APPLICATION AND LIMITATIONS OF DYNAMIC MODELS FOR SNOW AVALANCHE HAZARD MAPPING

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ABSTRACT: Dynamic models, initially based on fluid flow, have been used since the 1950s for modelling the motion and runout of extreme snow avalanches. The friction coefficients cannot be directly measured. They can, however, be calibrated to reproduce an extreme runout that was observed or statistically estimated in a particular path, and the resulting modelled velocity can be used to calculate impact pressures in the runout zone. Alternatively, the friction coefficients can be obtained from extreme avalanches in similar nearby paths and used, often with estimates of available snow mass, to estimate extreme runout in a path that threatens proposed development. This method is controversial because with average values of the friction coefficients, runout estimates from dynamic models are more variable than estimates from statistical runout models. However, uncertainty in the release mass and friction coefficients can be simulated with dynamic models, improving confidence in the runout, impact pressures and return intervals, all of which are required for risk-based zoning. Also, various scenarios can be modelled to see which yields reliable impact pressures for a given position in the runout zone. We argue that dynamic runout estimates can complement estimates from statistical models, historical records and vegetation damage, and be especially useful where some of these estimates are not available or are of low confidence. Limitations of dynamic models involving friction coefficients, snow mass estimates, number of variables and dimensions, entrainment and deposition as well as flow laws are reviewed from a practical perspective.

KEYWORDS: snow avalanche, hazard mapping, runout, dynamic model, uncertainty, friction coefficients

1. INTRODUCTION

Snow avalanches can affect residential areas, industrial sites, energy and transportation corridors as well as backcountry recreation (Fig. 1). For fixed sites, avalanche hazard or risk maps identify areas prone to extreme avalanches, typically with return periods between 30 and 300 years and specified impact pressures (BFF and SLF, 1984; Canadian Avalanche Association, 2002a, b) (Fig. 2). Often there is a red zone where construction of occupied structures is not allowed and a blue zone where construction of occupied structures is restricted, e.g. defence structures and/or evacuation plans required. Estimates of extreme runout, which are used to determine the hazard/risk zones, are based on terrain analysis, historical records, vegetation (including dendrochronology and old air photos) and models (Fig. 3). Two basic types of models are used: topographical-statistical models and dynamic models. See Harbitz et al. (1998) for a review of both types of models.

Coefficients for the statistical models are based on at least 20 measured extreme runouts in paths within a specific mountain range (e.g. Lied and Bakkehøi, 1980; McClung and Mears, 1991; Jóhannesson, 1998). The runout estimates cannot be used with confidence in other ranges. Using regression or distribution parameters from the dataset and a reference point in the lower path known as the Beta (β) point (Fig. 4), it is possible to estimate the probability of a given path having a runout a specified distance past the Beta point. The most commonly applied types of statistical

Figure 1: Photo of a house damaged by a snow avalanche in Valzur near Galtür, Austria, 1999, S. Margreth photo.

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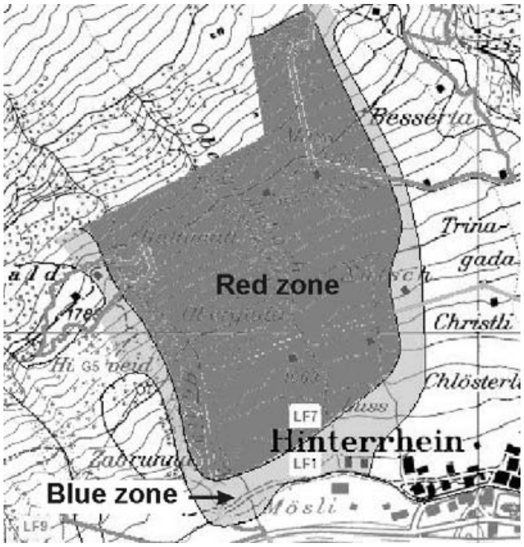

Figure 2: Hazard map showing red zone (dark gray) and blue zone (light gray) for part of the village of Hinterrhein, Switzerland. In a blue zone, residential construction is typically restricted, e.g. reinforcement and/or evacuation plan required. Typically, no new residential construction is permitted in a red zone. Source: http://www.wald.gr.ch/; Amt für Wald Graubünden, 2003.

