MITIGATION EFFECTS OF TREES ON ROCKFALL HAZARDS: DOES ROCK SHAPE MATTER?

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the date of receipt and acceptance should be inserted later

Abstract Does rock shape matter to the mitigation effects of trees on rockfall hazards? This question must be resolved in order to better quantify the protective role of mountain forests against rockfall. To probe this question we investigate a single rock-tree interaction using non-smooth, hard-contact mechanics that allows us to consider rock shape at impact. The interaction of equant shaped rocks with cylinder-like tree stems is modelled. The equant shaped rocks are close to spherical but have a certain shape variability governed by the rock's surface area ratio and aspect ratio. This work serves as an important follow-up study to the existing investigations from Toe et al. (Landslides 14: 1603-1614, 2017), where the effects of trees on block propagation are numerically investigated using spherical shaped rocks. The objective of our simulations is to understand how and to what extent, shape will influence energy dissipation and trajectory change. The primary results include: surface area ratio plays a more important role than aspect ratio in determining the rock's post-impact dynamics. The primary parameters governing the rock kinematics after impact (i.e. block's energy reduction, reflected rotational

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speed, and trajectory change) are impact velocity, impact eccentricity, and the tree stem diameter. The latter observation aligns well with previous findings and suggests that the shape factors, at least for nearly spherical rocks, can be integrated into the current block propagation models. However, from a statistical viewpoint, the anisotropic distribution of mass and hence the asymmetric moment of inertia of non-spherical rocks, leads to stronger or weaker spin effects compared to mass- and volume-equivalent spheres. Apparently, the rotational motion of an irregular object serves as a kinetic energy reservoir leading to subsequent rock-tree impacts, and therefore significant differences in energy loss and trajectory in comparison to spherical shaped rocks. This effect must be further investigated using elongated and flattened blocks and underscores the importance of measuring rockfall rotation in experimental investigations.

Keywords Rockfall · Rock-tree interaction · Non-spherical rock · Hazard mitigation · Computational modelling · Simulation

Nomenclature

English Alphabet

A	Closest point from tree stem to rock
B	Rock-tree contact point
c_d	drag coefficient (kg m^{-1})
C	Rock-tree contact frame
e_x^I, e_y^I, e_z^I	Ground inertial-frame coordinates
e_x^K, e_y^K, e_z^K	Rock eigen-frame coordinates
d_0, d_1	Tree stem bottom, top diameter (m)
δE_{rot}	Change rotational energy (J)
δE_{tot}	Change total kinetic energy (J)
δE_{tot}^{max}	Maximum loss total kinetic energy (J)
δE_{tra}	Change translational energy (J)

ΔE_{red}	Percentage reduction total kinetic energy
ΔE_{rot}	Percentage change rotational energy
ΔE_{tot}	Percentage change total kinetic energy
ΔE_{tra}	Percentage change translational energy
E^+	Rock energy after impact (J)
E^-	Rock energy before impact (J)
F_d^I	Drag force (N)
F_g^I	Gravitational force (N)
G	Contact penetration depth (m)
h	Tree stem height (m)
h_c	Rock-tree collision height (m)
I	Ground inertial-frame
K	Rock eigen-frame
l_X, l_Y, l_Z	Length of principal axis (m)
m	Rock mass (kg)
n	Rock-tree contact normal vector
N	Number of simulations
p_{IK}	Rock orientation indicator
q	Generalized coordinates
Q	Tree stem
r_0, r_1	Tree stem bottom, top radius (m)
r_{OS}^I	Tree stem positional vector
R	Tree stem bottom center
S	Rock center of mass
S'	Center of rock's bounding sphere
t_s	Contact separating axis
t'_s	Reference axis
T^K	Torque (N m)

u	Rock positional vector
v_{in}	Impact translational velocity (m s^{-1})
V_S^I	Rock translational velocity (m s^{-1})
$W(q)$	Matrix of generalized force directions
$\hat{\mathcal{X}}, \hat{\mathcal{Y}}$	PCE model input, output

Greek Alphabet

β	Error index
χ	Vertical impact angle (rad)
ϵ_n, ϵ_t	Normal, tangential restitution coefficient
γ	Rock aspect ratio
ι	Rock volume (m^3)
κ	Tree stem dimensional parameter
λ	Contact force vector
λ_n^C, λ_t^C	Normal, tangential contact force (N)
μ	Frictional coefficient
ν	Generalized velocities
ω_{in}	Impact rotational velocity (rad s^{-1})
ω_{out}	Reflected rotational velocity (rad s^{-1})
Ω^K	Rock rotational velocity (rad s^{-1})
ϕ	Horizontal trajectory change (rad)
ψ	Vertical trajectory change (rad)
ρ	Rock density (kg m^{-3})
τ	Sobol sensitivity index
θ_S^K	Rock inertia tensor
φ	Fixed angle $\pi/16$ (rad)
ϑ	Horizontal impact angle (rad)
ξ	Rock surface area ratio

$\Xi(q, \nu, t)$ Term of gravitational, gyroscopic forces

$\zeta_{BT}, \zeta_{BS'}$ Eccentricity indicators

1 Introduction

Rockfall, one commonly encountered form of landslide, has long been recognized as a major natural threat to human lives and public facilities in mountainous regions (Dorren 2003; Volkwein et al. 2011). Unlike rock avalanches and debris flows, rockfall is governed by discrete, collisional events that occur between a single rock and the environment, resulting in distinguishable, dynamic modes for the object i.e. free falling, bouncing, rolling, and sliding, and, in turn, an appreciable run-out distance from the release area (Dietze et al. 2017).

In order to reduce the losses caused by rockfall hazards, a robust design and management of technical (berms, dams, catching-nets) and natural (forests) countermeasures is essential (Crosta et al. 2015). The significance of the protective forest in rockfall hazards mitigation is widely recognized by scientists, engineers, and forest managers within the rockfall hazard community (Moos et al. 2018; Dupire et al. 2016a). When a rock collides with tree stands, the kinetic energy of this solid body will be (at least partially) dissipated, leading to a reduced speed, jump height and length, and run-out for the rock. As recently demonstrated by Radtke et al. (Radtke et al. 2014), the efficient management of coppice forests can maximize rockfall protection and therefore forests can act as a cost-effective protection measure. It has also been recently shown that even the biodiversity and the structural heterogeneity of the trees will affect the performance of the forest in alleviation of rockfall risks (Dupire et al. 2016b).

The logistical and experimental challenges of performing field-scale measurements of rockfalls in forests have hindered a contact scale understanding of the complex rock-tree interaction process (Bourrier et al. 2012). Advanced experimental techniques for controlled rockfall observations such as integrating a micro-

sensor into rocks to record their dynamics e.g. velocities and accelerations, have been applied only to open, non-forested rockfall slopes (Caviezel et al. 2018; 2019). Airborne laser scan instruments have been employed to sample spatial forest data e.g. stem density and diameter distribution, and then used as an input basis for rockfall simulation software (Monnet et al. 2017).

