

Calculating impact pressures in numerical avalanche and rockfall models

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ABSTRACT: Calculating contact/impact pressures with numerical finite element software remains a difficult task. It is especially difficult for highly compactable, plastic materials where the elastic fraction of the strain is very small. This is the case in most impact problems in natural hazards. One example is the impact of rockfall dams by large, rotating rocks. Another is the impact of pylons, walls and other obstacles by snow avalanches. We embed work-energy methods within numerical avalanche and rockfall models to make the calculation of impact pressures in natural hazards more applicable for engineering practice. Firstly, we can greatly reduce the computation time and secondly, the input parameters are much more intuitive for practitioners due to the use of purely plastic compactive material behaviour.

Keywords: Impact, natural hazards, avalanches, rockfall, work-energy, numerical methods.

1 INTRODUCTION

The calculation of impact forces in alpine mass movements is crucial for risk and hazard assessments of rockfall and avalanche events. These natural disasters, while distinct in their formation and propagation, share many similarities in their impact mechanics. The impact of an avalanche on a structure, for example, is similar to the impact of a rock on the ground in that both involve an interaction between a practically rigid object and a soft, compactible material. The main difference is that in the case of the avalanche impact, the rigid object decelerates the deformable material, while in the case of the rockfall impact, the soft material decelerates the rigid object. In both cases, energy dissipation occurs almost exclusively

through plastic, irreversible compaction of the deformable material, with elastic strains only making up a very small fraction of the total strains.

To calculate these impact forces using numerical methods, there are two general approaches: a continuum approach and a discrete approach (see Figure 1).

A continuum approach models the avalanche as a continuous mass using a set of mass and momentum balance equations. These are combined with a set of initial and boundary conditions. The governing equations are discretised into algebraic equations that can be solved numerically. This is typically done using the finite element or finite volume methods. Avalanche snow is a dense granular material and different constitutive relationships have been applied to model the internal stress

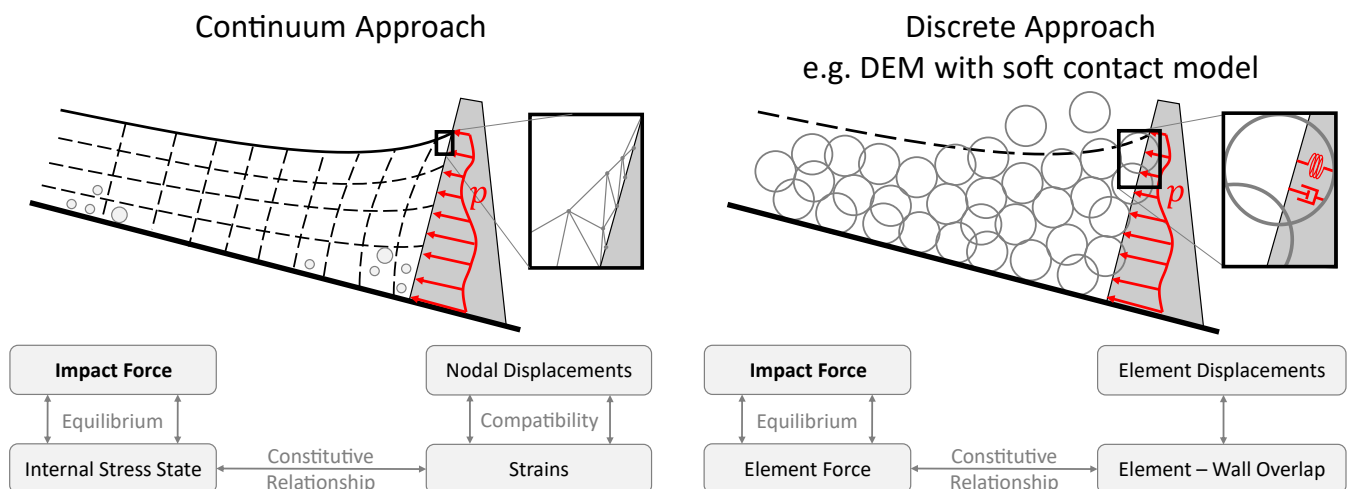


Figure 1. Two approaches to calculate impact forces exerted by a dense avalanche flow on a rigid obstacle with numerical methods. Avalanche snow modelled as a continuum (left) and with discrete elements (right).