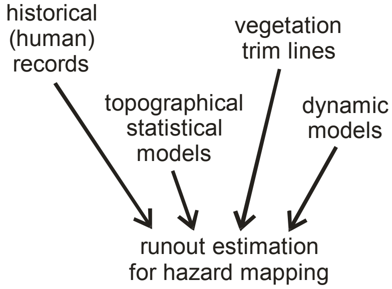

Figure 3: Types of inputs for estimating extreme avalanche runouts.

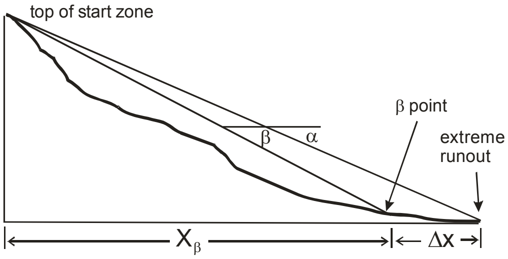

Figure 4: Geometry of an avalanche path as used for statistical runout models. For most of these models, the β point is defined to be where the slope angle decreases to 10° while descending the path.

runout models are known as Alpha-Beta ($\alpha-\beta$)(e.g. Lied and Bakkehøi, 1980) and Runout Ratio ($\Delta x/X_\beta$) (e.g. McClung and Mears, 1991). If the return period is known at a reference point such as Beta, and the parameters are known for the relevant Runout Ratio model, then the runout past the reference point can be estimated for a specific return period (McClung, 2005). Some key limitations of these models include: lack of availability for many ranges, run-up on the opposite side of the valley, and their runout estimates are independent of terrain anomalies, release zone area, release mass and confinement or gullies in the path to be mapped.

Dynamic models use physical laws (e.g. conservation of mass, conservation of momentum) to predict avalanche speed down a simplified representation of the topography of the path. There are many models for the lower dense flow, a few for the powder flow and a few that model the coupled motion of the dense and powder flow (Fig. 5). One-dimensional (1D) models predict the velocity of the centre of mass or the front along the centre line of the path. The earlier 1D models represent the terrain as a sequence of segments, each with constant slope angle, down the centre-line of the path. Two-dimensional (2D) models also estimate the flow depth or lateral extent whereas 3D models estimate both. Using the calculated velocity and estimates of flow density (based on published values and/or experience), impact pressures along the path can be estimated. There are many sources of uncertainty including the values and number of parameters for resistance and the particular flow law, which are discussed below.

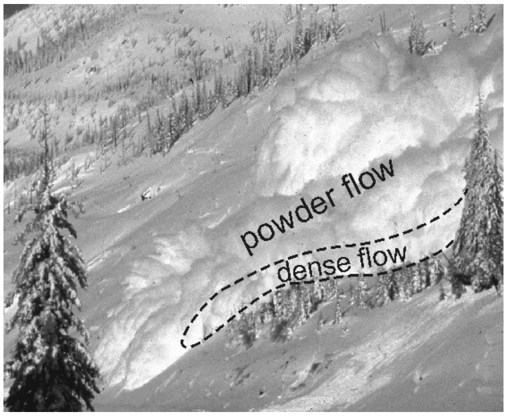

Figure 5: Photo of a large mixed motion avalanche annotating the powder flow and the dense flow, which is often hidden. Will Geary photo.