Numerical modelling has therefore become an indispensable tool to investigate on different kinematic levels the rock dynamics for rock-tree impact conditions, thanks to many existing computational mechanics methods (Li and Lan 2015; Leine et al. 2014; Toe et al. 2018b; Macciotta et al. 2015) and software calibration skills (Wyllie 2014; Corona et al. 2017; Volkwein et al. 2018; Bourrier et al. 2012). In this context, Toe et al. (Toe et al. 2018a) performed simulations of rockfalls on forested slopes within the discrete element modelling (DEM) framework (Lu et al. 2015). In particular, they constructed a non-spherical, approximately cubic rock shape and a virtual terrain having certain surface roughness composed by clustering of spheres, and approximated trees as deformable chains of interconnected cylinders of varying diameters. Their numerical results highlighted the important influence of forest spatial structure and tree basal area on rockfall mitigating efficiency. Similarly, Moos et al. (Moos et al. 2017) employed RockyFor3D (Dorren 2015), a three-dimensional, probabilistic, process-based rockfall trajectory simulation software, quantitatively investigating the effect of forest on reaching frequency and intensity of rockfalls. Different forest and non-forest scenarios were generated on a virtual slope, and a reduction up to 90% in reaching frequency and between 10% and 70% in intensity was found for rockfalls on forested compared to non-forested terrains. Three critical factors were emphasized for the risk assessment, i.e. the volume of rock, the basal area of trees, and the spatial distribution within the forest. Owing to the combined experimental and numerical modelling techniques, it has been reported that the protective effects of forest are very pronounced when the kinetic energy of a falling rock is below 200 kJ (Monnet et al. 2017).

Despite these achievements obtained from slope scale simulations, the energy dissipation and the trajectory change of a block after its impact with a single tree stand have been only poorly understood. This investigation is crucial because it provides from a contact scale perspective on how rocks behave when they meet an obstacle that stays on their moving paths. The findings are intrinsically valuable in developing models integrating the single-tree effect on rockfalls into the so-called block propagation models in forest (Toe et al. 2017). Again using DEM simulations, Toe et al. (Toe et al. 2017) were among the first to identify the leading parameters governing the rock kinematics, i.e. energy dissipation, reflected rotational speed, and trajectory change, after its collision with a tree stand: the impact velocities, the tree stem diameter, and the eccentricity at the impact instant. Their research implied a beneficial simplification to the description of block propagation in forest, because less important elements such as the impact height and the impact vertical incidence could be omitted. Nonetheless, DEM normally requires acquiring many mechanical parameters in fields to calculate the contact forces exerting at the rock's boundary. This adds additional difficulties to the system calibration (Dorren and Berger 2005; Bertrand et al. 2013) and to an accurate extraction of the key parameters that dominate rock kinematics upon impacts. Furthermore, so far only 'virtual' rocks represented by a spherical geometry (Toe et al. 2017) or by assembling many composite spheres (Toe et al. 2018a) have been used in the rockfall simulations, leaving the effect of rock shape (especially one having a sharp boundary) on rock-tree interactions largely unknown.

This study investigates non-spherical rocks interacting with single tree stems using the non-smooth mechanics dynamics coupled with the hard contact laws (Leine et al. 2014). As a start, we concentrate on the near-spherical, so-called equant rocks, meaning that the lengths of three principal axes of the rock are approximately identical. Two shape factors are then introduced to characterize a three-dimensional rock geometry: aspect ratio and surface area ratio. The former is given by the ratio between the two extreme lengths of a rock's principal axes;

the latter is defined as the ratio between the surface area of a rock and that of the volume-equivalent sphere. For the defined equant rocks, the aspect ratio falls between 0.7 and 1.3 and the surface area ratio is in the range from 1 to 1.3. Firstly, the mathematical framework adopted in this work for modelling rock-tree interactions will be established and the initial simulation results using spherical rocks will be validated against the existing researches. Subsequently, two groups of equant rocks, all having the same volume and mass, will be simulated against a determined tree stem. Test set 1 consists of a sphere together with two rocks of different surface area ratios and aspect ratio 1.0; test set 2 groups three non-spherical rocks of the same surface area ratio but different aspect ratios. These test sets allow us to separately investigate the effects of rock aspect ratio and surface area ratio on its energy dissipation and trajectory change for the studied rock-tree interaction cases. Finally, a total 110 artificial or natural equant rocks, falling into a pre-determined range of aspect ratios and surface area ratios, will be ‘thrown’ towards tree stems of varying dimensions. The objectives of these simulations are to answer the following questions: are the kinematic leading parameters arising from using a spherical rock shape (Toe et al. 2017) still valid for equant rock shapes? In addition, how important are a rock’s aspect ratio and surface area ratio to its dynamic behaviour during a rock-tree interacting process? Conclusions will be drawn on a practical level regarding the mitigation effects of trees on rockfall hazards. Outlooks for future research will also be provided.

2 Mathematical Framework Implemented for Modelling Rock-Tree Interactions

The non-smooth mechanics method coupled with the hard contact laws is adopted in this work for modelling rock-tree interactions. The main advantage of using this numerical algorithm over DEM is that the number of mechanical parameters is significantly reduced. This helps avoid many potential difficulties regarding system calibration and extraction of the leading parameters that govern rock-tree impacts.

One remarkable example is that the translational and rotational spring stiffness parameters, that are widely used in DEM simulations (but not in our method), are numerical rather than physical parameters. Excluding these parameters is greatly beneficial to simulation efficiency and model calibration. The reader is referred to Lu et al. (Lu et al. 2018) for a detailed comparison of the pros and cons of these two methods. The implementation of the non-smooth mechanics modelling for rockfall on an arbitrary slope was addressed elsewhere (Leine et al. 2014; Lu et al. 2019). The main aspect will only be shortly presented in the following sections. Nonetheless, readers are recommended to acquire the full details provided in the references.

2.1 Rock and Tree Stem Models

A rock can be modelled either as a sphere or an irregular non-spherical polyhedron. A sphere is determined via its geometrical center and radius. To build a polyhedron a point cloud is artificially provided or extracted from a natural rock shape using the software VisualSFM (Wu 2011) and Meshlab (Cignoni et al. 2008). The calculation of the rock’s mass and inertia properties follows the literature (Mirtich 1996). In this work a polyhedral rock requires 10-120 vertices for an accurate rock-tree contact detection. A rock is represented in its local, eigenframe K , which is attached to the mass center S of the object. The three axes e_x^K , e_y^K , and e_z^K overlap with the rock’s principal axes of moment of inertia. Choosing K to express a non-spherical rock’s inertia tensor is advantageous since the latter is timely invariant in the eigenframe K .

In contrast, a tree stem is described using the global, inertial-frame I , which is fixed to the origin O of the simulated terrain. The axes e_x^I and e_y^I span the horizontal plane, while the axis e_z^I extends along the vertical direction. Here we do not intend to model the root and crown system for a tree as done by Toe et al. (Toe et al. 2017). We model only the tree stem. A tree stem is simplified as a non-deformable, truncated cone, with the symmetry axis pointing along e_z^I .

Fig. 1 Tree stem model: a tree stem is abstracted into a non-deformable, truncated cone, characterized by the bottom circle diameter d_0 , the top circle diameter d_1 , and the height h . The dimensionless parameter κ , which correlates the tree stem dimensions, is normally negative.

As Fig. 1 shows, this simplification captures the major feature of a general tree stem. The center R at the bottom surface specifies the x^I and y^I locations of the tree stem, whose dimensions are given by the bottom circle diameter d_0 , the top circle diameter d_1 , and the height h . In order to perform rock-tree interaction calculations, one must set mechanical parameters, i.e. the coefficient of restitution ϵ_n and ϵ_t in the contact normal and tangential directions, respectively, and the coefficient of friction μ .

2.2 Equations Governing Rock's Motion

At each moment, the position and orientation of a rock is described using the generalized coordinates q :

$$q = \begin{pmatrix} r_{OS}^I \\ p_{IK} \end{pmatrix} \in \mathbb{R}^7 \quad (1)$$

where r_{OS}^I is the positional vector pointing from O to S , and p_{IK} is the rock's orientation indicator, i.e. quaternion that satisfies $\|p_{IK}\| = 1$ (Lu et al. 2015).