state (Norem et al., 1987, Schweizer et al., 2021, Anecy and Bain, 2015). These include standard geo-technical models – such as Coulomb-type approaches with active/passive pressures, combined with hydraulic models to simulate the rate-dependent stresses, see for example (Bartelt et al., 1999). Although flowing snow is indeed a particle ensemble, continuum approaches have some clear advantages over particle hydrodynamics or discrete element models. They provide a simplified representation of the behaviour of granular flow without the need to model each individual grain and allows for efficient computational analysis while preserving key features such as non-linear behaviour arising from changes in flow density and highly complex particle interactions (Bartelt and Christen, 2023).

The calculation of an impact force of the avalanche with continuum models, however, remains a difficult task (Zhong, 1993). Theoretically, the impact force exerted by the avalanche on the obstacle should be calculated by integrating the internal stresses of the deformed avalanche body over the impact area. The internal forces are in equilibrium with the impact force, via Newton's law of action and reaction. In the case of a high velocity avalanche impact, the avalanche flow body is severely distorted, especially close to the rigid obstacle. The granular material (clumps and clods of flowing snow) breaks up due to rapid shearing. They often splash upwards, indicating the lack of a continuum type behaviour. Modelling the interaction as a continuum results in very large plastic strains, with the boundary elements at the interface between the obstacle and the avalanche snow being highly distorted. These large strain localisations occur mainly at abrupt geometry changes on the obstacle, such as sharp edges. This results in large uncertainties in the impact forces which are calculated from these strains in the boundary layer in combination with a constitutive law. Even though it is possible to calibrate the constitutive parameters on the measured forces of an avalanche impact experiment, it is important to note that these parameters are only valid for the exact experimental setup and possibly even only for the mesh size used for the calibration. Often, the parameters determined have no relation to snow properties from literature or laboratory tests.

Modelling the flow with single discrete elements solves the problem of large deformations and distortions in high velocity impacts. Impact simulations with discrete element models (DEM) always require the definition of a contact model between the discrete elements and the impacting object as well as between the elements themselves (Li et al., 2020, Calvetti et al., 2017). In many cases, soft contact models are chosen because they provide more realistic results for materials such as snow. In soft contact models, the deformations of two colliding elements are represented by the distance they overlap. The contact force between the elements is then calculated by their overlap in combination with a constitutive

relationship such as a non-linear spring-dashpot system. The impact force exerted by the avalanche on the obstacle is calculated by the sum of the forces of all elements in contact with the obstacle. While the discrete element approach can produce very accurate flow behaviour even at high impact velocities on complex geometries, it still faces significant challenges in calculating the impact forces, as these forces are very sensitive to the chosen contact parameters (e.g., stiffness, damping ratio) at the avalanche-wall interface. As with the continuum approach, these parameters can be calibrated using experiments, but it is important to be aware that correct flow behaviour does not necessarily mean correct internal and impact forces. Furthermore, the calculated and calibrated stiffness in DEM simulations is a parameter adjusted to the flow behaviour of the material but not directly related to the stiffness of the actual material, which makes it very difficult to use this method as a predictive tool in real scenarios.

Hard contact models have recently gained great popularity in impact problems related to natural hazards, especially in the modelling of rock trajectories (Leine et al., 2014). In this case, the rock-ground contact is modelled as a rigid constraint. Using non-smooth dynamics, the change in momentum and energy dissipation of the rock during ground interaction can be modelled very accurately, even for rigid rocks impacting a rigid wall. However, when both materials are rigid, there are no deformations and thus undefined impact forces (Figure 2).

