CRITICAL LENGTH FOR THE ONSET OF CRACK PROPAGATION IN SNOW: RECONCILING SHEAR AND COLLAPSE

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ABSTRACT: The failure of a weak snow layer buried below cohesive slab layers is a necessary, but insufficient condition for the release of a dry-snow slab avalanche. The size of the crack in the weak layer must also exceed a critical length to propagate over a wide surface. In contrast to classical shear-based approaches, the anticrack model accounts for weak layer collapse and allows to better explain typical observations of remote triggering from flat areas. However, the latter model predicts that the critical length for crack propagation is independent of slope angle, a rather surprising and counterintuitive result. Our new mechanical model reconciles past approaches by considering for the first time the complex interplay between slab elasticity and the mechanical behavior of the weak layer including its structural collapse. The crack begins to propagate when the stress induced by slab loading and deformation at the crack tip exceeds the limit given by the failure envelope of the weak layer. We were able to reproduce crack propagation on flat terrain and the decrease of the critical length with slope angle observed in numerical experiments. Our new model agreed well with extensive field data of propagation saw tests and can easily be implemented into a numerical snow cover model.

KEYWORDS: Snow avalanche, crack propagation, critical length, slope angle, PST, slab, weak layer.

1 INTRODUCTION

Snow slab avalanches range among the most prominent natural hazards in snow covered mountainous regions throughout the world. The ability to reliably forecast avalanche danger is therefore of vital importance and requires a sound understanding of avalanche release processes.

Avalanches are the result of numerous factors and processes interacting over a large range of temporal and spatial scales (Schweizer et al., 2003). While snow slab avalanches can come in many different sizes, from a few meters to several kilometers, they all initiate within the snow cover by local damage processes at the grain scale. Indeed, the release of a dry-snow slab avalanche requires the formation of a localized failure within a so-called weak layer (WL) buried below cohesive slab layers (Fig. 1a). The initial failure – or crack – in the weak layer either forms in weak parts of the snowpack (Schweizer et al., 2008; Gaume et al., 2014), or below a local overload such as a skier or a snowmobile (van Herwijnen and Jamieson, 2005; Thumlert and Jamieson, 2014; Monti et al., 2016). Stress concentrations at the crack tip will then determine if crack propagation and eventually slope failure occurs (McClung, 1979; Schweizer et al., 2003), even if the average overlying stress is lower than the average weak layer strength (knock-down effect; Fyffe and Zaiser, 2004; Gaume et al., 2013; Gaume et al., 2014).

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These processes preceding avalanche release in the stratified snow cover can be described as crack propagation in a multilayered, elastic system under mixed-mode loading. For this complex fracture mechanics problem, theoretical and analytical approaches are not yet conceivable (Hutchinson and Suo, 1992).

The size of the initial crack at which rapid crack propagation occurs is called the critical crack length and represents an instability criterion for material failure. It is a crucial variable to evaluate snow slope instability (Reuter et al., 2015).

In the past, simplifying assumptions have been used to propose analytical models for the critical crack length. For instance, (McClung, 1979; Chiaia et al., 2008; Gaume et al., 2013; Gaume et al., 2014) assumed a weak layer without thickness which allowed solving the problem in the downslope direction only, by neglecting the effect of the structural collapse of the weak layer.