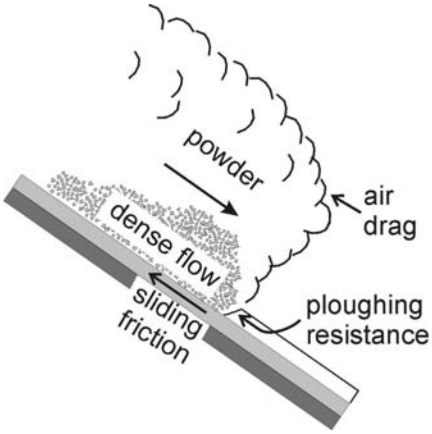

Figure 6: Diagram of mixed motion avalanche showing dense and powder flow. The flow is resisted by sliding friction, air drag and ploughing resistance

There are two basic ways to apply dynamic models in hazard mapping (e.g. Barbolini et al., 2000):

Direct calibration: For the path to be mapped, the friction coefficients and release mass or depth are adjusted so the dynamic model stops at an extreme runout taken from historical (human) records, vegetation damage and/or statistical models. This is sometimes called back-calculation of friction coefficients. With some expertise in the fitting of parameters and flow density, useful estimates of velocity and hence impact pressure along the path and, in particular, in the runout zone are possible.

Indirect calibration: Use resistance and flow parameters taken from extreme runouts in other paths and/or published values, sometimes supplemented with estimates of release area and mass or depth of released snow, and often adjusted with expertise or simulations to estimate extreme runouts in the path to be mapped.

There is little controversy on the direct calibration of dynamic models. In this paper, we focus on indirect calibration in which friction coefficients and sometimes other inputs are typically taken from other paths or published values and modified based on experience and/or simulation by the hazard mapper.

2. BRIEF AND INCOMPLETE REVIEW OF DYNAMIC MODELS FOR HAZARD MAPPING

Voellmy (1955) modeled the dense snow avalanche as a fluid using two parameters: μ for Coulomb (dry or sliding) friction at the base of the avalanche and ξ for "turbulence", which is multiplied by velocity squared (u^2) in the differential equation of motion. Although ξ represents turbulence in fluids, it can also include air drag or ploughing resistance (Fig. 6) in avalanche flow, which also resist motion according to u^2. Retaining the two friction coefficients, μ and ξ, the model was adapted to better fit observed runouts and include back-pressure due to deceleration in the runout zone (Salm et al., 1990) and became know as the Voellmy-Salm model. Given flow width, the model can also estimate flow depth. The calculations can be done by hand or using a simple computer program. This model was widely used in Europe and to a lesser degree in North America for developing avalanche hazard maps.

The PCM model (Perla et al., 1980) uses Coulomb friction μ and the mass-to-drag ratio M/D as the friction coefficients, where D/M is applied to velocity squared. In its derivation, PCM models the motion of the centre of mass from the start zone to the runout zone. However, the friction coefficients are fitted to avalanches flowing from the top of the release zone to the toe of the deposit. Sometimes with friction coefficients fitted to extreme avalanches in nearby similar paths, PCM is used to estimate extreme runout in the path to be mapped (i.e. close to proposed development). Lied et al. (1995) calibrated M/D in terms of the total fall height so that only μ had to be estimated from observed or statistically estimated extreme avalanches. The calculations can be done by hand but a spreadsheet or simple computer program is commonly used. This model has been used for hazard mapping in many areas in North America.

Various authors used the runout from extreme avalanches to estimate the friction coefficients of the Voellmy-Salm and/or PCM models (e.g. Schaerer, 1975, 1981; Martinelli et al., 1980; Buser and Frutiger, 1980; Mears, 1992; Lied et al., 1995). The sliding friction μ has the greatest effect on runout distance whereas ξ or M/D has greater influence on the maximum velocity. (Interestingly, Voellmy flow has been successfully used to analyze the runouts from landslides (Hungr, 1995; McKinnon et al., 2008)).

By dividing the flow mass into non-interacting "particles" and using the same friction coefficients as PCM, Perla et al. (1984) developed the PLK model which allowed for entrainment and deposition along the path (Fig. 7). Not all particles move at the same speed due to an additional term in the momentum equation which randomizes the speed of individual particles. A particle is deposited when its speed reaches zero. We have found PLK's deposits to be plausible for extreme avalanches. (Working with models similar to the Voellmy fluid, Barbolini et al. (2000) also found the modeled deposits useful for assessing the parameters.) Entrainment remains difficult to verify and predict (e.g. Sovilla et al., 2006).

