Two approaches to modeling the initiation and development of rills in a man-made catchment

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[1] Surface erosion rills are dominant structures in young developing ecosystems, and as such they fundamentally affect water flows, for example by channeling surface runoff or by locally changing the infiltration capacity of the soil. To successfully model the systems hydrology it is indispensable to take the development of erosion rills into account. The goal of this study was to model the emergence of an erosion rill network with total length of 6 km formed during the last few years in the 6 ha large artificial catchment Chicken Creek near Cottbus (Germany). The length and connectivity of the evolving network were quantified from aerial images. This analysis showed that the major rill network established already during the first year of ecosystem development. We adapted two model approaches to simulate the emergence of erosion rills. First, we used a self-organized critical network approach with soil erosion and sediment deposition governed by a local critical shear stress. The second approach, based on Manning's equation, was developed to compare the results of the self-organized critical network approach with results from a simple but more physically based approach. Erosion was triggered in the case of high local shear stress and deposition was calculated from settling velocity of the suspended particles. Both models were able to reproduce the observed rill network. Geometric characteristics such as network length and rill depth were simulated in the right order of magnitude. The models also managed to describe the position and temporal evolution of the main erosion rills in the catchment.

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1. Introduction

[2] Erosion on bare soil caused by concentrated ephemeral surface runoff leads to the formation of erosion rills with depths in the order of several centimeters (Soil Science Society of America, Glossary of Soil Science Terms, available at <https://www.soils.org/publications/soils-glossary#>, 2008). Many studies have shown the hydraulic and soil hydraulic characteristics in such surface erosion rills to be considerably different compared to interrill areas (for a general description of rill and interrill erosion the reader is referred to the reviews by Knapen *et al.* [2007] and Gumiere *et al.* [2009]). Slattery and Bryan [1992] and Brunton and Bryan [2000] investigated a selection of hydraulic parameters, such as runoff, Reynolds number (ratio of inertial to viscous forces), and shear velocity in laboratory flume experiments and documented significant changes of the parameters with channel incision and tributary development.

Poesen [1984] and Bryan and Poesen [1989] observed higher infiltration rates in rills than on interrill areas. This was confirmed by recent studies showing that infiltration rates at rill sites on two nonvegetated mining dumps were 5–42 times higher than on interrill sites [Biemelt *et al.*, 2005; Gerwin *et al.*, 2009a]. Higher infiltration rates in rills cause a reduction of downstream channel discharge [Bryan and Poesen, 1989]. Where rills incise down to the capillary fringe of a shallow groundwater table, hydraulic conditions in the rills are determined by seepage water significantly increasing soil erodibility [Huang *et al.*, 2001], hence inducing a positive feedback between rill incision and runoff. Furthermore, the incision of rills down to the capillary fringe causes water to flow off rapidly in the rills. This alters the local hydraulic gradients and promotes lateral flow from adjacent areas toward the rills.

[3] Because of the hydrological importance of surface erosion rills, a reasonable representation of the rill network is mandatory when modeling water percolation and runoff at the catchment scale. Thus, a model simulating the hydrology of a catchment prone to rilling should consider (1) the location of rills, (2) the geometric features of rills such as rill depth, (3) rill evolution with time, and (4) the appropriate scale (catchment scale) to account for the entire contributing area.

[4] However, most erosion models developed in the last years cannot satisfy all of these requirements. Rill characteristics, such as rill spacing, rill geometries or the location

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of rills often have to be specified a priori [e.g., *Parsons et al.*, 1997; *Ascough et al.*, 1997; *Desmet and Govers*, 1997], indicating that these models are static and only applicable to surfaces on which rills are already present. Conversely, dynamic rill evolution models are limited by their dependence on initial conditions of microtopography. Although some models allow erosion rills to grow in width and depth, the initiation and development, especially the lengthways extension of rills, is poorly modeled when compared to observed rill networks [*Favis-Mortlock*, 1998; *Favis-Mortlock et al.*, 2000]. Studies based on models accounting for the dynamic lengthways growth of drainage networks often investigate the evolution of virtual catchments. Here, the comparison of the simulated drainage networks with real networks is done only in terms of statistical means [*Kramer and Marder*, 1992; *Leheny and Nagel*, 1993; *Rinaldo et al.*, 1993; *Simpson and Schlunegger*, 2003]. There are surprisingly few studies, where erosion rills have been mapped and compared to simulated rill networks with observed erosion patterns [*Jetten et al.*, 2003].

[5] A further problem of current erosion models is the lack of accounting for changing microtopography caused by erosion and deposition processes [*Zobeck and Onstad*, 1987; *Nearing et al.*, 1997; *Planchon et al.*, 2000; *Planchon and Mouche*, 2010] and related alteration of surface flow paths during a rainfall event [*Favis-Mortlock et al.*, 2000]. As a realistic representation of the surface flow network is a prerequisite for modeling reasonable erosion patterns [*Jetten et al.*, 1999; *Takken et al.*, 2001] dynamic surface flow paths and the dynamic representation of erosion and deposition processes should be taken into account. Most research dealing with soil erosion by water has concentrated on erosion processes operating at the plot scale or is based on laboratory studies carried out in small flumes limiting the full development of the rill network [*Slattery and Bryan*, 1992; *Papanicolau et al.*, 2010]. Only relatively few studies have been conducted on rill erosion operating at larger spatial scales [*Poesen et al.*, 2003; *Chaplot et al.*, 2005].

[6] With this work we aim to overcome these shortcomings of existing models. We developed and tested a model that predicts the spatial location of rills and simulates the dynamics of rill growth. The model is based on the self-organized critical network model presented by *Rinaldo et al.* [1993]. Additionally, a second model was developed describing surface runoff with the Manning equation in order to compare the self-organized critical network approach with a simple but more physically based model approach. The models were validated against observed rill patterns and measured rill depths from a man-made 6 ha large catchment (Chicken Creek) in eastern Germany.

[7] Our study intends to answer the following questions: Are the models able to predict the location of evolving surface erosion rills in an artificial catchment? What are the controls for the development of surface rills in a new developing ecosystem? Does the rill network extension converge to a steady state? How do the surface flow paths change with developing erosion rills? The field site with evolving erosion rill network is characterized in section 2. The two model approaches are explained in section 3. The model results are presented and discussed in sections 4 and 5. After a summary in section 6 the study is closed with an appendix on the procedure used to calculate erosion rates.