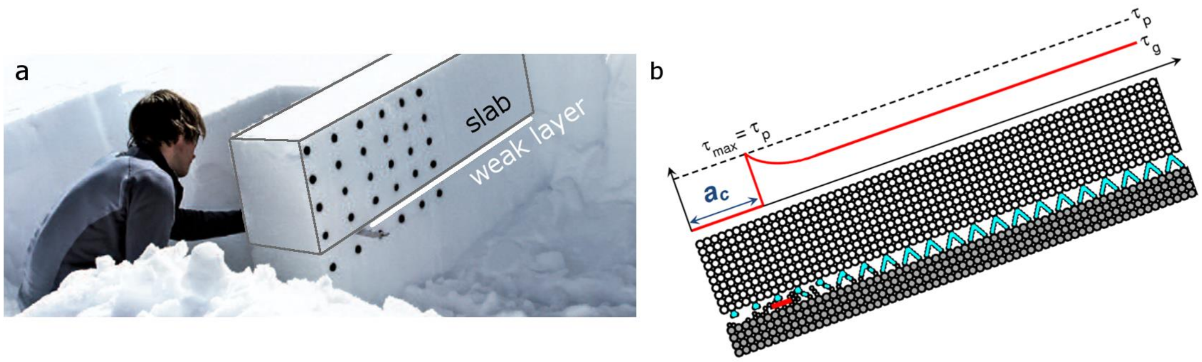

Fig. 1: (a) Propagation Saw Test. The weak layer is represented in white, the slab in grey. The black dots are markers used for particle tracking to measure slab deformation. (b) DEM simulation of the propagation saw test (PST). The plot on top represents an illustration of the shear stress (red line) at the onset of crack propagation in the weak layer. The red segment represents the saw.

On the other hand, (Heierli et al., 2008) assumed a rigid weak layer of finite thickness with a failure criterion that was independent of slope angle and a completely rigid behavior to neglect the elastic mismatch between the slab and the weak layer.

With the development of new field tests, in particular the propagation saw test (PST, Fig. 1a) (van Herwijnen and Jamieson, 2005; Gauthier and Jamieson, 2006; Sigrist and Schweizer, 2007), it is now possible to directly evaluate the critical crack length, and thus determine crack propagation propensity. Particle tracking velocimetry (PTV) analysis of PSTs has highlighted the importance of the bending of the slab induced by the loss of slab support while cutting through the weak layer (e.g., van Herwijnen et al., 2010).

To include weak layer collapse and slab bending – among other things – in the description of slab avalanche release mechanisms, Heierli et al., (2008) proposed the anticrack model. This model provides an analytical framework to estimate the critical crack length as a function of slab properties (thickness, density and elastic modulus) and the weak layer specific fracture energy – a material property quantifying the resistance to crack propagation. While some crucial features of the mechanical behavior of the weak layer, including elasticity and shape of the failure envelope are not included, the anticrack model provides a significant step forward as it accounts for various aspects that were left unexplained by previous theories, such as crack propagation on flat terrain and remote triggering of avalanches.

Clearly, the various methods to estimate the critical crack length all have their respective shortcomings, and a unified approach which incorporates all relevant processes is thus far missing. To overcome these limitations and take into account all the important physical ingredients, we evaluated the critical crack length for different snowpack stratigraphies using discrete element (DEM) simulations (Fig. 1b). On the basis of our numerical results (Gaume et al., 2016), we introduced a new expression for the critical crack length which accounts, for the first time, for the complex interplay between loading, elasticity, failure envelope of the weak layer (Reiweger et al., 2015) and its structural collapse.

2 NEW EXPRESSION OF THE CRITICAL CRACK LENGTH

2.1 Formulation of the model

Here we only present the new analytical model for the critical crack length; a full description can be found in Gaume et al. (2015, 2016). We consider a two-dimensional slab-weak layer system. The slab is characterized by its thickness D, density ρ, elastic modulus E, Poisson's ratio ν. The weak layer is characterized by its shear strength τ_p, its shear modulus G_{wl} and thickness D_{wl}. Slope angle is denoted ψ. Shear and normal stresses in the uncracked weak layer are assumed to be due to slab weight only and are given by $\tau_g = \rho g D \sin\psi$ and $\sigma_n = \rho g D \cos\psi$, respectively.