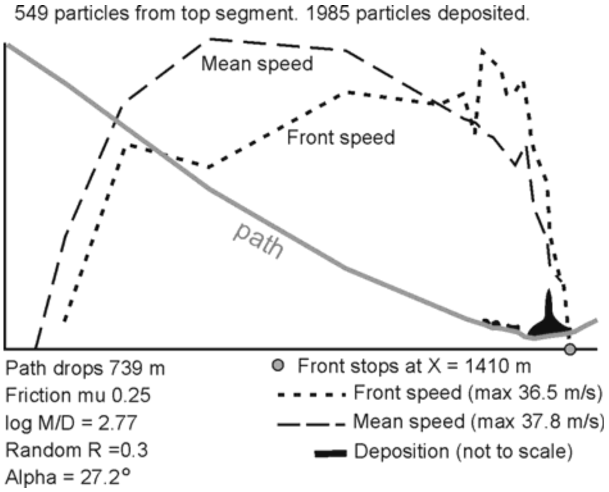

Figure 7: Example of PLK output. The gray line in this diagram represents the centre-of-flow for large avalanches down the path. Note that entrainment is simulated by an increase in the number of particles below the top segment.

In contrast to the Voellmy fluid models, the NIS model (Norem et al., 1987, 1989) treats an avalanche as a two-dimensional granular continuum, consisting of up to 600 elements. In addition to terrain data, the model requires the height and length of the released slab, sliding friction coefficient, and a viscosity parameter. For the dense flow, the 2D granular continuum is more realistic than the Voellmy fluid (Lied, 1998); however, less data are available for estimating the friction coefficients in NIS than for some simpler models such as Voellmy-Salm, PCM or PLK. The calculations require a finite difference program which runs on a PC.

Christen et al. (2002) used a finite difference scheme to solve the governing equations of mass, energy and momentum for "elements" of a flowing avalanche in the computer model AVAL-1D. As with the Voellmy-Salm model, the input parameters are μ, ξ, and the depth of the released slab *d* (traditionally called the "fracture" height), which can be calculated from the slab height and the slope angle. The AVAL-1D manual (v.1.4, SLF, 2005) guides the selection of the parameters μ and ξ based on the track type (unconfined, channeled or gully), the return period, the altitude and the volume. With different friction coefficients for the powder flow above the dense flow, the model can estimate velocity and impact pressure for powder avalanches. In recent years, the computer model has been sold commercially and is widely used, especially in Europe, for mapping snow avalanche hazards and for the calculation of impact loads on obstacles. There is a user's group of hazard mappers, which at their 2003 meeting, expressed general satisfaction with the model (2003 AVAL-1D Workshop in Davos, Switzerland).

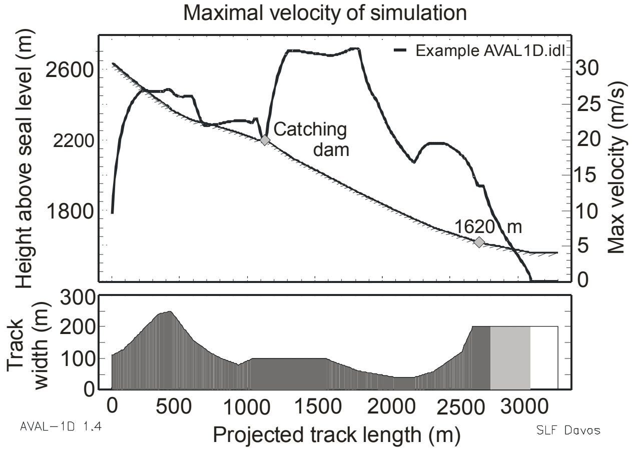

Figure 8: AVAL-1D output. Simulated velocity along the track profile. In the lower graph the track width is shown. The horizontal distance from the top of the starting zone to the end of the low pressure zone is 3095 m.