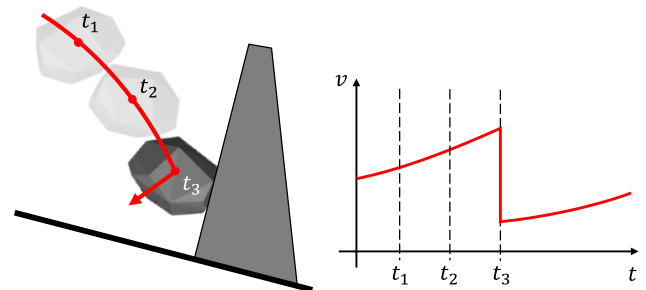


Figure 2. Impact of two rigid bodies with a hard contact model. While the momentum transfer and energy dissipation can be modelled very accurately, the velocity of the impacting object is not continuous over time and therefore the impact force is undefined.

The main challenge with all the above methods (continuum, DEM, hard-contact) for high-speed impacts is that the impact forces have to be calculated based on a very sensitive local deformation state at the boundary layer between the two impacting materials with calibrated constitutive parameters. To this end, much work has been invested in real scale field campaigns to actually measure impact accelerations, see (Caviezel et al., 2019, 2021). Moreover, it is not within the competence and intuition of engineering practitioners to discuss this sensitivity and thus evaluate the quality of the simula-

tion results in real cases. We therefore propose an approach in which a global deformation state is first defined in terms of a simplified kinematic mechanism. We then calculate the impact forces based on the chosen deformation using the principles of conservation of mass and work energy. This transforms the constitutive problem into a largely geometric problem. The resulting inertial forces fully represent the impact forces of the chosen mechanism. However, they represent the real forces only as well as the chosen mechanism represents the real flow behaviour.

This approach is an engineering solution, meaning that it requires the knowledge and experience of practitioners to define how an avalanche compacts and deforms upon impact with an obstacle. Since we define the mechanism on a global scale, the impact forces are averaged over the contact area, resulting in a loss of modelling resolution, but also avoiding the uncertainty of large strain localisations and highly sensitive constitutive parameters.

2 AVALANCHE PILE-UP WITH WORK-ENERGY

Figure 3 shows the simplest possible 2D mechanism for an avalanche impacting a rigid wall. We consider a dense avalanche with velocity v_0 , flow height h_0 and bulk density ρ_0 . The structure has a width w and a height so it cannot be overtopped. For simplicity, we assume that the avalanche has a mean depth-averaged velocity and density but can vary in streamwise direction and therefore time. All the incoming snow is compacted and deposited in a pile with velocity $v_1 = 0$, length s , height $h_1 \geq h_0$ and density $\rho_1 \geq \rho_0$. Due to the constant height and density of the piled-up mass, its border to the incoming avalanche snow moves against the flow direction of the avalanche with velocity \dot{s} .

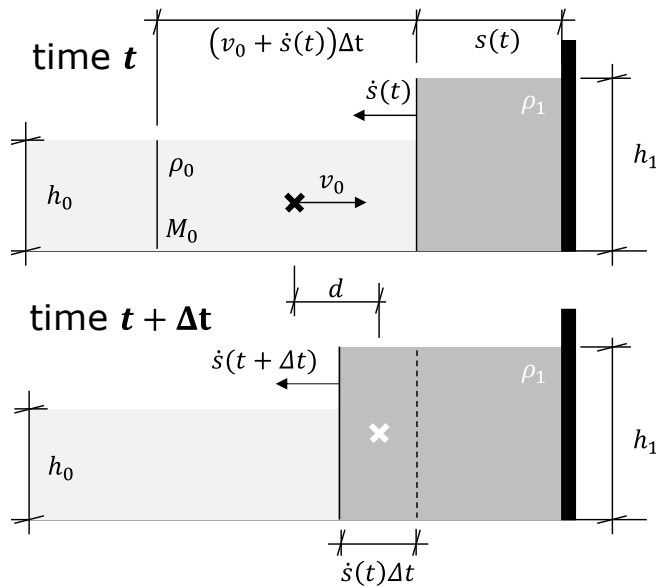


Figure 3. Pile-up mechanic of an incoming avalanche flow at two time steps t and $t + \Delta t$.