DEM simulations in Gaume et al. (2016) revealed that the critical crack length can be expressed as

$$a_c = \Lambda\left(\frac{-\tau_g + \sqrt{\tau_g{}^2 + 2\sigma_n(\tau_p - \tau_g)}}{\sigma_n}\right) \qquad (1)$$

The lengthscale $\Lambda = (E' D D_{wl}/G_{wl})^{1/2}$ where $E' = E/(1-\nu^2)$ is the plane stress elastic modulus of the slab. Thus, our new formulation accounts for the interplay between slab elasticity and the me-

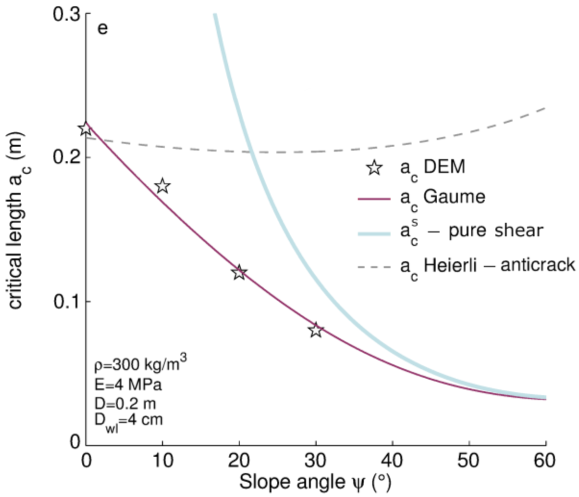

Fig. 2: Critical length a_c for crack propagation as a function of slope angle ψ. The symbols represent the critical crack length obtained from the DEM simulations and the red solid line represents the critical length modeled with Eq. 1. The green line represents the pure shear model. The dashed line indicates the critical length obtained with the anticrack model assuming w_f=0.1 J/m^2.

chanical behavior of the weak layer including its structural collapse. For flat terrain, the critical crack length $a_c = \Lambda\sqrt{2\tau_p/\sigma_n}$ is a function of the normal stress σ_n.

2.2 Sensitivity analysis

To investigate the influence of snow cover parameters on the critical crack length a_c we performed a sensitivity analysis by varying the system parameters (slab, weak layer and slope angle) independently (Gaume et al., 2016). Overall, a_c increased with increasing elastic modulus of the slab E and with weak layer thickness D_{wl}. On the contrary, a_c decreased with increasing slab density ρ and with increasing slab thickness D. More importantly, the critical crack length decreased with increasing slope angle ψ (Fig. 2). The rate of decrease was strongly influenced by the elastic modulus E and thickness D of the slab. Low values of E and/or D lead to a gentler decrease of a_c with ψ (Gaume et al., 2016). In addition, for steep slopes (>30°) our model gives very similar results as the so-called pure shear model – assuming an infinitely thin weak layer so that no bending occurs. As the model (Eq.1) was derived from the DEM simulations, it agrees well with DEM results in Fig. 2.

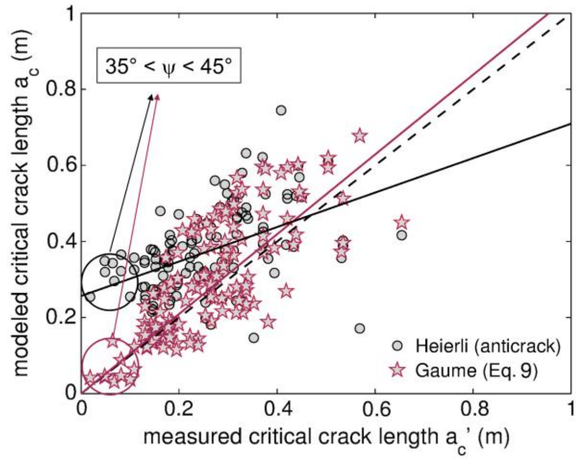

Fig. 3: Comparison between measured and modeled critical crack lengths using the anticrack model (Heierli et al., 2008) (black circles) and our new model (Eq. 2, red stars). The continuous lines represent linear fits. The dashed line represents the 1:1 line.