2. Site Description

2.1. General Site Characteristics

[8] The artificial catchment Chicken Creek is located in the postmining landscape of the open-cast lignite mine Welzow-South, 150 km south from Berlin (Germany). The catchment was constructed by the mining company Vattenfall Europe Mining AG in cooperation with the Brandenburg University of Technology in Cottbus. It was established as a joint research site of the Transregional Collaborative Research Centre (SFB/TRR 38), a project investigating the structures and processes of the initial ecosystem development phase in the artificial catchment. After completion in late 2005 the catchment was left to an undirected and undisturbed development [*Kendzia et al.*, 2008].

[9] The extension of the Chicken Creek catchment is about 450 m in northwest and 140 m in north–northeast direction resulting in a catchment area of approximately 6 ha (Figure 1). The catchment has an elevation difference of 15 m and an average longitudinal inclination of 3.2%. The catchment is composed of three major sections [*Gerwin et al.*, 2009b]: the backslope with low inclination (2.7% in average), the footslope with a steeper inclination (5.4% in average) and the lake basin. During the past five years the basin was filled gradually by catchment runoff and direct precipitation input and evolved to a small lake with a surface of 3600 m² and an average depth of 1 m. The backslope of the catchment is gently v-shaped with two parallel sides facing together with an inclination of 0.3 – 2.1% [*Schneider et al.*, 2011] causing surface runoff to concentrate at the centerline.

[10] The soil material, ranging from sand to loamy sand and sandy loam (USDA soil texture triangle), was dumped by a stacker on top of an impermeable clay layer. Soil texture measurements from 316 samples taken from soil core probes from different soil depths throughout the research site [*Gerwin et al.*, 2009b] showed that the mean composition of the soil consists of 83% sand of which 13% were classified as coarse sand (0.63–2 mm), 44% as middle sized sand (0.2–0.63 mm), and 26% as fine sand (0.063–0.2 mm). The clay and silt fractions account for 7 and 10%, respectively. The soil surface was leveled by caterpillars and tracks were removed by final flattening at the end of the construction works [*Gerwin et al.*, 2009a]. Dominant structures evolving in the last five years were erosion rills on the surface (Figure 1). The rill network began to form immediately after the construction was completed.

2.2. Rill Network Assessment by Remote Sensing

[11] To investigate the evolution of the catchment surface in time microdrone- and helicopter-borne aerial images were taken once per year between September 2005 and April 2010. The assessment of erosion rills by remote sensing techniques has proven to be a successful method to determine the geometry of erosion rills [*Bobrovitskaya and Vorozhbitov*, 1979; *Hancock et al.*, 2008; *Waythomas et al.*, 2010], their dynamics [*Daba et al.*, 2003] and the volume of eroded soil on a slope [*Watson and Evans*, 1991]. From the aerial images, we visually digitized the backbone of the rill network of the different years (Figure 2a). The main rill network connecting the slope with the lake area established already during the first year (between October 2005 and

Figure 1. Overview of the artificial catchment Chicken Creek (aerial image from June 2007) with gray lines marking the boundary of the field site. Note the erosion rills visible at the foothills. Insets: Erosion rills developed in the catchment (left) June 2006; (right) September 2006) with white and orange lines emphasizing contour and depth of rills. The large rill on the right-hand side of the left image is about 0.5 m deep, whereas the depth of the smaller rill in the left image and the rill in the right image are only a few centimeters deep.

September 2006). A major rill formed in the middle of the catchment, running around a weir facility at the lower end of the slope, and spanning the entire area in northwest-southeast direction. The network density was high on the foothills, and, apart from the main middle rill, negligible on the backslope. The second year (September 2006 to June 2007) was characterized by elongation and further branching of the existing erosion rills but no new major rills reaching down to the lake formed. In the following years (July 2007 to April 2010) only minor rill expansions and short new tributaries were detected and the rill network approached a steady state. Figure 2b quantifies the network evolution over the years: 63% of the maximum rill network (the network in 2010) formed in the first year, 25% in the second year, 8% in the third year, and 3% and 1% in the fourth and the fifth year, respectively. As the major part of the rill network formed in the first year and afterward only developed marginally, we confined our study to the rill network evolution of the first year. The rill network extracted from the aerial image of September 2006 will be further referred to as the “measured” rill network.

[12] Rill depth measurements from September 2008 are available for four transverse transects in the catchment. The data were obtained by a Riegl LMS-Z420i laser scanner mounted on a 6 m high tower on various positions inside of the catchment. The four transects were extracted from the data and a raster based minimum filter was applied to remove the vegetation from the data. To reconstruct the surface without the rills, a spline with 30 degrees of freedom

was fitted using a 90%-quantile regression [R Development Core Team, 2008; R. Koenker, quantreg: Quantile Regression, R package version 4.67, available at <http://CRAN.R-project.org/package=quantreg>, 2011]. The distances from the measured minimum points to this spline was used as rill depth. By applying a local minimum finder (“peak” from A. Ruckstuhl et al., IDPmisc: Utilities of institute of data analyses and process design, R package version 1.1.10, available at <http://CRAN.R-project.org/package=IDPmisc>, 2010] the major individual rills were extracted. The depth measurements of totally 18 rills (Figure 2a) were used to compare the simulated with the observed rills. The measured rill depths are between 0.07 m and 0.65 m.

2.3. Digital Elevation Models

[13] The initial state of the catchment surface and the developed state in 2008 were reproduced by Digital Elevation Models (DEMs). The DEMs have a spatial resolution of 1 m and were produced by means of automated digital photogrammetry based on aerial images taken in November 2005 and August 2008 (courtesy of Vattenfall Europe Mining AG, Cottbus). The DEMs were improved by filtering out systematic errors and artifacts [Schneider et al., 2011].