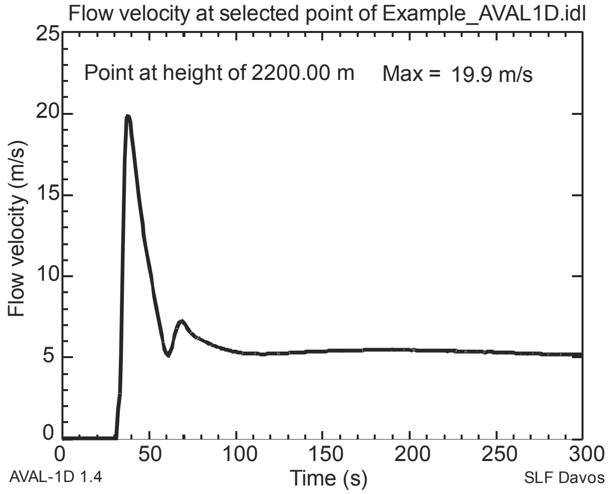

Figure 9: AVAL-1D point information. The diagram shows the flow velocity at the elevation of 2200 m (location of planned catching dam) over time. The maximal calculated flow velocity is 19.9 m/s.

While the dense flow and powder component are modeled separately in AVAL-1D, the dense (granular) flow and the powder flow (turbulent fluid) are coupled in the SAMOS computer model (Sampl and Zwinger, 2004). In contrast to all other models mentioned in this section, the basic friction coefficients are fixed, leaving the user to adjust release height, release area and initial density, as well as flow density and particle size. Given detailed terrain inputs, the model can predict downslope and lateral runout as well as impact pressures in three dimensions. Along with other methods, SAMOS is currently used for hazard mapping in Austria (Sauermoser, 2006).

For more information on the many other dynamic models potentially applicable for estimating avalanche runout, see Harbitz et al. (1998).

3. MODELING UNCERTAINTY IN INPUTS TO ACHIEVE CONFIDENCE IN OUTPUTS

3.1 Sensitivity analysis

The results of avalanche dynamic models depend strongly on the choice of the input parameters. The input parameters such as release volume or track type have to be assessed carefully by an expert. In this section, we demonstrate the sensitivity of the runout distance calculated with AVAL-1D for a parabolic avalanche path with a slope angle of 35° in the starting zone and 2.9° in the runout. Figure 10 shows the influence of different μ/ξ combinations proposed according to the AVAL-1D manual for different altitudes. For a μ/ξ combination of 0.16/2500, which is proposed for an altitude of more than 1500 m, the runout measured from point *P* is 90 m longer, than the runout due to a μ/ξ combination of 0.20/1750, which is proposed for an altitude of less than 1000 m. The effect of the release ("fracture") depth on the runout distance is demonstrated in Figure 11. An increase of the release depth of 20 cm results in a 40 m longer runout. A difference of 20 cm corresponds approximately to one standard deviation for a release depth with a return period of 100 years. Finally, Figure 12 shows the influence of the slope angle in the runout zone on the runout distance calculated with AVAL-1D. If the friction value μ is close to the critical slope, the runout is very sensitive. If the runout slope is, for example, increased from 8.7° to 9.2° the calculated runout is 320 m longer. It is very important that the sensitivity of the different input parameters be checked to get reliable runout estimates. The key input parameters, which strongly influence the runout, can be found by sensitivity analysis.

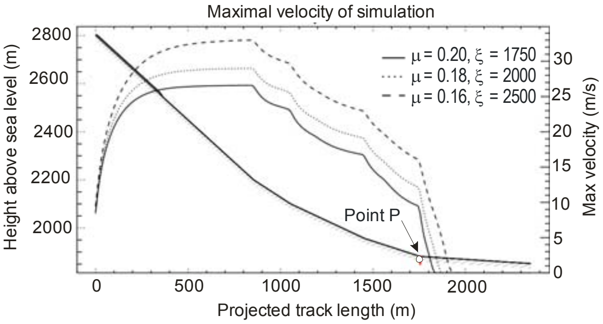

Figure 10: Maximal velocity of AVAL-1D simulations along the track for three μ/ξ combinations. The avalanche volume is considered to be larger than 60'000 m^3 and the avalanche flow is unconfined. The slope angle in the runout zone past point P is 2.9°.