We calculate the velocity of this compaction wave with mass conservation in a given time step Δt between the incoming and piled-up snow. The mass M_0 that arrives at the pile-up front is

$$M_0 = (v_0 + \dot{s})\Delta t w h_0 \rho_0 \quad (1)$$

The mass conservation criterion then states:

$$M_0 = \dot{s}\Delta t w h_1 \rho_1 \quad (2)$$

$$\Rightarrow \dot{s} = v_0 \frac{h_0 \rho_0}{h_1 \rho_1 - h_0 \rho_0} \quad (3)$$

The kinetic energy loss ΔE_k of the incoming snow during this pile-up process is

$$\Delta E_k = \frac{1}{2} M_0 v_0^2 = \frac{1}{2} (v_0 + \dot{s})\Delta t w h_0 \rho_0 v_0^2 \quad (4)$$

The braking distance d for this energy change is given by the distance travelled by the centre of mass of the incoming snow when piled up and compacted:

$$d = \frac{(v_0 + \dot{s})\Delta t}{2} - \frac{\dot{s}\Delta t}{2} = \frac{v_0}{2} \Delta t \quad (5)$$

Combining equations (1-5), we define the average pile-up pressure on the rigid structure with the work-energy principle:

$$p = \frac{\Delta E_k}{d w h_1} = \rho_0 v_0^2 \frac{h_0 \rho_1}{h_1 \rho_1 - h_0 \rho_0} \quad (6)$$

Note that if the avalanche snow can neither pile up in height ($h_1 = h_0$) nor compact ($\rho_1 = \rho_0$), the velocity of the compaction wave \dot{s} and therefore also the pile-up pressure p become infinity. Equation (6) can be formulated into a viscous drag law with a dimensionless drag coefficient C_d , which characterises the influence of the relative compaction of the avalanche snow ρ_1/ρ_0 as well as the relative pile-up height h_1/h_0

$$p = \frac{1}{2} \rho_0 v_0^2 C_d \quad (7)$$

$$C_d = 2 \frac{\rho_1/\rho_0}{\frac{h_1}{h_0} \frac{\rho_1}{\rho_0} - 1} \quad (8)$$

The Swiss guidelines (SIA 261/1) suggest for the design loads of avalanche impacts the same viscous drag law with a drag coefficient of $C_d = 2$. Figure 4 below compares the drag coefficients for various compactibilities and pile-up heights to the Swiss standards.

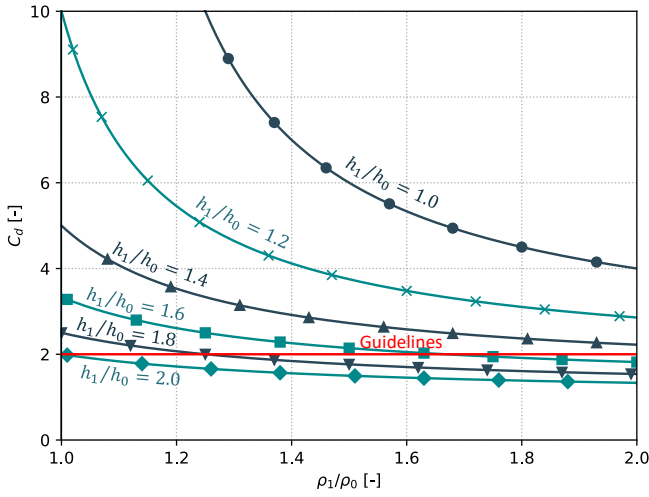


Figure 4. Comparison between drag coefficients of the pile-up model for various snow compactabilities ρ_1/ρ_0 and relative pile-up heights h_1/h_0 to the Swiss guidelines.

The comparison shows that in the case of low pile-up height (e.g., geometrically restricted flow) or poorly compactable snow (e.g., high initial density, wet snow avalanches), the impact pressures can exceed the design pressures by a multiple. On the other hand, if the snow can pile up considerably in relation to the flow height (e.g., seen in avalanches with a very low flow height) or if the initial density is low, the guidelines overestimate the impact pressures.