The time derivative of q leads to the generalized velocities ν for the rock:

$$\nu = \frac{\partial}{\partial t} q = \begin{pmatrix} \dot{r}_{OS}^I \\ \dot{p}_{IK} \end{pmatrix} = \begin{pmatrix} V_S^I \\ \Omega^K \end{pmatrix} \in \mathbb{R}^6 \quad (2)$$

189 where V_S^I is the translational velocity expressed in I , and Ω^K is the rotational
 190 velocity addressed in K . The time evolution of V_S^I follows:

$$\frac{\partial}{\partial t}(mV_S^I) = m\dot{V}_S^I = F_g^I + F_d^I \quad (3)$$

191 where m is the mass of rock. F_g^I and F_d^I is the gravitational force and the external
 192 drag force exerting at S , respectively. Accordingly, the time evolution of Ω^K obeys
 193 Euler's equation:

$$\theta_S^K \dot{\Omega}^K + \Omega^K \times \theta_S^K \Omega^K = T^K \quad (4)$$

194 where θ_S^K defines in K the inertia tensor for the rock, and T^K is the additional
 195 torque generated by contact forces acting on the rock's boundary faces. If there is
 196 no detected rock-environment collision, T^K equals to zero. Hence, the equations
 197 governing rock's motion are summarized as:

$$\left\{ \begin{array}{l} M\dot{\nu} - \Xi(q, \nu, t) = W(q)\lambda \\ M = \begin{bmatrix} mI_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & \theta_S^K \end{bmatrix} \\ \Xi(q, \nu, t) = \left(F_g^I + F_d^I, -\Omega^K \times \theta_S^K \Omega^K \right)^T \end{array} \right. \quad (5)$$

198 where λ is the contact force vector, $W(q)$ is the so-called matrix of generalized
 199 force directions transferring all the contact forces λ from the rock's boundary faces
 200 to the rock's center of mass S . The term $\Xi(q, \nu, t)$ contains the gravitational and
 201 gyroscopic forces generated by the non-spherical rock's rotation.

202 Generally, for a rock-tree impact a contact is reported if at least one vertex of
 203 the rock 'penetrates' into the body of tree stem, generating an effective penetration
 204 depth $G > 0$. At every contact point, a local contact frame C is constructed within
 205 the frame I specifying the contact normal and tangential directions, such that the
 206 contact force can be decomposed onto these directions. In the contact normal
 207 direction the Signorini condition correlates the normal contact force λ_n^C and the

penetration function G following:

$$\lambda_n^C \geq 0 \perp G \leq 0 \quad (6)$$

This formula indicates $\lambda_n^C \geq 0$ for a closed contact characterized by $G = 0$, and $\lambda_n^C = 0$ for a case of no contact with $G < 0$. In the contact tangential direction, the spatial Coulomb's friction law is applied:

$$-V_t^C = \begin{cases} \{0\} & \text{if } \|\lambda_t^C\| < \|\mu\lambda_n^C\|, \quad \text{sticking} \\ \Re_{\geq 0}\lambda_t^C & \text{if } \|\lambda_t^C\| = \|\mu\lambda_n^C\|, \quad \text{slipping} \end{cases} \quad (7)$$

where μ is the sliding frictional coefficient. The mechanical structure switches from the sticking to the slipping mode if the frictional force λ_t^C increases and reaches the sliding frictional force $\mu\lambda_n^C$. The negative sign in Eq. 7 implies that λ_t^C acts along the counter-direction of the tangential velocity V_t^C measured at this contact point.

2.3 Rock-Tree Contact Detection Models

The contact detection routine is distinguished into two paths: DCT (Direct Contact Test) and SAT (Separating Axis Test), depending on the spherical or non-spherical rock shape. The interaction between a rock and a tree stem is considered as hard contact (Leine et al. 2014; Lu et al. 2019), preventing the rock from further penetrating into the tree stem; thus the energy dissipation of the rock caused by the uprooting and the swaying of tree stem/crown is not incorporated. In order to efficiently identify collisions between non-spherical rocks and tree stems, a global contact detection phase is in general required by performing AABB (Axis Aligned Bounding Box) tests (Lu et al. 2012). The AABB for a tree stem is determined only once at the start of simulation, while the position and the orientation of the AABB for a non-spherical rock needs to be updated every time step. A bounding

sphere is built on top of a non-spherical rock's bounding box to further speed up the contact detecting procedure (Lu et al. 2012).

2.3.1 DCT

Fig. 2a shows the principle of the rock-tree contact detection model DCT. To determine the contact between a sphere and a tree stem, the point A that is closest to the center of sphere S must be identified on the surface of the tree stem. If A is found inside the spherical body, a closed contact must be generated and the corresponding contact forces should be calculated. The contact forces are acting at the unique point B on the spherical surface which has penetrated the deepest into the tree stem. Apparently the line AB passes through the center of sphere S and defines the contact normal n , which points towards the outside of the tree stem body.

To efficiently locate the point A , the spatial region that is outside the tree stem Q is further divided into five non-overlapping parts from P_1 to P_5 . Note, in this study a rock cannot contact Q in the area of P_2 and P_4 because the tree stem will be rooted on a horizontal plane. In real slope simulations, the rock might fall into contact within these two regions as Q is generated vertically and a slope is normally inclined. Thus a 'gap' will be generated between the bottom of the tree stem and the slope surface. Adding P_2 and P_4 simply addresses any potential zone for rock-tree contacts. The remaining task is to identify which region the center of sphere S falls into. As shown in Fig. 2b, if S is found in P_1 , one can immediately determine A as the projection of S along the vertical direction onto the top surface of the tree stem. If S is in P_3 , A is the crossing point given by the plane passing through the symmetry axis of the tree stem and the center of sphere S , the top surface, and the side surface of the tree stem. Similar algorithms can be performed if S locates in P_2 or P_4 . To determine A when the center of sphere S is in P_5 , one needs to project S to the tree stem body Q along a line that is perpendicular to the side surface of the tree stem. Note, for visualization convenience Fig. 2b

Fig. 2 Scheme of the contact detection model DCT: (a) definition of the contact point B , the penetration depth $G = AB$, and the contact normal n for a sphere that is in ‘touch’ with a tree stem Q ; (b) the spatially divided parts $P_1, P_2, P_3, P_4,$ and P_5 that are outside a tree stem body Q , allowing efficiently identifying the contact point B and the closest point A from the tree stem surface to the center of sphere S .

only shows the contact detection model in two-dimensional space; the algorithms follow the same procedures for the three-dimensional case.

2.3.2 SAT

If the rock shape is non-spherical, the well-known SAT method (Gottschalk et al. 1996) is invoked for the contact detection model taking into account the specific geometry of the tree stem. Taking a two-dimensional case as an example, two convex polygons do not intersect in the SAT state if and only if a line is present, so that the projections of the two polygons do not intersect on the line. This line is called a separating axis. Fig. 3 illustrates the cases in two-dimensional plane that two rectangles are and are not intersecting with each other, respectively. The same principle can be extended to three-dimensional conditions.

The contact detection between a polyhedron and a truncated cone has several steps (Fig. 4): broad phase, narrow phase, and evaluation phase. The objective of the broad phase is to perform geometrical analysis demonstrating that no intersection between objects is formed. Thus, the cases in which a contact is evidently impossible can be ruled out. As shown in Fig. 4a, the bounding sphere of a rock is examined in coordinates relative to R against (i) the top and the bottom surfaces,

Fig. 3 Scheme of the SAT method in two-dimensional space: (a) Separating axis exists between two rectangles when a line can be found such that the projections of the two rectangles on this line do not intersect; (b) Separating axis does not exist as the projections of the two rectangles intersect on all of the potential separating axes.