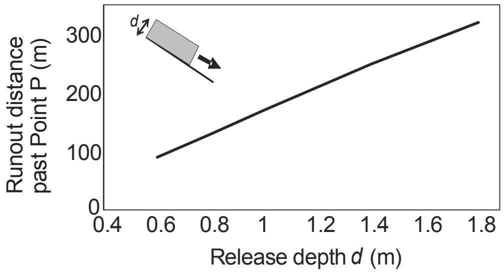

Figure 11: Calculated effect of the release depth, d, on runout distance past Point P. The track profile is shown in Fig. 10.

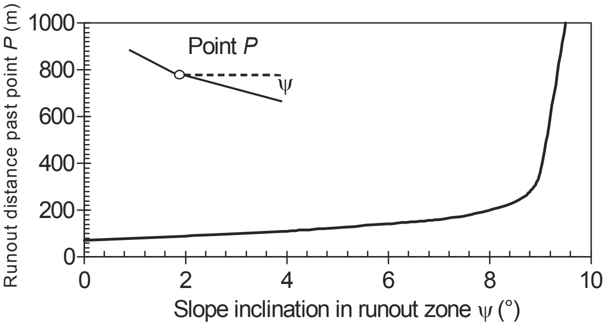

Figure 12: Runout distance in relation to the slope angle in the runout zone. The track profile is as shown in Fig. 10. The applied sliding friction μ is 0.16. The simulated runout is very sensitive to the slope angle at around 9° since $\tan^{-1}$ 0.16 = 9.1°.

3.2 Simulations with dynamics models - concepts

For each unique combination of inputs such as friction coefficients, release depth and release area, a deterministic avalanche dynamic model such as Voellmy-Salm will yield a runout and associated decay of impact pressure in the runout zone. Hazard mappers can try out various combinations of the inputs such as release depth *d*, and friction coefficients μ and ξ and observe the effect on predicted runout. Instead of trying out only a few combinations of inputs, computers and Monte-Carlo simulations allow the analyst to try out many thousands of combinations. For example, suppose the plausible range of release depth for extreme avalanches in a particular path is 1 to 2.2 m with 1.8 being the most likely value. For a particular simulation or run, the release depth could be selected according to the triangular distribution shown in Figure 13, with values near 1.8 m being chosen most often and values near 1 or 2.2 chosen less often. In cases where the values of the input are expected to follow some other statistical distribution such as a normal or Gumbel distribution, then the specific distribution should be used in place of the triangular distribution.

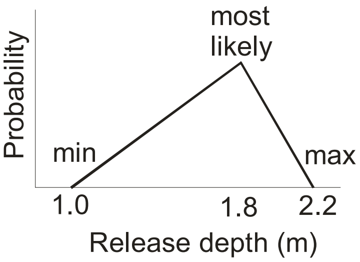

Figure 13: Hypothetical triangular distribution for release depth.

Suppose we allow the release depth, both friction coefficients and slab density to vary according to plausible distributions and do 10,000 runs. The hypothetical distribution of simulated stopping positions (runouts measured horizontally from the top of the start zone) might look like Figure 14, with 95% of the simulated avalanches stopping at or before 1550 m. In this example there is a highly unlikely combination of inputs yielding an extreme avalanche which stops beyond 1600 m, but the simulations might give us confidence that runouts past 1550 m are sufficiently unlikely. This does not mean that only about 5% of *all* avalanches would stop beyond 1550 m. Since the input parameters were based on knowledge of—or experience with—extreme avalanches, there is only a small probability that an *extreme* avalanche will run past 1550 m.

The distribution of impact pressures can also be calculated in the runout zone, as shown in the next section.