3 AVALANCHE PILE-UP WITH WORK-ENERGY AND VOELLMY

The work-energy approach strongly relies on the prediction and knowledge, how much the snow compacts during an impact, which can be a difficult task. We therefore propose to combine this method with the approach by (Voellmy, 1955). Voellmy began his derivation by assuming that high-speed compression of avalanche snow is controlled by how fast air can escape the pore space during compaction. Due to the short impact duration, a conservative assumption is that initially, all air remains trapped inside the pores of the dense avalanche snow. He further assumed that the deformation of the snow ice-matrix provides little to no resistance against compaction, in comparison to the force required to compress and expel the pore air. He did not consider the compressibility (elasticity) of solid ice, or any water existing in the pore space. The compaction process was considered to be completely plastic and given by the irreversible reduction in pore space. Based on these modelling assumptions, the decisive pressure during the early stages of an impact is due to the isothermal compression of the trapped pore air according to Boyle's law. This state also defines the final compacted density of the avalanche snow even after pressure relieve due to the assumption of pure plasticity. The relationship between initial snow density ρ_0 , compacted density ρ_1 ,

impact pressure p , atmospheric pressure p_{atm} and maximum snow density ρ_{max} is:

$$\rho_1 = \rho_0 \frac{1 + \frac{p}{p_{atm}}}{1 + \frac{\rho_0}{\rho_{max}} \frac{p}{p_{atm}}} \quad (9)$$

Voellmy provides the values $\rho_{max} = 800 \text{ kg/m}^3$ for dry snow and $\rho_{max} = 1000 \text{ kg/m}^3$ for wet snow. Note that these maximum densities can only be reached with infinitely high pressure according to Equation (9). Combining Voellmy's density relation with Equation (6) from the work-energy approach results in a quadratic equation that can be solved for the impact pressure p , and the compacted snow density ρ_1 .

$$p = \frac{b + \sqrt{b^2 + 4ac}}{2a} \quad (10)$$

with:

$$a = \frac{h_1}{h_0} - \frac{\rho_0}{\rho_{max}} \quad (11)$$

$$b = p_{atm} + \rho_0 v_0^2 - \frac{h_1}{h_0} p_{atm} \quad (12)$$

$$c = \rho_0 p_{atm} v_0^2 \quad (13)$$

A comparison to the Swiss guidelines shows that a typical dry avalanche of initial density $\rho_0 = 400 \text{ kg/m}^3$ matches the design pressures if the avalanche piles up behind the obstacle with height $h_1 = 1.5 h_0$. If the initial density is lower or the pile-up height higher, the guidelines will overestimate the impact pressures.

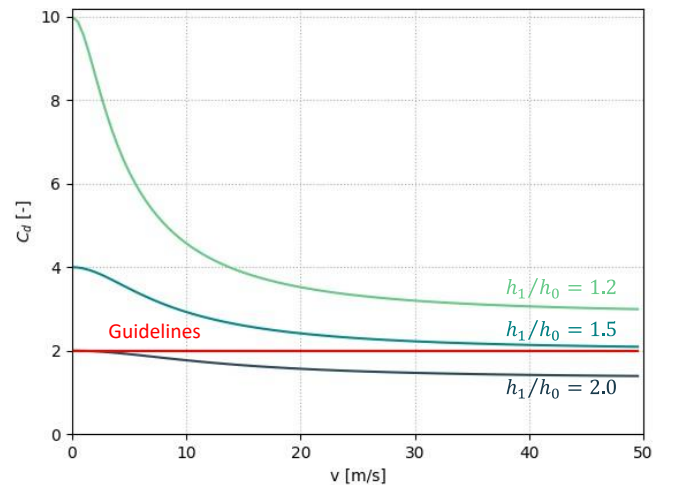


Figure 5. Comparison between drag coefficients of the work-energy approach combined with Voellmy for a dry avalanche flow of $\rho_0 = 400 \text{ kg/m}^3$ and various relative pile-up heights h_1/h_0 to the Swiss guidelines.