(ii) the bounding cylinder, and (iii) the side surface of the tree stem. If any of the No-Collision conditions highlighted by the black boxes in Fig. 4a is met, the contact detection routine will directly exit and report no contact; otherwise the narrow phase will be performed, i.e. the tree stem and the rock will be projected onto several ‘candidates’ of separating axes to evaluate the potential overlap between the two objects. In Fig. 4b, these critical separating axes t_s may refer to (i) the symmetry axis of the tree stem, (ii) the normal vectors of the rock faces, and the specific vectors in space determined by the rock’s (iii) vertices and (iv) edges. Note, t_s that is designated to the latter two situations is perpendicular to the side surface of the tree stem. In case that non-zero overlaps are found through (i) to (iv), we only record the minimum overlapping length and the corresponding direction along that axis t_s ; a subsequent evaluation phase is consequently required to generate the contact points between the rock and the tree stem.

For the above unique axis t_s on which the minimum non-zero overlap has been found, a contact evaluation phase is performed where the defined contact points are added to the contact set. In Fig. 4c, depending on the spatial orientation of the axis t_s , rock-tree contacts can occur to the tree stem on either (i) the inner region of the top surface, (ii) the side surface, or (iii) the boundary circles of the top and the bottom surfaces. To distinguish these circumstances, a reference axis

Fig. 4 Scheme of the contact detection model SAT: (a) a broad phase where the bounding sphere of a rock is checked with respect to (i) the top and the bottom surfaces, (ii) the bounding cylinder, and (iii) the side surface of the tree stem. The vector u is the relatively positional vector pointing from the tree stem's root R to the center S' of the rock's bounding sphere. The No-Collision conditions are given in the black boxes; (b) a narrow phase where the spatial geometries of the tree stem and the rock are projected onto several 'candidates' of separating axes t_s , i.e. (i) the symmetry axis of the tree stem, (ii) the normal directions of the rock faces, and the specific directions determined by the rock's (iii) vertices and (iv) edges. This procedure aims to identify if there is a non-zero overlap existing along the direction given by t_s . Note for (iv) that for a given truncated cone and an edge of rock aligning with a line that is not contained in the cone, there exists two tangential planes which contain the rock's edge and attach at two different side surface lines on the tree stem. The two subsequently obtained t_s are perpendicular to these tangential planes; (c) an evaluation phase where the contact points are generated on (i) the inner region of the top surface, (ii) the side surface, and (iii) the boundary circles of the top and the bottom surfaces of the tree stem. The threshold angle φ between the reference axis t'_s and the symmetry axis of tree stem is fixed to $\pi/16$. The key conditions that the cases (i) and (ii) should satisfy are highlighted in the black boxes. Note that t_s is a normalized unit vector in these equations.

t'_s is introduced which forms a small threshold angle $\varphi = \pi/16$ with respect to the tree's symmetry axis. Here the angle φ is chosen as a constant value which does not depend on κ but simplifies the contact evaluation phase. Because the modelling of the tree stem geometry as a truncated cone is already an abstraction, using a fixed φ would not significantly affect the calculated rock behaviour near the top of a tree stem. Considering now the case (i), we pinpoint the potential contact points by firstly performing iterations over all the rock edges and capturing those which intersect with the tree stem's top surface. Then we loop through all the rock vertices and record those which lie inside and are sufficiently close (± 0.01 m) along e_z^I to the tree stem's top surface. To speed up the numerical solver (Leine et al. 2014) for calculating contact forces, the number of the final contact points is filtered down to a maximum size of 5 by only considering the above potential contact points having relatively smaller coordinates along e_z^I . If no contact point can be identified using the above routine, the crossing point formed by the symmetry contact plane (the plane which passes through the tree stem's symmetry axis and the axis t_s), the top surface, and the side surface of the tree stem is referred to as a default return. Turning now to case (ii), we firstly iterate over all the rock edges and intersect them with the symmetry contact plane. Note, chances exist that all the rock vertices locate outside the tree stem body but some rock edges are still intersecting with the latter. The intersecting points that are located inside the tree stem body are treated as some potential contact points. Then, we check all the rock vertices and collect those nodes which are falling into the tree stem body and are close enough (± 0.01 m) to the symmetry contact plane in distance. The selected threshold 0.01 m only aims to control the contact points generated between the rock and the tree stem body, which should not be too far away from the symmetry contact plane. Finally, the resulting contact set is sieved to a maximum of 2 contact points which are extreme with respect to their coordinates along e_z^I . If no contact point can be found through the above routine, a default contact point is selected, which is obtained by sorting the rock vertices

and identifying the one having the shortest distance to the side surface of the tree stem measured along t_s . In the end, if both (i) and (ii) are not the case, we switch to case (iii) where a contact point can be allocated on the bounding circle of the top or the bottom surface of the tree stem, the same as the default contact point appointed in (i). The contact evaluation phase guarantees that at least one ‘contact’ point will be generated once an effective overlap is found between the rock and the tree stem. Close attention must be paid that the contact detection model SAT is specifically tailored for the rock and tree stem geometries considered in this work, and one must be adaptive when applying the method on other applications.

2.4 Numerical System Setup and Simulation Steps

The numerical system is established for simulating interactions between one rock and one tree stem. Some simplifications to the studied problem are made in order to highlight the effect of rock shape on its energy dissipation and trajectory change after impacting with a tree stem: (1) Following the investigation of Toe et al. (Toe et al. 2017) and the truncated cone model used here, it is presumed that the rock-tree collision height h_c does not govern the rock behaviour; thus the height of the tree stem is chosen as a constant parameter $h = 10$ m; (2) It is presumed that the tree stem can absorb any kinetic energy from the rock, i.e. the tree stem is a rigid body perfectly fixed to the ground; (3) The same as the rock-terrain interactions (Leine et al. 2014; Lu et al. 2019), the scarring drag force is introduced to consider the additional energy dissipation mechanisms due to the actual deformation of the contact surface. This external drag force F_d^I acts at the rock’s center of mass and equals to the drag coefficient c_d times the squared magnitude of rock’s velocity v_{in} . Here c_d is set to a constant value 100; (4) Again the same as the rock-terrain interactions (Leine et al. 2014; Lu et al. 2019), the coefficients of restitution in the rock-tree contact normal and tangential directions are both set to zero. Note that here the coefficients of restitution are defined at the contact points between the rock and the tree stem body. Although the post-impact velocities are zero

Table 1 Ranges and values of the major input parameters describing rock and tree stem properties and rock-tree interaction processes. The values of γ and ξ define the ‘equant’ rock geometries.

Tree		Rock	
d_0 (m)	0.05-0.6	ρ (kg m ⁻³)	1600-2800
κ (-)	-0.01	ι (m ³)	0.1-1.0
μ (-)	0-1.2	γ (-)	0.77-1.30
ϵ (-)	0	ξ (-)	1.02-1.27
Rock-Tree			
v_{in} (m s ⁻¹)	5-40	ϑ (°)	0-180
ω_{in} (rad s ⁻¹)	0-60	χ (°)	-67.5-53.3
ζ_{BT} (%)	0-100	$\zeta_{BS'}$ (%)	50-120

at the contact points, the contact forces may generate additional torques acting upon the non-spherical rock body, changing the rock’s translational and rotational motion afterwards. Consequently, the remaining mechanical parameters that we plan to inspect are: the rock density ρ , volume ι , aspect ratio γ and surface area ratio ξ , the bottom diameter d_0 and the top diameter d_1 of the tree stem, the coefficient of friction μ , the incoming translational/rotational velocities v_{in} and ω_{in} of the rock that is before the impact moment, the eccentricity indicators ζ_{BT} and $\zeta_{BS'}$, and the impact angle ϑ and χ measured in the horizontal plane $e_x^I e_y^I$ and vertical plane $e_y^I e_z^I$, respectively (Fig. 5a). The ranges and values of the major input parameters used in the simulations are obtained from Toe et al. (Toe et al. 2017) and are listed in Tab. 1. Note that d_0 and d_1 are regulated in such a way that $\kappa = (d_1 - d_0)/(2h) = -0.01$ is a constant.