3. Model Descriptions and Methods

[14] As stated in the introduction, the objective of this study is to explore if various characteristics of a measured rill network can be represented by model approaches. For

Figure 2. (a) Visually digitized backbone of the rill network extracted from aerial images. The dark brown lines denote the rill network formed in the first year (October 2005 to September 2006). The orange lines represent the rills, which developed after the first year until April 2010. The circles indicate the positions of the rill depth measurements obtained by laser scanning. The positions are enumerated from West to East along the transects. The first position of each transect is marked in the figure with the name of the transect and the number 1 (e.g., A1: A for A-transect and 1 for the first position in the transect). The green star shows the location of the weir facility. (b) Evolution of total erosion rill length, L_r . Rill evolution in the figure starts from the unrilled state of the surface in October 2005 and ends with the most actual aerial image available in April 2010. Rill growth was intense in the first year and successively decreased afterward. The network length in 2010 is considered as maximum total rill length. In the first year 63% of the total network length developed.

the evaluation of the model simulations the length of the rill network detectable in the airborne images and the rill depth measurements derived from the laser scanning are available. However, we must be aware that the “true” structure of the erosion network including complete depth information and location of small rills too shallow to be revealed by airborne imaging is unknown. This uncertainty is a challenge for the comparison between measured and modeled network. For that reason we made two different types of analysis: (1) We compared geometric characteristics of modeled and measured erosion rills like depth, position, length and the spatial density of the rills. Simulated rill depths were compared with laser scanning measurements. A rill was selected from a laser scanning transect if it was detectable on the aerial image and if both of the models also simulated the rill. The resulting 18 rill points are displayed in Figure 2a. The difference between the DEM 2005 and the DEM 2008 was used to infer the maximum rill depth in the catchment. We are aware that rill depths obtained from the laser scanning in 2008 and from the difference of the DEMs may be higher than at the date of the aerial image (September 2006). However, since rilling activity decreased after 2006 and based on field inspections

and photographs from 2006 the rill depths measured in 2008 are considered to be comparable to the rill depths in 2006. (2) In addition, we investigated if the modeled flow network matches the rill network deduced from airborne images. Based on the assumption that deep rills occur where surface flow is the most erosive, we hypothesize that the imaged rills correspond to large flow paths and that the measured rill network may be used as a surrogate of the true flow field. This assumption seems reasonable, especially for the footslope, where most of the single segments of the surface flow network are supposed to have enough energy to produce erosion rills due to the steeper slope and the higher contributing area. On the backslope the congruence between the rill network and the true flow field is expected to be less pronounced since the erosive power of surface flow is limited due to flatter slopes and smaller contributing areas.

[15] Two different model approaches were used to simulate the emergence of rill networks in the Chicken Creek catchment. They are presented in section 3.1 and 3.2, respectively. A short sensitivity analysis of the free parameters of the models is included in the discussion. The spatial domain of both models is the DEM of the smooth

catchment (no rills) imaged in 2005 divided into 53'519 cells with a resolution of 1 m. The overall shape of the catchment generally assures that each model cell has a defined surface flow direction and is connected to the lake or the catchment boundary following a strictly decreasing path. Artifact depressions resulting from the DEM generation procedure were removed using an algorithm developed by *Planchon and Darboux* [2001]. The neighborhood of a center cell i is defined by the eight cells surrounding i (Moore neighborhood). Four neighbors are located in the orthogonal direction at distance 1 m and four neighbors are situated in the diagonal direction at distance $\sqrt{2}$ m.

3.1. Self-organized Critical Network Model

[16] The first approach is an adaptation of the self-organizing critical network (SOCN) model proposed by *Rinaldo et al.* [1993] based on the principle of minimum energy dissipation in the network [*Rodríguez-Iturbe et al.*, 1992]. The authors used the model to compare fractal and multifractal network statistics of randomly generated virtual DEMs with optimal channel networks and with characteristics of naturally evolved large-scale river networks [*Rinaldo et al.*, 1993; *Rodríguez-Iturbe et al.*, 1994]. We applied the model to the surface of the Chicken Creek Catchment in order to investigate the evolving erosion rill network at the hillslope scale. As explained in more detail in the next paragraph, the original model was adapted to handle physically based threshold values required in erosion modeling and to describe particle deposition.

3.1.1. Representation of the Surface Flow Paths in the SOCN Model

[17] For the SOCN model the surface flow pattern is represented by the contributing area. *Rodríguez-Iturbe et al.* [1992] have shown that the mean annual discharge from a cell i , Q_i , is proportional to the contributing area A_i , which is defined as the areal sum of all cells eventually draining through cell i . The surface flow direction from a cell i was computed in the direction of steepest slope S_i between cell i and its neighborhood. This method is commonly known as the deterministic eight-neighbors (D8) algorithm [*O'Callaghan and Mark*, 1984; *Fairfield and Leymarie*, 1991].

3.1.2. Erosion and Deposition

[18] Shear stress τ is one of the key quantities guiding the erosion process [*Torri et al.*, 1987]. For approximately steady, uniform equilibrium flow shear stress is quantified as product of fluid density ρ_f , gravitational acceleration g , slope S and flow depth d_f :

$$\tau = \rho_f g S d_f. \quad (1)$$

[19] Erosion occurs if a critical threshold value τ_{ce} is exceeded. Shear stress is largely determined by runoff Q , which has been shown to scale with flow depth as $d_f \propto Q^{0.5}$ [*Rodríguez-Iturbe et al.*, 1992]. Thus, at a given grid cell i the shear stress τ_i is proportional to $Q_i^{0.5} S_i$. Runoff in turn is proportional to the contributing area A_i . Thus, following *Rodríguez-Iturbe and Rinaldo* [1997] and setting the coefficient of proportionality to 1, τ_i can be estimated as:

$$\tau_i = A_i^{0.5} S_i. \quad (2)$$

[20] The critical shear stress for erosion τ_{ce} was determined following *Alberts et al.* [1995], who empirically defined it for cropland surface soils containing 30% or more sand as