Unlike in the pure work-energy method (see Equation 8), the drag coefficient is now also a function of the impact velocity. This drag coefficient increases for lower impact velocities because with a low impact velocity,

the snow cannot be compacted as much, leading to a smaller braking distance and thus a higher force according to the work-energy principle. This is also shown in Figure 6 below. At low flow velocities (< 5 m/s), the snow undergoes barely any compaction, while the density of the deposited snow approaches the maximal snow density ρ_{max} for $v \rightarrow \infty$. For the typical dry snow avalanche shown in Figure 5, the drag coefficients of the relative pile-up heights $h_1/h_0 = 2$ and $h_1/h_0 = 1.5$ form a lower and upper bound for the design pressures in the guideline.

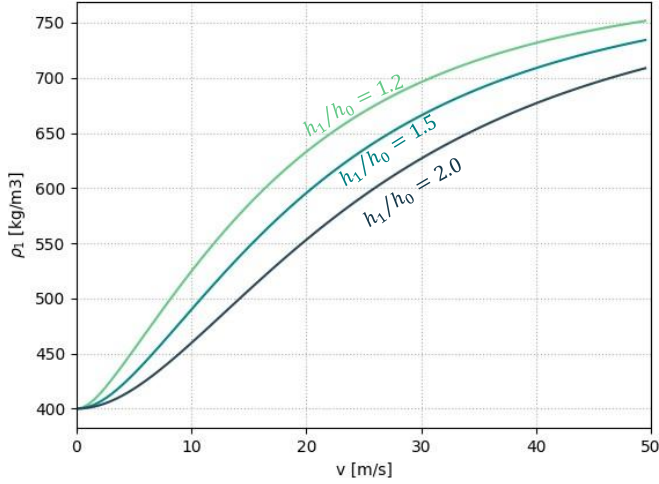


Figure 6. Densities of the piled-up snow ρ_1 of the work-energy approach combined with Voellmy for a dry avalanche flow of $\rho_0 = 400$ kg/m³ and various relative pile-up heights h_1/h_0 .

4 COMPARISON TO DEM SIMULATIONS

We compare the analytical pile-up solution to the DEM simulations by Calvetti et al., 2017. They derived an empirical equation to calculate the impact forces based on DEM simulations of a dry granular flow.

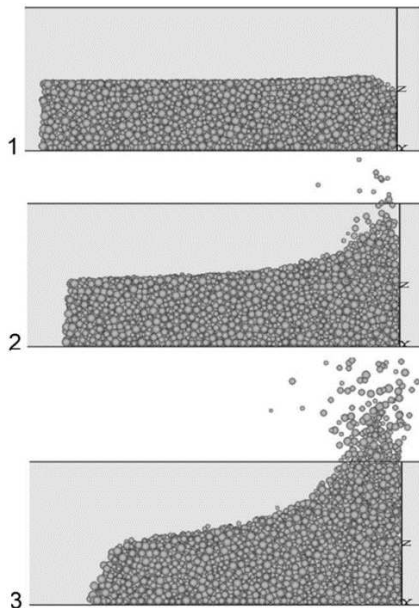


Figure 7. Three time steps of the DEM simulation of a dry granular flow from (Calvetti et al., 2017).

The maximal impact pressure derived by Calvetti for the simplified geometry shown in Figures 3 and 7 is

$$p_{max} = \rho_0 v_m v_0 + \rho_0 v_0^2 \quad (14)$$

where ρ_0 is the initial density of the assembly, v_0 is the initial flow velocity and v_m is the velocity of propagation of (elastic) compression waves within the impacting medium. Even though neither the analytical solution nor the DEM solution are specifically derived for snow avalanches, their results can be compared, firstly because both methods calculate the maximum depth-averaged impact force. Additionally, the DEM method in this particular example assumes an initial porosity of 0.45, which agrees with the assumptions of the proposed analytical work-energy model. Figure 8 below compares both methods to the values suggested by the Swiss guidelines.