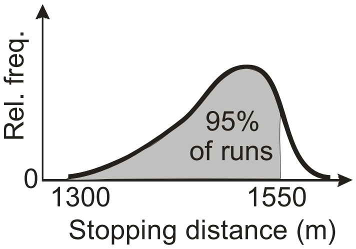

Figure 14: Hypothetical example of runout distribution based on simulation.

3.3 Simulations with dynamics models for an Icelandic path

Avalanches become less frequent farther along/down the runout zone. If the runout is expressed as a runout ratio, then the decrease tends to follow a particular statistical distribution (McClung and Lied, 1987; McClung and Mears, 1995). The runout ratio for any point in the runout zone is the horizontal distance past a reference point divided by the horizontal distance of that reference point from the top of the start zone (Fig. 4). Based on an annual frequency of avalanches (*F*) in an Icelandic path, Barbolini and Keylock (2002) calculated a return period which increases in the runout zone, e.g. the 300 year avalanche stops farther down the runout zone than the 30 year avalanche. However, *F* is not known accurately, so they simulated triangular distributions of return periods around the 30 and 300 year periods used for land use planning to get a distribution of runouts. Further, different sizes of avalanches can reach the same runout with different impact pressures along the runout zone. They adjust the parameters in a Voellmy-fluid model (Bartelt et al., 1999) so that the avalanches stop at points with the required return periods. This gives a distribution of impact pressures in the runout zone from which confidence intervals can be obtained for land use zoning.

4. SOME LIMITATIONS OF DYNAMIC MODELS FOR HAZARD MAPPING

- The friction coefficients must be calibrated, preferably from numerous extreme avalanches in the same range. Using the published range for the friction coefficients results in a range of impact pressures in the runout zone that exceeds the common values of the factor of safety for engineering (Mears, 1992, p. 29).
- When calibrating extreme avalanches with the Voellmy-Salm model (and perhaps PCM or PLK), the *calibrated* values for sliding friction μ

are sometimes between 0.15 and 0.20, which is well below *measured* values for dry snow sliding on dry snow, roughly 0.25 to 0.45 (Perla, 1980). This indicates that the physics of avalanche motion are not that well represented by the model.

- Using a Voellmy fluid model, Barbolini et al. (2000) found that a 15% variation in μ could change the runout by 10% and the impact pressure by up to 50%.
- Two or more friction coefficients cannot be independently validated (Jóhannesson, 1998; Lied et al., 1995).
- For 80 avalanche paths in the Austrian Alps, Lied et al. (1995) showed that with average values of the friction coefficients in the PCM model, runout estimates were more variable than with average values of the parameters in the α–β model for statistical runout estimation.
- In the practical dynamic models, the flow is greatly simplified. Many are 1D with depth-averaged flow, excluding flow characteristics that influence velocity, runout and impact pressure (Harbitz et al., 1998).
- Entrainment can increase the mass of a flowing avalanche by a factor of four (Sovilla et al., 2006)—and hence affect velocity, impact pressure and runout—yet entrainment is currently omitted from most practical dynamic models.
- Many dynamic models require the release area, release height/depth or mass of extreme avalanches, yet these measurements are seldom documented and hence infrequently available for calibrating models.
- The better calibrated models are 1D and hence do not estimate the lateral extent of extreme avalanches, which is important for hazard mapping. (Of course, the common statistical models also do not estimate the lateral extent of the runout.)