3 COMPARISON WITH FIELD DATA

To compare the prediction of our model with field data, we used a dataset consisting of 93 PST experiments (Gaume et al., 2015). It includes the average slab density ρ, slab thickness D, slope angle ψ and WL thickness D_{wl}. Other properties required for the comparison, such as the weak layer specific fracture energy w_f, the weak layer shear strength τ_p and shear modulus G_{wl}, the elastic modulus E and Poisson's ratio ν of the slab, were either derived or evaluated from literature. Please refer to Gaume et al. (2016) for more details.

The predictions of Eq. 1 compare well with field measurements (Fig. 3). Overall, our model provides very good estimates of the measured critical crack lengths, as demonstrated by the proximity of the data to the 1:1 line despite substantial scatter (R^2 = 0.53). The predictions of the anticrack model overestimate the critical crack length, in particular for steep slopes.

4 DISCUSSION AND CONCLUSIONS

We proposed a new analytical expression to assess the conditions for the onset of crack propagation in weak snowpack layers. The formulation was developed based on discrete element simulations; it accounts for crucial physical processes involved in crack propagation in snow, namely the complex mechanical behavior of the weak layer and the mixed stress states in the slab induced by slab tension and bending resulting from weak layer failure. A critical parameter in the formulation is the length scale Λ, accounting for the elastic mismatch between the slab and the weak layer. The analytical expression for the critical crack length convincingly reproduced field measurements obtained from 93 propagation saw test experiments.

Our new model reconciles the shear- and collapse-based approaches. For example, our model can describe crack propagation in flat terrain and thus remote triggering, similar to the anticrack model. However, it also predicts a decrease of the critical crack length with increasing slope angle, in line with shear-based models and in contrast with the anticrack model. This implies that skier-triggered avalanches are more likely on steep rather than on flat slopes, a rather intuitive result. The discrepancy with the anticrack model arises from the fact that the latter assumes (i) that the failure behaviour of the weak layer is slope independent, (ii) disregards weak layer elasticity, and (iii) does not properly account for the interplay between tension and bending in the slab as also suggested by the finite element simulations by (van Herwijnen et al., 2016).

Heierli et al. (2008) used field experiments by Gauthier and Jamieson (2008) to support their finding that the critical crack length does not depend on slope angle. However, these PST experiments were performed on a non-persistent weak layer and measurements made on the flat were performed one day before the experiments made on slopes (Gauthier, 2007). We argue that the reported trend with slope angle may have been influenced by the burial time of the weak layer since sintering and settlement effects can strongly affect snowpack properties within one day, especially with the layer of precipitation particles which was tested (Szabo and Schneebeli, 2007; van Herwijnen and Miller, 2013; Podolskiy et al., 2014). Furthermore, Heierli et al. (2008) assumed snow cover properties independent of slope angle, which is somewhat questionable since snowpack properties can also change with slope angle, thus obscuring the true slope angle dependency.

Gaume et al. (2016) showed that the almost constant trend of the critical crack length observed in the field (Gauthier and Jamieson, 2008; Bair et al., 2012) is due to variations in snowpack properties with slope angle, geometrical effects and to the fact that these studies were performed on storm snow leading to very low slab elastic moduli thus inducing a reduction in the decreasing rate of the critical crack length with slope angle.

Several field studies mainly using the Extended Column Test (ECT) reported even increasing scores with increasing slope angle (e.g., Heierli et al., 2011; Bair et al., 2012). These observations can actually be explained if one considers the vertical faces of the test columns. In fact, Gaume et al. (2016) predicted an increasing trend of the critical crack length with slope angle for PSTs performed with vertical faces and a constant slab depth.

Finally, our new model was implemented in the snow cover model SNOWPACK (Gaume et al., 2016). If the critical crack length is calculated for each layer in the simulated snow stratigraphy, the deeper critical weak layers nicely show. Their modeled critical crack lengths were in fair agreement with critical crack lengths measured in the field (PST). This opens promising perspectives to improve avalanche forecasting.

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