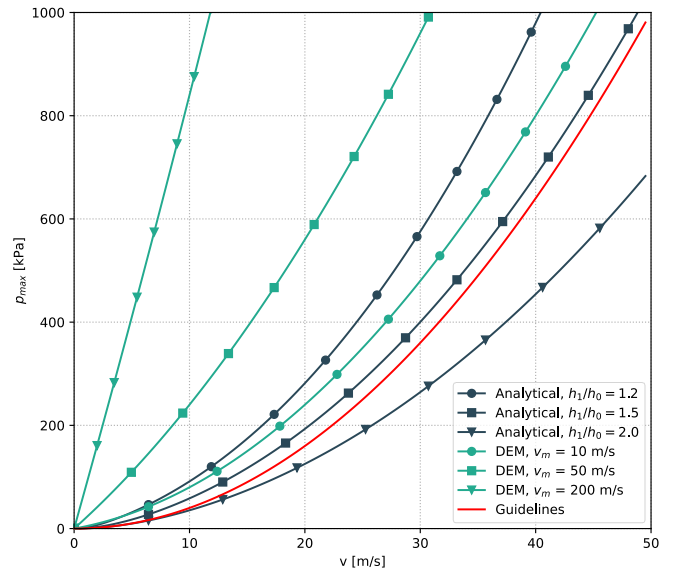


Figure 8. Comparison between the empirical formula derived by DEM simulations, the analytical work-energy approach combined with Voellmy and the Swiss guidelines ($\rho_0 = 400$ kg/m³, $\rho_{max} = 900$ kg/m³)

The comparison shows that the resulting impact pressures from the DEM simulations heavily depend on the compression wave velocity v_m , which is a function of the normal contact stiffness between the discrete particles. Calvetti et al., 2017 chose a contact stiffness of 100 MN/m, which led to a measured wave propagation velocity of 200 m/s. This corresponds well to a soft avalanche snow with an E modulus of 10 MPa (Gerling et al., 2018), which leads, combined with the introduced avalanche density of $\rho_0 = 400$ kg/m³, to an elastic wave propagation velocity of

$$v_m = \sqrt{E/\rho_0} \approx 160 \text{ m/s} \quad (15)$$

Interestingly, for $v_m = 0$, the DEM model replicates the Swiss guidelines (see Equation 14 and Figure 8). This

corresponds to an ideal plastic model with an elastic modulus of $E = 0$. However, as the contact between the discrete elements is purely elastic, it cannot describe this plastic material behaviour. Therefore, the calibration of a DEM model may yield non-physical material parameters. In this specific example, the impact pressures overestimate the expected pressures with a correct elastic modulus of $E = 10 \text{ MPa}$ and best matches the expected pressures with a non-physical elastic modulus of $E = 0$.

5 CONCLUSIONS

Calculating impact pressures with numerical finite element software is a challenging task, especially for highly compactable, plastic materials. While both continuum and discrete element models are powerful tools to accurately model the kinematic behaviour of high-speed impacts of avalanches or rocks, the calculation of impact forces depends on a very sensitive local state of deformation in the boundary layer between the two impacting materials. Calibrating constitutive parameters to match impact forces of experiments often results in an incorrect flow behaviour and vice versa. While the usage of these numerical tools is of great importance for the future research of the dynamics of natural hazards, they still provide little help to engineering practitioners facing design problems. To address these challenges, an engineering approach is proposed, where a global deformation state is defined in terms of a simplified kinematic mechanism, and the impact forces are calculated based on the chosen deformation using principles of conservation of mass and work energy. The approach refrains from a high degree of modelling, but thus avoids high sensitivities and uncertainties and provides reliable results, whose interpretation and discussion is within the competence of engineers. A comparison with the Swiss guidelines SIA 261/1 shows that the obtained impact pressures lie within the same range of the current design pressures. However, depending on the expected impact behaviour defined by the engineer (snow compaction, pile-up height), the design pressures may not be reached or may be exceeded, which offers the possibility to optimise the structure towards either a more conservative or more economical design.

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