In each simulation, rocks are released in front of the tree stem from the lattices of a 1 m² square area (Fig. 5b). The distance between the neighbouring grid points in the e_x^I and e_y^I directions is 0.01 m, respectively. From each grid point 50 identical rocks are released, with varying orientations, and randomly assigned translational and rotational velocities falling into the ranges of parameter values suggested in Tab. 1. This procedure results in 5500 simulations in total for the simulated rock geometry. The initial release height is 5 m above the horizontal terrain surface, and the minimum distance between the mass center of the rock and the symmetry

Fig. 5 Definitions of the parameters for rock-tree interaction simulations: (a) the two angles ϑ and χ and the two eccentricity indicators ζ_{BT} and $\zeta_{BS'}$ which are used to describe the spatial impact position of a rock (blue) against a tree stem (green). The contact point B represents the ‘mean’ impact position in case of multiple contact points; (b) the initial release conditions for rocks impacting with a tree stem, shown in the $e_x^I e_y^I$ plane; (c) the two angles ϕ and ψ measured in $e_x^I e_y^I$ and $e_y^I e_z^I$ planes describing the spatial change of a rock’s trajectory due to impacting with a tree stem.

axis of the tree stem is 2 m. The time step for the simulations is 0.002 s in order to maintain the accuracy of the numerical results (Lu et al. 2019), and 5 s simulations are performed covering the whole rock-tree interaction process. For the simulated trajectories, only the ones corresponding to rocks that have collided with the tree stem are collected, from which a further 2750 trajectories are randomly sampled for analyses. Since the total number of effective trajectories between rocks and trees varies among the simulation cases, only half of the trajectories is sampled ensuring that the sample size remains sufficiently large and unchanged for differently shaped rocks.

The simulations are performed as follows. Firstly, we performed a group of 5500 simulations and randomly selected 2750 of them in order to compare with the state-of-the-art work (Dorren and Berger 2005; Jonsson 2007; Toe et al. 2017). The rock simulated has a spherical shape, volume 0.5 m^3 , and density 2800 kg m^{-3} ; while the tree stem has a bottom diameter 0.35 m, a top diameter 0.25 m, and the surface coefficient of friction 0.6. The rock-tree impact velocity along e_y^I is approximately controlled to be 15 m s^{-1} (Dorren and Berger 2005), and additionally 24 m s^{-1} (Jonsson 2007). A small velocity fluctuation of $\pm 1 \text{ m s}^{-1}$ along e_x^I is set to the rocks.

Secondly, simulations are performed and the results are analyzed on a contact scale level, i.e. the effects of rock surface area ratio ξ and aspect ratio γ on its energy dissipation and trajectory change are separately investigated during interaction with a given tree stem. We are particularly concerned with the change of rock kinematics behaviour from its first ‘touch’ with the tree stem to the first moment when there appears a complete separation between the rock and the tree stem. Based on Tab. 1, the rocks are further constrained by a fixed set of mechanical parameter values: $\rho = 1600 \text{ kg m}^{-3}$ and $\iota = 0.1 \text{ m}^3$; similarly for the tree stem we have $d_0 = 0.6 \text{ m}$ and $\mu = 1.2$. In the first sub-group simulations γ is kept unchanged as 1.0 whereas ξ is altered from 1.0 (sphere), 1.05 (eota), to 1.24 (cube). Here the eota rock has a geometry following the regulations issued by the official European Technical Assessment Approval Guidelines (Caviezel et al. 2019). For the second sub-group simulations three box-shaped rocks are investigated, where ξ remains the same, i.e. 1.24, while correspondingly γ is 0.9 (tabular), 1.0 (cube), and 1.1 (columnar). The name ‘tabular’ indicates that the middle length of a rock’s three principal axes is closer to the maximum one, leading to a relatively platy rock shape. Correspondingly ‘columnar’ refers to a relatively elongated rock shape where the middle length of the rock’s three principal axes is closer to the minimum one. Tab. 2 lists the geometries and the dimensions of the above mentioned 5 rocks.

Thirdly, more simulations are performed using varying properties for rocks and tree stems. By studying the rock-tree interactions it is intended to examine whether the leading energy- and trajectory-influencing factors found for impacts between spherical rocks and tree stems (Toe et al. 2017) are still valid for generally equant rocks. In this context in total 110 rocks having different ξ and γ values are either artificially created or extracted from real rock shapes. For the former we have employed MATLAB function ‘sphere(25)’ to initiate 676 nodes on a spherical surface, and from them a few pre-defined numbers of nodes in the range of 10 to 120 have been randomly selected and used for constructing convex hulls for the polyhedral rocks. This procedure allows for the introduction of some shape

Fig. 6 Workflow of rock-tree interaction simulations in this study.

variability in the simulated rocks. One can refer to Fig. 7 for the detailed rock shapes modelled. Meanwhile, to restrict the total amount of computations, for each rock shape 10 rocks are released from each grid point shown in Fig. 5b, resulting in 1100 simulations in total and correspondingly 550 selected effective trajectories for analyses.

Finally, as a validation for the results obtained from the third step, another 12100 effective trajectories are randomly selected from the simulations of 110 rocks for analysis. The entire workflow of our simulations is described in Fig. 6.

2.5 Metrics Defining Rock Energy Dissipation and Trajectory Change

We evaluate several critical variables which characterize a rock's post-impact behaviour in rock-tree interactions. The change of total kinetic energy is defined in percentage as:

$$\Delta E_{tot} = \frac{E_{tot}^- - E_{tot}^+}{E_{tot}^-} \quad (8)$$

where the sign ‘-’ and ‘+’ refers to the moment that is before and after the impact, respectively. Correspondingly, the change of translational kinetic energy

Table 2 Rocks simulated for undermining the effects of rock surface area ratio ξ and aspect ratio γ on its energy dissipation and trajectory change after impacting with tree stem.

Group 1 $\gamma=1.0$				Group 2 $\xi=1.24$			
Shape	Name	$l_X/l_Y/l_Z$ (m)	ξ (-)	Shape	Name	$l_X/l_Y/l_Z$ (m)	γ (-)
sphere	0.58/0.58/0.58	1.0		tabular	0.48/0.48/0.43	0.9
eota	0.52/0.52/0.52	1.05		cube	0.46/0.46/0.46	1.0
cube	0.46/0.46/0.46	1.24		columnar	0.45/0.45/0.49	1.1

and rotational kinetic energy can be individually defined in percentage as:

$$\Delta E_{tra} = \frac{E_{tra}^- - E_{tra}^+}{E_{tra}^-} \quad (9)$$

and

$$\Delta E_{rot} = \frac{E_{rot}^- - E_{rot}^+}{E_{rot}^-} \quad (10)$$

These energy changes can be defined in magnitudes:

$$\delta E_{tot} = E_{tot}^- - E_{tot}^+ \quad (11)$$

and

$$\delta E_{tra} = E_{tra}^- - E_{tra}^+ \quad (12)$$

and

$$\delta E_{rot} = E_{rot}^- - E_{rot}^+ \quad (13)$$

Fig. 7 Rock shapes modelled in this work shown by their surface area ratio ξ and aspect ratio γ . The rocks listed in Tab. 2 are marked in red. The dashed line separates columnar and tabular rocks in general. Several block shapes are shown in the insets using a reference volume 0.1 m^3 . Note that depending on how spherical this rock is, the total number of the rock's vertices is ranging from 10 to 120.

