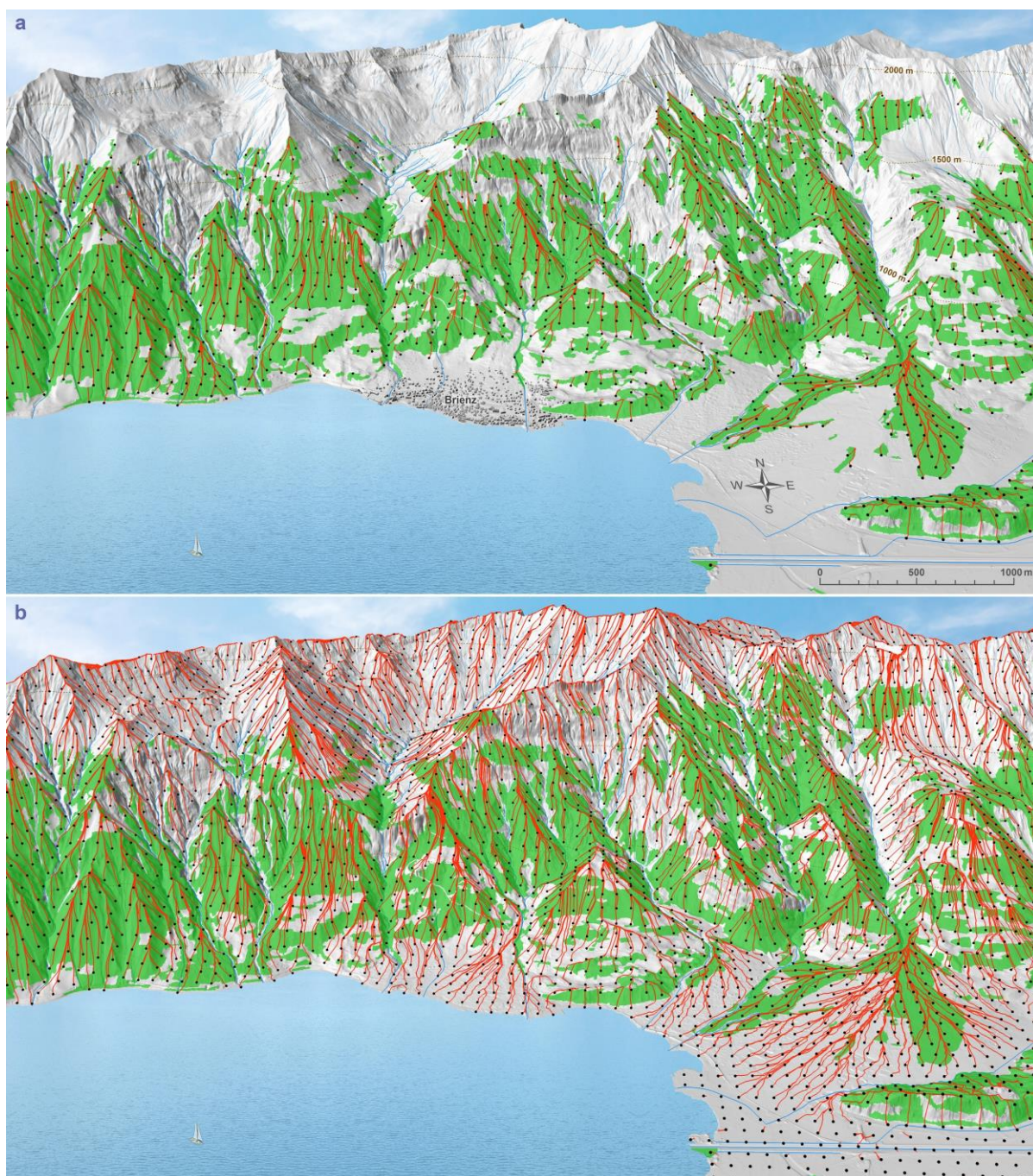


Supplementary Material 2 - Calculation of the steepest path

When calculating the steepest paths, we take the time of fire ignition to be $t=0$ and then proceed in (dimensionless) time steps of length Δt ($\Delta t=10$ in our case), where at each step the fire has the possibility to spread or – in case of insufficient slope gradient or lack of fuel - to be extinguished. The position of the fire at time t will always be modeled as a point p_t , so that over time, the fire will trace out a line which we will call the trajectory of the fire. At $t=0$, the fire starts at the user-defined point of ignition p_0 . After this has been set, we increment t by Δt and update the position of the fire by moving it along the direction of steepest ascent (the gradient) of the slope at point p_0 to reach the point $p_{\Delta t}$ (Fig. 2). We then repeat the procedure, starting at $p_{\Delta t}$ and moving it along the direction of steepest ascent to obtain $p_{2\Delta t}$. In this context, it is important to note that the value Δt only influences the pace and precision with which the position of the fire is calculated, thus usually with little effect on the overall trajectory.

After repeating this procedure for a user-defined number N times ($N = 400$ in our case), the algorithm checks the progress of the polyline representing the fire trajectory in terms of horizontal and vertical displacement (at least 2 m in both cases, in our case). If the fire has moved by less than 2 m either in the horizontal or vertical direction, then it has reached a flat region of the terrain, which with our choice of parameters correspond to a slope of less than ~2%. To further check if a major mountain top or a terrace has been reached (in contrast to a small local maximum/flat region of limited spatial extent), we search for existing points within a radius R (100 m in our case) in the surroundings, displaying an altitude of at least 2 meters above the current altitude. If this is the case, we let the fire front “jump” to the nearest location (within a radius of 100 m in our case) which has an altitude at least 2 m higher and we then continue to run the algorithm from this new point. If not, we “extinguish” the fire and stop the path. If more than 2 consecutive jumps occur without the fire front moving more than 2 m during the normal evolution after the second jump, the fire again extinguishes, and the path ends.

As illustrated in the following Fig. SM2.1, the output of the steepest path algorithm can then be further elaborated to produce a first set of three-dimensional polylines representing the path lines stopping where they leave the forest and a second set representing the uninterrupted path lines considering the topography only.



green area = forests; black dots = simulated ignition points (100 x 100 m square grid); red lines = steepest paths

Figure SM2.1: Three-dimensional view of the mountainside above the village of Brienz (Canton Bern, Switzerland) with represented two types of lines resulting from our algorithm.

- a) The *forest steepest paths*, i.e., the 3D polylines that start from the points of ignition, run up the slope following the direction of the maximum gradient within the woody vegetation and stop where the path leaves the forest for a horizontal distance of at least 40 meters.
- b) The *2500 full steepest paths*, i.e., the steepest path lines assuming that the whole territory turned to forest (hypothetical totally forested landscape) and truncated at 2500 m asl.