$$\tau_{ce} = 2.67 + 6.5c - 5.8vfs, \quad (3)$$

where c and vfs are the fractions of clay and very fine sand (0.05–0.1 mm), respectively. For $vfs < 0.40$ the authors recommend to use $vfs = 0.40$ in equation (3). Using the averages of the clay and fine sand texture measurements from the Chicken Creek catchment, a threshold value τ_{ce} of 0.81 Pa is assigned to all grid cells. This value is in agreement with values for sand surfaces reported in other studies [*Mitchener and Torfs*, 1996; *de Linares and Belleudy*, 2007]. We also chose 0.81 Pa for the critical shear stress for deposition τ_{cd} implying that deposition activities in the catchment only take place for $\tau < \tau_{cd}$. Model simulations with $\tau_{ce} \neq \tau_{cd}$ were also conducted and are discussed in section 5.4.1. For each cell i the shear stress is calculated based on equation (2). Between the cells with shear stress exceeding the critical value for erosion ($\tau > \tau_{ce}$) the cell with maximum exceedance is determined and its elevation is decreased by the value of the erosion parameter d_e . Because erosion rills of maximum depth of about 1 m were generated during about 120 days of rainfall, we set $d_e = 0.01$ m (effect of other values will be discussed in section 5). For all cells where erosion takes place the threshold is increased by 4% to account for soil erodibility decreasing with increased soil bulk density in lower soil depth [*Bradford and Grossman*, 1982; *Mouzai and Bouhadeh*, 2011; *Amos et al.*, 1992]. After erosion at the cell with maximum exceedance, the contributing area for all cells is recomputed to account for the modified surface topography. Downslope of the eroded cell, material is deposited at the nearest cell with $\tau < \tau_{cd}$. The amount of deposited material d_d was calculated as $d_d = ((\tau_{cd} - \tau)/\tau_{cd})d_e$. For $0 < \tau < \tau_{cd}$ material is left for deposition and will be deposited at the next cell with $\tau < \tau_{cd}$ in downslope direction. As soon as all eroded material is deposited, the contributing area of all cells is recomputed to account for topography modified by deposition. Erosion and deposition is repeated until a steady state with $\tau < \tau_{ce}$ is reached for all cells. It should be emphasized here that time in the SOCN model is not represented explicitly and that a single iteration step during the simulation cannot be related to time in terms of day, hours or seconds. The simulation algorithm is summarized in the flowchart shown in Figure 3.

3.1.3. Delineating the Surface Flow Network in the SOCN Model

[21] Later on we will compare modeled and measured flow and rill networks. To determine the flow network in the SOCN approach we neglected cells with contributing areas smaller than a threshold value A_{crit} [*Gómez et al.*, 2003]. The threshold value was chosen according to the density D_f of the surface flow network and based on the assumption that on the footslope the measured rill network corresponds well with the surface flow field. The density D_f was defined as the ratio between the total length of the surface flow paths L_f and the catchment area of the footslope A_f . First, we computed the measured rill density for the

Figure 3. Flowchart of the self-organized critical network (SOCN) model where τ_{ce} is the critical shear stress for erosion, τ_{cd} is the critical shear stress for deposition, H is the erodibility factor, d_e is the erosion parameter, A is the contributing area, and i_e and i_d are cells where erosion or deposition occur. The loop stops when all cells are exposed to shear stress below the critical value.

footslope. Thereafter, we calculated the modeled flow network density for increasing values of A_{crit} until measured and modeled rill density for the footslope were identical (for $A_{crit} = 35 \text{ m}^2$). All cells with contributing areas smaller than A_{crit} were considered not to be part of the flow network.

3.2. Manning-Based Model

[22] The model accounts for depth and velocity of the ephemeral surface runoff. For that purpose we have to interpret rainfall data from the Chicken Creek catchment in terms of height and volume of available water at the surface. Rainfall dynamics in the temporal resolution of a few minutes or even seconds would be necessary to exactly describe the dynamics of surface runoff. Unfortunately, such high-resolution rainfall data were not available. In the model we use a constant rainfall intensity, although we know that this reduces the dynamics of surface runoff in the catchment. Of course, the larger surface flow depths generated by higher simulated rainfall intensities would increase the erosive power of the surface water and thus would have an impact on the simulated rill depths. However, since the major zones of surface flow convergence are considered to be relatively independent on the surface flow depth, we argue that mainly rill deepening is affected by the damping of surface runoff dynamics. The location of

the flow paths, as well as the position, lengthwise extension and spatial density of the main rill network are less affected. Additionally, using a constant rainfall intensity allows a better comparison with the SOCN model, which is based on mean runoff. The rainfall intensity used in the model was determined as follows: For the time series October 2005 to September 2006 12 rainfall events with a total of 324 mm of rainfall could be distinguished. Rainfall events were separated from each other based on the criterion of minimum four days without rainfall between events. As shown in a previous study [Hofer *et al.*, 2011] more than a total of 9 mm of rainfall per event is needed to generate runoff in the Chicken Creek catchment. We assign these 9 mm to soil sorptivity and subtract its amount from the cumulated rainfall amount of each event. The remaining total rainfall (216 mm) is equally divided between all the days of the twelve events with rainfall larger than zero ($T = 120$ days) resulting in the rainfall intensity of 1.8 mm per day.

3.2.1. Modeling Surface Water Flow

[23] The flowchart of the MB model (Figure 4) gives an overview of the algorithm described in detail in sections 3.2.2 and 3.2.3. Precipitation of 1.8 mm is added to each grid cell as rainfall excess. Surface water flow is driven by the elevation difference between ponding water levels in

Figure 4. Flowchart of the Manning-based (MB) model. T is the number of days with rainfall. $T_{\max}$ is 120 and represents the last day at which precipitation (1.8 mm) is added as source term.