Note that the relatively small change of a rock's potential energy before and after the instantaneous impact is not taken into account. In order to capture the trajectory change of a rock in space, two angles ϕ and ψ are defined in the $e_x^I e_y^I$ and $e_y^I e_z^I$ planes, i.e. the included angles measured between the rock's incoming and outgoing translational velocities in these two planes (Fig. 5c).

2.6 Sensitivity Analyses Built upon Meta-Models for Leading Influencing Parameters on Rock Dynamics

A two-step procedure is applied on the simulation results of 110 differently shaped rocks, aiming to determine the leading parameters influencing on the rock-tree impact dynamics.

Following Toe et al. (Toe et al. 2017), a meta-model is built upon our direct simulations using the generalized polynomial chaos expansion (PCE) method (Sudret 2008) in the MATLAB environment Uqlab (Marelli and Sudret 2014). The PCE model M_{PCE} links the input $\hat{\mathcal{X}}$ and the output $\hat{\mathcal{Y}}$ parameters of the simulations in a polynomial space composed by orthogonal polynomials. The sparse PCE Least Angle Regression algorithm (Blatman and Sudret 2011) is adopted to calculate the polynomial coefficients in the PCE model. To quantify the accuracy of the established meta-model based on the obtained N simulations, an error index β is introduced as:

$$\beta = \frac{\sum_{i=1}^N \delta_i^2}{\text{Var}[\hat{\mathcal{Y}}]} \quad (14)$$

where δ_i is given by:

$$\delta_i = \hat{\mathcal{Y}}^i - M_{PCE \setminus i}(\hat{\mathcal{X}}^i) \quad (15)$$

According to the leave-one-out cross-validation principle (Toe et al. 2017), $\hat{\mathcal{X}}^i$ and $\hat{\mathcal{Y}}^i$ belong to the simulation i , and $M_{PCE \setminus i}$ is the reduced PCE model established using all the $N - 1$ simulations excluding the simulation i . Eq. 14 evaluates how close the sum of δ_i^2 is to the variance of $\hat{\mathcal{Y}}$ and thus gives an estimation to the accuracy of M_{PCE} .

Then, also following Toe et al. (Toe et al. 2017), a global sensitivity analysis is performed on top of the M_{PCE} using the so-called Sobol decomposition (Sudret 2008). Here the total variance of any output parameter in $\hat{\mathcal{Y}}$ is decomposed as a summation of the partial variances contributed by each input parameter in $\hat{\mathcal{X}}$, taking into account of the coupling among all the input parameters. Thus the algorithm calculates the global Sobol sensitivity index τ , i.e. ratio of the partial

variance to the total variance, for each input parameter. Higher values of τ correspond to leading influencing input parameters on the targeted output parameter. In order to validate the above global sensitivity analysis, we additionally build a regressor using the Random Forest algorithm, such that the relative importance of every input parameter in $\hat{\mathcal{X}}$ to each output parameter in $\hat{\mathcal{Y}}$ can be quantified (Micheletti et al. 2014). The outcomes are compared to those from the Sobol decomposition confirming the leading influencing parameters.

3 Insights Gained from Equant Rocks Impacting with Tree Stems

In this section we demonstrate the validity of our rock-tree interaction model by comparing experimental measurements with simulation results. Afterwards, the results of new simulations are presented for rock shapes with given surface area ratios and aspect ratios (Tab. 2). The results of all the investigated equant rocks are also discussed with respect to the methods explained in Sec. 2.5 and Sec. 2.6. Finally, a short discussion is given regarding how to apply the obtained findings to the practical rockfall hazard analyses.

3.1 Rock-Tree Interaction Model Validation

Fig. 8 shows ΔE_{red} as a function of ζ_{BT} , where ΔE_{red} is defined as:

$$\Delta E_{red} = \frac{\delta E_{tot}}{\delta E_{tot}^{max}} = \frac{\forall(E_{tot}^- - E_{tot}^+)}{\max(E_{tot}^- - E_{tot}^+)} \quad (16)$$

where δE_{tot}^{max} , belonging to a frontal impact case, is the maximum dissipation of the total kinetic energy obtained from 5500 simulations. From Fig. 8 we confirm that the block energy reduction decreases with increasing ζ_{BT} . The post-impact behaviour of the simulated rocks well obeys the so-called T1 decrease pattern (Jonsson 2007), normally reported for rocks impacting with trees of relatively large diameters, i.e. the ones that tend to consume more impact energy. Here, and in (Jonsson 2007), the tree stem is presumed to be a rigid and unbreakable

Fig. 8 Rock-tree interaction model validation: ΔE_{red} shown as a function of ζ_{BT} . The continuous curves are the extracted experimental (Dorren and Berger 2005) and finite element modelling (Jonsson 2007) results from literature. The rock-tree impact velocity is about 15 m s^{-1} (Dorren and Berger 2005) and 24 m s^{-1} (Jonsson 2007). The blue, solid circles are obtained from the DEM simulations performed by Toe et al. (Toe et al. 2017). In correspondence, the ΔE_{red} obtained from the current simulations are displayed in dots.

body. Therefore, the T1 decrease type can be expected for the current tree stem diameter. The difference observed between our results and the work of Dorren and Berger (Dorren and Berger 2005) can be attributed to the approximately spherical rock shapes used in the field experiments, which potentially affect their energy dissipating processes. Interestingly it is noticeable that the rocks' energy losses produced from our simulations are higher than those of Toe et al. (Toe et al. 2017). Again, this can be explained by the different modelling methods implemented by Toe et al. (Toe et al. 2017) in which the tree stem is modelled as a 'flexible' body takes into account bending and swing motions. Overall the above validations demonstrate that our rock-tree interaction model is able to capture the rock's energy dissipation pattern and is thus applicable to the studied problem.

Fig. 9 Change of rock's (a) total kinetic energy, (b) translational kinetic energy, and (c) rotational kinetic energy due to rock-tree interaction, shown in percentage and as a function of rock shape (sphere, eota, cube) and rock-tree impact regime (frontal, lateral, scratch). PDF: Probability Distribution Function; CDF: Cumulative Distribution Function.

3.2 Effects of Rock Surface Area Ratio and Aspect Ratio on Rock-Tree

Interactions

3.2.1 Surface Area Ratio

Fig. 9 depicts the Probability Distribution Function (PDF) and Cumulative Distribution Function (CDF) of ΔE_{tot} , ΔE_{tra} and ΔE_{rot} for the rocks sphere, eota and cube for three different impact regimes. From Fig. 9a one observes that the most frequently occurring ΔE_{tot} decreases from approximately 95% for frontal impact, to about 80% for lateral impact, and to around 55% for scratch impact. This simulation result aligns with our intuition that a direct ‘face-to-face’ collision with the tree stem tends to dissipate more kinetic energy. For a frontal impact we have

$$\| \Delta E_{tot_{sphere}} \| > \| \Delta E_{tot_{eota}} \| > \| \Delta E_{tot_{cube}} \| \quad (17)$$

whereas for a scratch impact we find the opposite

$$\| \Delta E_{tot_{sphere}} \| < \| \Delta E_{tot_{eota}} \| < \| \Delta E_{tot_{cube}} \| . \quad (18)$$

In this context lateral impact acts as a transition phase between these two cases. Likewise, shown in Fig. 9b ΔE_{tra} exhibits similar trends for the three simulated rocks in the three impact regimes. Turning to Fig. 9c we find that rocks can gain different reflected rotations when the impact regime alters. For example, for frontal impact we have

$$\| \Delta E_{rot_{sphere}} \| < \| \Delta E_{rot_{eota}} \| < \| \Delta E_{rot_{cube}} \| \quad (19)$$

while for scratch impact there exists

$$\| \Delta E_{rot_{sphere}} \| > \| \Delta E_{rot_{eota}} \| > \| \Delta E_{rot_{cube}} \| . \quad (20)$$

Note the opposite trends of ΔE_{rot} in comparison to ΔE_{tra} . It seems that δE_{tra} can be partially ‘converted’ to δE_{rot} . The rotating capability of the rock gained due to impact, or the translational/rotational energy partition during the impact, is closely linked to its surface area ratio.

