation. A false estimate of snow strength could present a problem if used for snow stability evaluation [Schweizer et al., 2008] [Reuter et al., 2015]. Independent mechanical tests should therefore be carried out to assess the reliability of the both approaches to predict bulk mechanical parameters.

In conclusion, in this study we presented a continuum cone-penetration model for inferring macroscopic snow mechanical properties. A comparison between the CEM and the standard MMM showed the CEM is capable of estimating high elastic moduli values of denser snow that the MMM is not capable of doing. Given the value and simplicity of cone penetration measurements, the study proposes the CEM as an alternative for practical applications, capable of providing estimates for layered snow and for different penetration rates (both not tested in this study). An interesting synergy could be gained by linking MMM microstructural inferences with the CEM to improve interpretation of bulk mechanical properties (by both) in order to constrain the CEM and expand the range of predictability by the MMM.

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