neighboring cells. For each grid cell with ponding water the flow velocity v was calculated based on the Manning equation:

$$v = \frac{d_f^{3/2} S_w^{1/2}}{m}, \quad (4)$$

with hydraulic radius estimated as water depth d_f [Parsons and Fonstad, 2007], downslope gradient of water level S_w , and a roughness coefficient m . Water depth d_f is determined from rainfall input and from eventual lateral surface input calculated by equation (6). The slope S_w is defined by the difference in ponding water level and the distance l between the centroids of adjacent neighbors, that is 1 m in orthogonal and $\sqrt{2}$ m in diagonal direction. As in the case of the SOCN approach, water is flowing only in the direction of the maximum gradient. A low value (0.03) was chosen for m to represent the unvegetated and rather smooth sandy surface [Dingman, 2008]. Although widely used in erosion models the application of the Manning equation to describe rill flow was criticized [Govers et al., 2007]. Giménez and Govers [2001] and Giménez et al. [2004]

noticed that flow velocities in rills tend to be independent of slope due to a feedback between rill bed morphology and flow conditions. Thus, for models using a static topography the assumption of a constant hydraulic roughness m in the Manning equation was considered inappropriate when simulating eroding rills [Govers et al., 2007]. However, the use of a constant m in the MB model was justified by the description of a dynamic surface topography during a model simulation. Since slopes are gradually adapted according to the erosive power of the surface flow (section 3.2.2) the feedback between rill bed morphology and flow conditions is considered. In the case of lake formation at the lower end of the catchment the velocity of water flow on the equalizing water table is described with a wave velocity formula [Garrison, 2009]:

$$v = \sqrt{g d_f}, \quad (5)$$

where g is gravitational acceleration. Within each time step t the water volume V flowing in downslope direction is

$$V = Q t = d_f s_{oct} v t, \quad (6)$$

with surface runoff Q and the width of the stream between two cells determined as side length of an octagon s_{oct} with the same area as the grid cell (1 m^2). The characteristic time of the model is represented by a dynamic time step t , which was determined based on the condition that a water volume cannot be transported beyond the length l (corresponding to the distance between cells) and is thus limited by the maximum flow velocity $v_{\max}$ in the system:

$$t = \frac{l}{v_{\max}} \quad \text{for } v_{\max} > 0. \quad (7)$$

[24] In the course of time, surface water is redistributed in downslope direction toward the lake and the volume of ponding water in upslope direction is decreasing. Because according to equation (4) velocity approaches zero for small water depths d_f , we stopped water redistribution when the cumulated surface flow of all cells dropped to less than 5% of the initial value after adding 1.8 mm water. When this criterion was fulfilled the next portion of 1.8 mm water was added to each cell. The simulation stopped after 120 precipitation events, each contributing 1.8 mm, were distributed.

3.2.2. Erosion and Deposition

[25] By using a constant average rainfall intensity surface runoff peaks will be damped and runoff is assumed to be comparable to mean runoff. The erosion rate, e , was calculated as a threshold-dependent function based on shear stress, τ (equation (1)). For the comprehensive derivation of e from τ the reader is referred to Appendix A.

[26] The eroded material at a time step t is transferred in suspension to the downstream cell. The suspended sediment is equally distributed along a vertical profile in the water column. Deposition was calculated according to the Stokes formula:

$$v_s = \frac{2r^2 g(\rho_s - \rho_f)}{9\eta}, \quad (8)$$

with sediment velocity v_s of grains with particle radius r , sediment density ρ_s (here 2650 kg m^{-3}) and dynamic fluid viscosity η ($0.001 \text{ kg s}^{-1} \text{ m}^{-1}$ for water at 20°C). Because medium and fine sand fractions were dominant soil texture classes we used $r = 0.2 \text{ mm}$ as a representative value for the particle size. The sink velocity determines the maximum change of height d_s of a particle in its settling movement during the time step. The amount of deposited sediment between successive time steps equals the fraction d_s/d_f with d_f being the height of ponding water. For $d_s > d_f$ all suspended sediment is deposited.

3.2.3. Delineating the Surface Flow Network in the MB Model

[27] The procedure to delineate the flow network in the MB model is very similar to the one used in the SOCN approach. However, instead of using the critical contributing area, a threshold of total discharge from a cell was used in the MB model to decide whether a cell belongs to the flow network or not. The total discharge from each of the model cells was calculated by adding 120 times 1.8 mm of rainfall and successively letting the water flow off according

to the model described above. As in case of the SOCN model, the threshold was iteratively increased until measured and modeled flow network density for the footslope of the Chicken Creek catchment were identical. This was the case for the discharge threshold of $0.2 \text{ m}^3 \text{ d}^{-1}$. All cells with lower discharge were considered not to be part of the flow network. The resulting binary image consisting of cells with flow above threshold was skeletonized according to the algorithm proposed by *Vogel et al.* [2005] to obtain the backbone of the predicted flow network, which was then compared to the flow network of the SOCN model and to the measured rill network.

3.3. Surface Flow Network

[28] To estimate the quality of the simulated surface flow networks we compared the simulations with the measured rill network of the Chicken Creek catchment. As explained in the beginning of section 3, we assumed that the measured rill network is a first-order estimate for the unknown ephemeral surface flow pattern of the Chicken Creek catchment. Two methods to characterize the simulated surface flow networks were applied. First, we used the Strahler order [*Horton, 1945; Strahler, 1957*], a numerical measure of the networks branching complexity. We compared the total length of the flow branches of individual Strahler orders normalized by the total length of flow branches for all orders. The surface flow branch length corresponding to the different Strahler orders was compared with that of the measured rill network (serving as first-order estimate of the flow network). By this method not only the topology of the flow network is investigated, but also the lengths of the flow branches are considered. As a prerequisite of this comparison, the Strahler orders of the different flow branches must be quantified. This is straightforward in case of the SOCN model where the flow network equals the contributing area that is characterized by a treelike structure without loops because each cell has just a single outlet. However, for the Manning approach and the measured rill network closed branches (loops) can occur in the network and we used the following procedure to determine Strahler order: at a bifurcation of a surface flow branch in downslope direction, the main flow branch is defined as the most direct branch toward the lake. The less direct branch is disconnected and is treated as individual tributary. The second method to validate the simulated surface flow networks is based on the relationship between flowpath length ratios. For a cell i belonging to the flowpath network the relative flowpath length L_p is defined as L_i , the flowpath length between the lake and i , divided by the maximum flowpath length. The relative flow network length L_n is defined as the total flow network length of all individual flow paths up to the distance L_i divided by the total flowpath network length. The relationship between the relative flowpath length and the relative flow network length was used to compare the simulated flowpath networks with the measured one.