Fig. 10 Transfer of rock's kinetic energy between the translational and the rotational components due to rock-tree interaction in different impact regimes: (a) δE_{rot} as a function of δE_{tra} shown for sphere, eota and cube. The lines are linear regressions for the corresponding data points; (b) δE_{rot} as a function of δE_{tra} shown for cube. The data points are coloured according to the parameter $\zeta_{BS'}$.

To understand how δE_{tra} is mechanically transferred to δE_{rot} in Fig. 9, Fig. 10a plots δE_{rot} as a function of δE_{tra} using the 2750 selected samples for each rock shape. A linear regression line is added on top of the corresponding data points. This confirms that, due to rock-tree interactions, the rock's translational energy can be partially transferred to the rotational energy, leading to the reflected rock rotations shown in Fig. 9c. Again, frontal impact cubes have the highest capability to gain rotations and scratch impact spheres acquire much more rotational energy after impact with the tree stem. The rotational energy behaves as an energy

Fig. 11 Change of rock's trajectory in the (a) $e_x^I e_y^I$ and (b) $e_y^I e_z^I$ planes, shown in angles (a) ϕ and (b) ψ , and as a function of rock shape (sphere, eota, cube) and rock-tree impact regime (frontal, lateral, scratch). PDF: Probability Distribution Function; CDF: Cumulative Distribution Function.

reservoir, influencing the rock's total energy partitioning. The results indicate that the partitioning depends on the rock's surface area ratio and impact regime. We assume that this phenomenon correlates with the relative position of the contact point to the rock's center of mass, and also the anisotropic distribution of the moment of inertia for non-spherical rocks. Using a cube as an example, Fig. 10b plots δE_{rot} as a function of δE_{tra} . The varying colours represent the parameter $\zeta_{BS'}$. Recall that $\zeta_{BS'}$ quantifies how close the contact point is to the rock's center of mass (scaled by the radius of the rock's bounding sphere). However, as no clear boundaries can be observed among these data points, no straightforward correlation exists between $\zeta_{BS'}$ and the rock's instant capability to gain rotations. Other factors at the impact moment, such as the rock's rotational velocity, may affect the energy transfer.

Fig. 11 displays the changes of rock's trajectory after impact viewed in the horizontal $e_x^I e_y^I$ and the vertical $e_y^I e_z^I$ planes. Clearly, the dispersion of rocks in

space increases with increasing surface area ratio, independent of the specific impact regime. This phenomenon was also found before in avalanche of particulate flows (Lu et al. 2017). For non-spherical rocks the normal contact force does not necessarily pass through their centers of mass. This adds an additional influencing factor to the rock's reflected rotation. The rotation of a non-spherical geometry may lead to subsequent contacts between the rock and the tree stem, which tend to significantly alter the rock's moving direction.

3.2.2 Aspect Ratio

We continue to examine the effects of rock's aspect ratio on its energy dissipation and trajectory change during a rock-tree interaction. Following the definition of equant rocks, the aspect ratio has been altered $\pm 10\%$ away from one such that the rock's surface area ratio remains the same (Tab. 2). For such a variation of aspect ratio, the resulting change in rock's behaviour is quite small. However, we do observe for scratch impacts that the tabular and columnar rocks gain less reflected rotations in comparison to a cube (Fig. 12a). This can be attributed to the anisotropic distribution of moment of inertia for the flattened and elongated rock shapes, i.e. it is relatively difficult for these rocks to acquire rotation about their major principal axis. Fig. 12b further differentiates the gained rotations for the rock tabular using the parameter $\zeta_{BS'}$. Similar to Fig. 10b, $\zeta_{BS'}$ alone cannot separate the rock's instant 'rotating capability'. On the other hand, as shown in Fig. 13, the tabular and columnar rocks exhibit more significant impact-induced changes in their moving directions, quantified on the $e_x^I e_y^I$ plane. As explained in the previous section, this phenomenon correlates with the acting directions of contact forces relative to the rock's center of mass, as well as the irregular rock geometry, which likely trigger subsequent impacts driving the rock into a different moving path.

Fig. 12 Transfer of rock's kinetic energy between the translational and the rotational components due to rock-tree interaction in different impact regimes: (a) δE_{rot} as a function of δE_{tra} shown for tabular, cube and columnar. The lines are linear regressions for the corresponding data points; (b) δE_{rot} as a function of δE_{tra} shown for tabular. The data points are coloured according to the parameter $\zeta_{BS'}$.

3.3 Identification of Influence Parameters on Block Kinematics after Impact

Following Sec. 2.6, we have established the PCE model based upon the simulations of 110 differently shaped equant rocks corresponding to an entity of in total 60500 rock-tree impact cases. Eleven parameters are used as inputs: the bottom diameter of the tree stem d_0 , the coefficient of friction μ , the block volume ι and density ρ , the rock's incoming translational velocity v_{in} and rotational velocity ω_{in} , the vertical impact angle χ , the two eccentricity indicators ζ_{BT} and $\zeta_{BS'}$, and the rock surface area ratio ξ and aspect ratio γ . Four key kinematic parameters are chosen

Fig. 13 Change of rock's trajectory in the (a) $e_x^I e_y^I$ and (b) $e_y^I e_z^I$ planes, shown in angles (a) ϕ and (b) ψ , and as a function of rock shape (tabular, cube, columnar) and rock-tree impact regime (frontal, lateral, scratch). PDF: Probability Distribution Function; CDF: Cumulative Distribution Function.

to quantify the rock's post-impacting behaviour (Toe et al. 2017): the kinetic energy reduction δE_{tot} , the reflected rotational velocity ω_{out} , and the two angles ϕ and ψ characterizing the trajectory changes. The error index β of the PCE model obtained for these four targeted parameters is 5.9%, 56.0%, 57.1% and 58.4%, respectively. Noticeably, β is below 10% only for δE_{tot} . As further validations, we have tested our PCE model on another 12100 impact data acquired from the simulations of 110 rocks. We find that β is very comparable to the obtained values, i.e. the relatively larger errors seen are not due to the abnormal distribution accidentally being encountered in the input. In addition, the coefficient of correlation that is calculated for any among the possible 55 input parameter pairs is below 0.5, with the maximum value being 0.48 and in total 7 correlational coefficients being larger than 0.1. This shows that there are no strong interactions among the input parameters, which satisfies the requirement for PCE meta-models. The

Table 3 Comparison of the leading influencing parameters obtained for spheres (Toe et al. 2017) and ‘equant’ rocks impacting with trees, quantified by the Sobol decomposition where the total Sobol indice is larger than 10%.