5. ADVANTAGES OF DYNAMIC MODELS FOR HAZARD MAPPING

- In contrast to statistical runout models, dynamic models allow the effect of start zone area and release mass or depth as well as friction coefficients to be modeled for different scenarios. Different sets of friction coefficients can be used in different parts of the path where the ground roughness changes or flow conditions are expected to change. Further, the uncertainty in these inputs can be simulated and used to increase confidence in certain extreme runouts. See Section 5 below.
- In contrast to statistical models, the effect of run-up and terrain anomalies on velocity and hence runout can be modeled.
- Velocities and flow heights and hence the impact pressure can be calculated at any point along the path.
- They provide an extreme runout estimate that can complement estimates from vegetation, human records and statistical models (Fig. 3).
- They can be used when other estimates are of low confidence or unavailable. For example, dynamic models have proven indispensable in the arctic where vegetation records are non-existent, human records are missing or very short (< 20 y), and calibrated statistical models do not exist.
- AVAL-1D has been commercially distributed since 1999, and now has more than 100 installations worldwide. The model parameters can be easily adapted to a specific situation, and parameter studies can be performed quickly. The results can be documented with tables and diagrams. In Switzerland, AVAL-1D is currently the standard model for hazard mapping.
- Dynamic models, because they model the velocity and flow height at terrain anomalies, are essential for designing defence structures. However for this application, friction coefficients are usually taken from extreme avalanches in the same path.
- Dynamic models of *powder* avalanche motion can assist in estimating the boundary between the blue and white zones. In the Canadian definition of the white zone, even extreme avalanches with a return period of 300 years are not expected to produce impact pressures greater than 1 kPa (Canadian Avalanche Association, 2002a). Many jurisdictions expect or require that public and unreinforced residential buildings only be located in the white or similar low hazard zones (BFF and SLF, 1984; Mears, 1992; Canadian Avalanche Association, 2002a),
- Dynamic models are a primary method for hazard mapping in Switzerland. During the extreme avalanche winter of 1999 in Switzerland, 97% of hazard maps proved effective (Gruber and Margreth, 2001). This suggests the uncertainty in dynamic models can

be mitigated with complementary methods and experience.
- Simulations can increase confidence in extreme impact pressures and zone boundaries (e.g. Barbolini et al., 2002).

> *Among avalanche researchers, there are two opposite attitudes towards [the] problem: One group considers that our knowledge of avalanche dynamics will always be insufficient and therefore advocates the use of the simplest models with three or fewer adjustable parameters that are to be calibrated extensively. The price to pay is a very wide range of these parameters that are moreover nearly devoid of precise physical meaning. Prime examples are the Voellmy-Salm and PCM models.*
>
> *The opposite attitude is to try to construct models that correctly capture the main physical processes in avalanche flow and contain parameters with a clear physical meaning. Advocates of this approach argue that the parameters can in principle be measured in experiments and their probable range of values can be guessed in advance.*
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> From invited talk by Deiter Issler at "L'ingegneria e la neve" of the Associazione Georisorse e Ambiente, Politecnico di Torino, Torino (Italy), 21 February 2006.

6. SUMMARY

Hazard mapping benefits from runout estimates from largely independent methods such as vegetation, human records, statistical models and dynamic models (e.g. Margreth and Gruber, 1998; Canadian Avalanche Association, 2002a, b).

Runout estimates from dynamic models involve substantial uncertainty due to simplifications and uncertainty involving the release area, release mass or depth, equations of flow including depth averaging, terrain in the track and runout, friction coefficients, entrainment and deposition, and lateral spreading. Some of the uncertainty can be simulated, increasing the confidence in estimates of extreme runout. In view of the uncertainty, highly sophisticated models with many poorly confined inputs are presently not practical for hazard mapping (Salm, 2004).

Before applying dynamic models, the expert must identify the critical scenario(s) for the extreme avalanches, e.g. the area of the starting zone(s) likely to release, flow type (dry dense, powder or wet), whether the flow will separate or leave the track taken by more frequent avalanches, etc.

The user of avalanche dynamic models should be able to approximate the endangered area independent of the model output and to recognize the most relevant input parameters. The results of the calculations help the expert achieve a more reliable and systematic hazard assessment.

Dynamic models using friction coefficients from sources other than extreme avalanches in the path to be mapped can provide useful runout estimates for hazard mapping but application of the models requires knowledge of their limitations and experience. Or as Peck (1980) wrote regarding geotechnical engineering "Judgement is required to set up the right lines for scientific investigation, to select appropriate parameters for calculations, and to verify the reasonableness of the results. What we can calculate enhances our judgements, permits us to arrive at a better engineering solutions."

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