3.4. Quantifying the Rill Network

[29] The rill networks simulated with the SOCN and the MB model were compared with measurements of total rill network length, rill density of the total catchment, the back-slope and the footslope, and with rill depths. The “true” maximum rill depth was estimated from the difference

Figure 5. Comparison of (a) the measured erosion rill network, serving as proxy for the surface flow paths in the catchment and surface flow paths simulated by (b) the SOCN model, and (c) the MB model, respectively. The dark brown arrow indicates the main rill emerging at the back-slope and flowing around weir facility (green star). The western part of the catchment is not drained by the main rill but by an independent rill network (orange arrow). These main characteristics are represented by model approaches.

between the DEM of 2005 and the DEM of 2008. The simulated rill networks were also compared visually with the measured network in order to verify the location of the simulated rills.

4. Results

4.1. Surface Flow Networks

[30] The measured rill network representing the surface flow paths of the Chicken Creek catchment and the simulated flow patterns of both models are displayed in Figure 5. Note that for the MB model not all the flow paths of a network do always conduct water during a model simulation. For example flow paths at the catchment boundary are “dry” for a longer time due to a limited lateral water inflow. However, the main flow paths at the footslope draining large parts of the catchment are almost always “wet.” This reflects the ephemeral character of the flow paths also observed in nature. The imaged flow network consists in two structures: a main drainage pathway emerging in the center of the back-slope that is fed by smaller inflows on the footslope and an additional independent flow network in the western part of the catchment. The models match these main characteristics of the observed flow network.

[31] Figure 6a shows the total length of flow branches of Strahler order ω normalized by the total surface flow network length on they axis and Strahler order on the x axis. The values show the expected decay of flow branch length with increasing Strahler number. The simulations are in good agreement with the measured rill network, although both models overestimated the length of flow branches of order 1. The MB model slightly underestimated all the lengths of flow branches of higher orders whereas for the SOCN model no trend of overestimating or underestimating the lengths of flow branches of higher orders was observed. Since very fine rills in the catchment are not detectable on the aerial image, we expected the number of order 1 flow branches to be underestimated. Figure 6b displays the relationship between the relative flowpath length L_p and the relative flow network length L_n . Figure 6 quantifies the differences of the flowpath networks shown in Figure 5. The increase of L_n with L_p is relatively constant for

Figure 6. Quantification of simulated surface flow branches (SOCN and MB model, respectively) and measured erosion rills (MEAS). (a) Total flow branch length of a given Strahler order ω normalized by total flow branch length. The topological relations of the simulated networks are in good agreement with the measured network. (b) Relationship between the relative flowpath length L_p and the relative flow network length L_n . The homogeneous distribution of flow paths in the catchment for the two simulations is represented as uniform increase of L_n with L_p . The deviation of the curve representing the measurements is explained by the difference of the flowpath density between the footslope and the backslope.

the two simulations. This is the result of a quite homogeneous distribution of flow paths in the catchment, which is confirmed by the visual inspection of Figures 5b and 5c. The increase of L_n with L_p is much less uniform for the measured flow paths and deviates from the simulations especially for $0.2 < L_p < 0.8$. The deviation can be explained by the difference of flowpath density between the footslope and the backslope of the measured network, which is directly affecting the relative network length L_n .

4.2. Rill Networks

[32] The rill network digitized from the aerial image taken in September 2006 is shown in Figure 7a. The main rills were annotated with a number. The total rill network length accounted for 4066 m, which results in a rill density of 0.09 m^{-1} for the entire catchment. Rill density on the footslope is about three times higher (0.17 m^{-1}) than on the backslope (0.05 m^{-1}). The maximum measured rill depth is located in the erosion rill North from the weir and is a result of the combination of steep terrain and high-flow depths in this area (Figure 7a). The measured rill depths derived from the laser scanning were between 0.07 m and 0.65 m (Table 1). Generally, two trends in rill depths are observable. Rills on the footslope are deeper than rills on the backslope and rills besides the main middle rill are more shallow than the main rill.

Figure 7. Comparison of (a) the measured rill network with (b) the rill network simulated with the SOCN model, and (c) the rill network simulated with the MB model, respectively. The numbers indicate the main rills developed in the first year. The main 12 erosion rills are represented by the models. In the Manning-based approach the rill network is not continuous due to the high impact of deposition of material. The orange arrows indicate the location of maximum rill depth, which is measured and simulated near the weir (green star). The circles indicate the positions of the rill depth measurements obtained by laser scanning (A1-D1 mark the first position of each transect).

Table 1. Comparison of Measured and Simulated Rill Depths at the Positions Indicated in Figures 2a and 7a^a

Position	MEAS	SOCN	MB
A1	0.28	0.42	0.15
A2	0.15	0.31	0.03
A3	0.58	0.65	0.17
A4	0.65	0.31	0.53
B1	0.23	0.30	0.01
B2	0.28	0.37	0.02
B3	0.11	0.03	0.06
B4	0.21	0.16	0.01
B5	0.60	0.52	0.38
B6	0.23	0.23	0.09
B7	0.26	0.03	0.01
C1	0.13	0.27	0.02
C2	0.10	0.21	0.03
C3	0.16	0.17	0.01
C4	0.47	0.43	0.25
C5	0.21	0.23	0.02
D1	0.07	0.05	0.07
D2	0.22	0.10	0.10
Max	0.81	0.77	0.89

^aMeasured values (MEAS) are from laser scanning, except the maximum rill depth (Max), which was derived from the difference between the DEMs of 2005 and 2008. Values for the main middle rill are given in bold. Simulated rill depths are in the right order of magnitude, especially for the SOCN model. The MB model generally underestimated the rill depths.

[33] Visual comparison between the simulation of the SOCN model and the measurements of the individual rills showed that the main rills were captured well by the model (Figures 7a and 7b). Generally, the length extension of the rills is simulated realistically, although for some of the rills (3 and 7) the simulated rill was too short. The total simulated rill length for the SOCN approach is 3365 m (83% of the measured total network length). The modeled rill density for the entire catchment is 0.07 m^{-1} (78% of measurements), whereas the backslope accounts for 0.04 m^{-1} (80% of measurements) and the footslope for 0.11 m^{-1} (65% of measurements), respectively. Average rill depth and maximum simulated rill depth are 0.18 m and 0.77 m (measured value 0.81 m), respectively. The SOCN model predicted the deepest rill segment in the western part of the weir (Figure 7b), where the measured rill depth is smaller, but still more than twice the average (0.38 m). On the other hand, at the location where the deepest rill was measured, the rill depth simulated by the SOCN model is only 0.3 m. Comparing the rill depth simulated by the SOCN model with the measured rill depths (Table 1) it became clear that the model was able to reproduce the correct magnitude of the rill depths. For the 18 selected points the model simulated rill depths between 0.05 and 0.65 m. Also the general characteristics of decreasing rill depth with increasing distance from the lake and large depths for the main rill and smaller depths for rills beside were well represented.