	δE_{tot}	ω_{out}	ϕ	ψ
sphere	d_0 (43%) ζ_{BT} (35%) v_{in} (14%)	ω_{in} (85%)	d_0 (68%) ζ_{BT} (15%)	ζ_{BT} (52%) d_0 (32%)
‘equant’	v_{in} (64%) ι (40%)	ω_{in} (52%) v_{in} (37%) ι (16%)	ζ_{BT} (68%) $\zeta_{BS'}$ (33%) ξ (14%) χ (11%)	ζ_{BT} (75%) χ (21%) v_{in} (20%) $\zeta_{BS'}$ (11%)

reason we obtain the relatively larger β might be attributed to the non-smooth or discontinuous structure lying inside the data. For instance, shape especially surface area ratio affects the rock’s reflected rotations (Fig. 10) and, in turn, causes more scattering to the moving paths (Fig. 11). This limits the accuracy of the PCE meta-models in representing the rock’s rotation and trajectory dynamics.

Nonetheless, global sensitivity analyses, aiming to the identification of the influencing parameters governing equant rock-tree impacts, have been performed using the Sobol decomposition and the Random Forest algorithm. The total Sobol indice (i.e., the relative importance of each individual input parameter to each output parameter) measured within the two frameworks is shown in Fig. 14. Using 10% as a threshold value for the Sobol decomposition, we have found that δE_{tot} is mostly affected by v_{in} (64%) and ι (40%), ω_{out} is governed by ω_{in} (52%), v_{in} (37%) and ι (16%), ϕ is mainly influenced by ζ_{BT} (68%), $\zeta_{BS'}$ (33%), ξ (14%) and χ (11%), and ψ is controlled by ζ_{BT} (75%), χ (21%), v_{in} (20%) and $\zeta_{BS'}$ (11%). We have also observed some agreement between the results of the two sensitivity analysis frameworks. For example, both methods confirm that, in Fig. 14a δE_{tot} is largely affected by v_{in} and ι , in Fig. 14b ω_{in} , v_{in} , ι and ζ_{BT} are prevailing, and in Fig. 14c and Fig. 14d ζ_{BT} is dominating. These observations have demonstrated the effectiveness of our sensitivity analyzing procedures.

Tab. 3 further compares the leading influencing parameters obtained for spheres (Toe et al. 2017) and equant rocks impacting with trees. Here only the

Fig. 14 Global sensitivity analysis using the Sobol decomposition based on the PCE meta-models for (a) the kinetic energy reduction δE_{tot} , (b) the reflected rotation ω_{out} , and (c, d) the two angles ϕ and (d) ψ indicating trajectory changes of the rocks. The insets show the corresponding analyses using the Random Forest algorithm in order to compare the results with those of the Sobol decomposition.

Sobol decomposition is taken into account. Several interesting observations can be made from Tab. 3. Firstly, unlike spheres d_0 seems to be a less important factor for equant rock-tree impacts. However, this might be caused by the assumption made for our simulations (the tree stem is a rigid body which can absorb any kinetic energy from rocks). In reality, the diameter of a tree stem quantifies how strong and stable this tree is; thus it directly affects the maximum energy that the tree stem can draw from the rock. Because of the assumption made in this work, the rock volume ι shows a higher importance in δE_{tot} and ω_{out} . Secondly, for both spheres and equant rocks, v_{in} and ω_{in} play a key role in δE_{tot} and ω_{out} , and ζ_{BT} significantly influences ϕ and ψ . Lastly, the rock shape parameters ξ and γ do not directly, or strongly correlate with the rock's post-impacting behaviour, except for the indirect parameter $\zeta_{BS'}$ (measuring the relative position of the contact point to the rock's 'center'), which exhibits some influences on ϕ and ψ . Overall, we can conclude that the most critical input parameters for the equant block kinematics after impact are: the impact translational and rotational velocities v_{in} and ω_{in} , the impact eccentricity ζ_{BT} , and, potentially, the tree diameter d_0 . In addition, the contact eccentricity indicator $\zeta_{BS'}$, which is specific to the non-spherical rock shapes, plays a minor however non-ignorable part in the rock's trajectory changes after impact with tree stems.

3.4 Application of New Findings on Rockfall Hazards Analyses

A critical issue that we have not addressed is how to apply our results to rockfall mitigation practice. In particular, how can the results be managed in an existing code for 3D stochastic trajectory analysis? One possible approach is to employ machine learning techniques, i.e. training a machine to learn the intrinsic patterns underlying the rockfall input and output parameters. A recent example is using a K-nearest neighbors (KNN) algorithm to investigate the major factors governing rockfall run-out distance in order to establish a rockfall trajectory prediction model (Huang et al. 2019). In our case, the fact that rock shape plays only a secondary role

in equant rock-tree interactions is valuable because it suggests that it is possible to construct simplified machine learning algorithms that link input parameters (e.g. Tab. 1) of the contact scale simulations to the rock's kinematics after impact (e.g. block energy reduction, reflected rotation, and trajectory change). These meta-models can be integrated into 3D stochastic trajectory codes to improve the current Monte Carlo based modelling routines (Macciotta et al. 2015) and to speed-up rockfall simulations on terrain surface with forest covers. Thus, slope scale simulations can be more easily managed by engineers. With this approach it should also be possible to perform stochastic analyses of rock run-out distances at regional scale.

4 Conclusions and Outlook

This work combines the non-smooth mechanics method with hard contact laws to study the interaction process between a falling rock and a tree stem. The aim is to better understand the mitigation effects of trees on rockfall hazards.

Unlike previous studies using spherical rocks (Toe et al. 2017), equant rocks are modelled as three-dimensional polyhedrons. Two shape factors (surface area ratio and aspect ratio) are introduced to quantify how different a block is from a pure spherical geometry. Tree stems are abstracted as non-deformable, truncated cones. Two contact detection routines (DCT and SAT) are developed using the results of simulations with rocks of either spherical or non-spherical bodies. The effects of rock's surface area ratio and aspect ratio on its energy dissipation and trajectory change are investigated separately. Global sensitivity analyses employing the Sobol decomposition and the Random Forest algorithm are carried out using the numerical results of 110 'equant' rocks colliding with tree stems. The validation of the models and the range of the input mechanical parameters follows the procedure developed by Toe et al. (Toe et al. 2017).

For equant rocks we find that neither surface area ratio nor aspect ratio are the leading indicators to describe the block kinematics after impact, i.e. block energy

reduction, reflected rotation, and trajectory change. Essentially, the governing parameters for rock-tree impacts are the impact velocity, the impact eccentricity, and the tree stem diameter, which seem to be independent of rock shape. However, we observe that surface area ratio and aspect ratio have a secondary effect on the rock's energy dissipation and trajectory change. In particular, surface area ratio plays a more important role than aspect ratio in these processes. From the global sensitivity analyses, we identify the shape-specified impact eccentricity indicator as an additionally critical measure for the equant rocks' trajectory change after interacting with tree stems. This is linked to the anisotropic nature of the mass and moment of inertia distribution for non-spherical rocks, which influences the capabilities of rocks to instantly gain rotational motion at impact. This induces further contacts with trees. Thus, more 'uncertainty' is introduced between non-spherical bodies (rock and tree stem) at impact.

Our research implies that the existing spherical block propagation models can be applied at the regional forest scale to approximate equant rocks. However, shape factors must be somehow introduced to account for the effects of rock shape. Nonetheless, this needs to be further tested utilizing rock-forest, slope-level studies where the 'predictions' of the block propagation models are validated against in-field experimental tests. Furthermore, how a rock's three-dimensional, rotational motion serves as an energy reservoir that influences the rock's dynamics must be studied using more elongated and flattened rock shapes. Finally, the role of aspect ratio in non-equant, non-spherical rock's behaviour during tree impact requires more detailed investigation.

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