[34] Comparing the results of the Manning approach with the measurements (Figures 7a and 7c), it could be seen that the locations of the main rills were simulated reasonably well. However, two main differences between measured and simulated rill network exist: first, the simulated rill network is not continuous, meaning that not every rill cell is connected with the lake. This is also apparent in the reduced total rill network length of only 2459 m (60% of measured length). The reason for this is the strong

erosion-deposition interplay for this modeling approach. It became especially evident in the lake-near part of the catchment, where simulated rills only formed scarcely, because deposition preponderated over erosion in this area. Because of the noncontinuous network the allocation of an individual rill number was not always trivial (e.g., 3, 4, and 5). However, for most of the simulated rills the corresponding measured individual could be found. Second, the simulated rill density of the backslope was equal to the measured rill density (0.05 m^{-1}). This was in contrast to the visual impression of a higher backslope rill density for the simulation. However, the impression only occurred because the simulated rill cells did not form a continuous structure and were distributed loosely on the backslope area. The low-rill density on the footslope (0.06 m^{-1}) could also be explained by the noncontinuous rill network leading to a shorter total rill length. As the location of the measured maximum rill depth, the MB model simulated the location of the deepest incision in the North of the weir (Figure 7c). Although the simulated maximum rill depth of 0.89 m was well in accordance with the measured value (0.81 m), the average rill depth for the Manning approach (0.07 m) was much smaller than for the SOCN approach. This was also visible from 3-D plots of the simulated rills (Figure 8) and from the comparison of rill depths between laser scanning measurements and simulation (Table 1). Although the range of simulated rill depth was reasonable (0.01–0.53 m) rills were generally simulated too shallow. However, the trend of shallower rills besides the main middle rill was reproduced and the rill depths for the main rill are in the right order of magnitude. The characteristic decrease of rill depths with increasing distance from the lake is less obvious than for the SOCN model and for the measurements.

4.3. Simulated Rill Network Evolution

[35] Figure 9 shows different snapshots in the evolution of the rill network for both models. In the MB model an evolution step corresponds to the addition and runoff of 1.8 mm of precipitation, whereas in the SOCN model a step

represents an iteration in which the elevation for the cell with maximum excess shear stress was decreased. For the SOCN model the maximum rill depth developed rather quickly when the network was only short resulting in a high average rill depth. Network length increased and average depth decreased with proceeding iterations until a steady state ($\tau < \tau_{ce}$ for all cells) was reached. The increasing erosive power in deepening rills (due to steeper slopes and larger contributing areas) is gradually reduced according to the model rule accounting for the consolidation of soil with depth.

[36] The network evolution for the MB model was quite different. The extent of the network established relatively quickly after adding about 20 times the rainfall amount of 1.8 mm. However, maximum rill depth gradually increased with increasing rainfall amount, and so did average rill depth, until the end of the simulation when the total of 216 mm of rainfall had been added.

4.4. Change of Surface Flow Patterns

[37] The change of surface flow patterns due to erosion rill development was assessed by comparing the contributing areas of the original surface (DEM of 2005) with the contributing areas of the simulated evolved DEMs, i.e., after reaching a steady state for the SOCN simulation and after adding 120 times 1.8 mm of rainfall for the MB simulation, respectively.

[38] The same threshold of contributing area ($A_{crit} = 35 \text{ m}^2$; see section 3.1.3) was applied to delineate the flow networks of the original surface and the evolved surface. The change of surface flow paths due to the development of erosion rills is visualized in Figure 10. The main change in the surface flow network of the SOCN simulation is a reduction of the number and the length of flow paths. The erosion of a model cell, that is the source of a rill, likely attracts the drainage direction of neighboring cells, due to the new topographic gradient. This increases the contributing area of the rill cell. However, due to the redirection of flows the contributing areas of some cells next to the rills will drop below the threshold to delineate flow paths (A_{crit})

Figure 8. The 3-D plots (vertical scale-up 5x) of the rill network (a) derived from the DEM 2008, (b) simulated with the SOCN model, and (c) simulated with the MB model. The visual comparison reveals that the average rill depth simulated with the SOCN model is in good agreement with the average rill depth observed in the catchment. The average rill depth simulated with the MB model apparently is underestimated and is, apart from the major middle rill, difficult to identify on this scale. The inset in Figure 8c displays a detail view of the MB simulation (vertical scale-up 10x) in which also smaller rills are shown (indicated by arrows).

Figure 9. Simulated network evolution starting from an unrilled state, where I is the iteration step, P is precipitation, RD_{max} is maximum rill depth and ϕRD is average rill depth. (a) Results of the SOCN model. The network in the SOCN model evolves until reaching a steady state, i.e., until $\tau < \tau_{ce}$ for all model sites (at $I = 70'127$). (b) Results of the MB model. For the MB model the final state is reached after the application of 120 times 1.8 mm rainfall (at $\Sigma P = 216$ mm). Note the difference and the nonlinearity of the rill depth scales.

and consequently some flow paths will not be present anymore after the evolution of erosion rills. For a few cells the evolving rill structures lead to the diversion of surface flow paths. The changes between the initial contributing area and the contributing area of the evolved surface simulated by the MB model are much more pronounced. Although the reduction of surface flow paths is observed, the diversion of the flow paths is dominant. Whereas for the SOCN model the majority of the eroded material was deposited in the lake at the end of the simulation, more material was sedimented on the slope of the catchment for the MB model and caused flow paths to change or be interrupted. This becomes especially evident in areas of high deposition as the area between the lake and the lower footslope, where many initial flow paths disappeared and new ones evolved.

5. Discussion

[39] We used two different model approaches to simulate rill initiation and further growth of the rill network of the

man-made catchment Chicken Creek. The catchment characteristics at the starting point of the project in 2005, a slightly sloping, unvegetated sand soil with a flattened, rill-free soil surface, provided excellent conditions to test erosion rill models on a larger scale than the usually studied flume or plot scale. The first model was based on the principle of minimal energy expenditure in the entire surface flow network [Rodríguez-Iturbe and Rinaldo, 1997], whereas the second model was based on well-established physical and empirical equations for surface water runoff and sediment entrainment. Both models display dynamic erosion phenomena, which finally trigger the geomorphology of the rill network.

5.1. Surface Flow Network

[40] Both methods for delineating the surface flow network, the contributing area on the one hand and the method based on the Manning equation on the other, were able to reproduce the main features of the measured rill network on the catchment's footslope and on the backslope for high

flow convergence areas around the major middle rill. Comparing the initial contributing area presented in Figure 10 (dark brown and blue lines together) with the measured rill network it is obvious that the contributing area is a good indicator for predicting rill locations in steeper parts of the catchment (on the footslope) and in areas with high-flow convergence (along the major rill on the backslope). However, using the contributing area (above a certain threshold) derived from a DEM as proxy for the rill network has several disadvantages when applied to the Chicken Creek catchment. (1) It overestimated the rill density found in flatter parts of the slope with low-flow convergence. (2) It is not able to provide information on rill depths, and (3), the method does not account for the dynamics of an altering surface flow pattern due to the elevation change induced by erosion. To overcome these limitations the representation of flow paths has to be combined with erosion and deposition processes.

5.2. Erosion Rills

[41] Both models account for the dynamic change of topography induced by water erosion and sediment deposition as the topography is successively updated at the end of each iteration step and flow paths at the following iteration step are newly calculated based on the updated topography. Rill erosion in the SOCN model is a function of contributing area and local slope. Thus, for the rather flat areas on the backslope with low-contributing areas only few rills are simulated and the rill density and rill depths on the backslope as well as on the footslope are properly reproduced. However, for the MB model the simulated rill network was very similar to the flow network, thus the rill density of the backslope was clearly overestimated. Additionally, the simulated rills were generally too shallow. A possible reason for these problems could stem from the way precipitation was

represented in the MB model. For each precipitation step the same amount of rainfall was added (1.8 mm). Higher amounts of rainfall would cause higher-flow depths, especially in flow convergence areas, resulting in increased erosion. Although precipitation data on a high temporal resolution (hours) are available for the Chicken Creek catchment there is limited information on the distribution of infiltrating water and surface runoff on a high temporal scale and how these are related to the rainfall threshold to produce runoff observed in the Chicken Creek. Further research is needed to clarify infiltration-runoff relationships for temporally high-resolution rainfall data in order to estimate the real amount of water available for erosion relevant surface runoff. Nevertheless, the MB model was able to locate the evolving rills in the catchment and to distinguish zones of high and low-erosion activity based on only very few general catchment characteristics (topography and average texture). The simulated evolving rill network changed surface flow patterns in many areas of the catchment, but only on a rather small spatial scale. Thus, the basic surface flow pattern did not alter fundamentally with erosion and deposition processes. Therefore, the initial topography of the Chicken Creek catchment was considered as the main control on determining the location of the erosion rills.

5.3. Rill Network Evolution and Stability

[42] The development steps of the SOCN model shown in Figure 10 reveal the transition of the initial system from nonequilibrium to steady state, where no further erosion takes place. The evolution of rills in the SOCN model does not necessarily progress strictly by headward growth, but is rather driven by local slope instabilities [Rodríguez-Iturbe and Rinaldo, 1997]. However, the catchments topography with a steeper footslope where contributing areas are high (unstable slopes) favors the initiation of the rills in the area near the lake from where they propagate headward. The good agreement between measured rills and those simulated with the SOCN model indicates that, until September 2006, the rill network of the Chicken Creek catchment formed according to the underlying process of the SOCN model, i.e., the minimization of energy in the entire network. It is, however, not clear to what extent the steady state of the rill network reached in 2010 is controlled by energy minimization. It is likely that the upcoming pioneer vegetation in spring 2007 and the increase of vegetation patterns thereafter played a major role for the decrease of rill formation. The increase and distribution of vegetation patterns in the Chicken Creek catchment is currently assessed and will allow for a more detailed investigation of the relationship between upcoming vegetation and rill formation.

[43] For the MB model rill initiation started in zones of high-flow convergence and developed along the main runoff paths of the surface flow network. The final state of the rill network is a product of the amount of precipitation added to the catchment. Network evolution will continue with proceeding time, i.e., additional precipitation. However, the series of evolution steps presented in Figure 10 indicates that further rill development is likely to occur especially as rill deepening and less as elongation of the rill network. Because of the increasingly important role of vegetation after the appearance of pioneer species in spring 2007, further rill deepening beyond the year 2006 cannot

Figure 10. Comparison of the contributing area (representative for the surface flow paths) of the initial, unrilled surface topography and the evolved surface topography after (a) the SOCN model run and (b) the MB model run. Blue lines denote flow paths, which did not experience change due to the simulated erosion and deposition processes. The dark brown lines represent flow paths occurring on the initial topography, but not on the evolved surface anymore. The orange lines are flow paths, which were not present on the initial surface, but evolved due to the erosion and deposition induced change of topography.

be understood without taking the vegetation and its effect on soil erosion (stabilization of soil surface, change of infiltration capacities, retention of sediment) into account. However, regarding the spatial rill network extension the state in September 2006 can be considered as steady state.

5.4. Sensitivity of Free Parameters

[44] An advantage of both models is the use of only a small number of model parameters. Most of the parameters were derived from data or from empirical studies. This constrained the number of free parameters and thus increased the reliability of the models. However, it should be mentioned that the performance of the models may be limited when applied to other environments with different parameter settings, for instance with other texture composition and thus with different critical shear stress for erosion (equation (3)). In sections 5.4.1 and 5.4.2 the free parameters of the two models are investigated in more detail.

5.4.1. Parameters in the SOCN Model

[45] Using the SOCN model three free parameters had to be defined: d_e describes the elevation reduction caused by erosion per iteration step for the model cell with maximum shear stress. In the original model of *Rinaldo et al.* [1993] d_e was chosen dynamically at each iteration step so that the elevation of the cell was eroded to a value which yielded $\tau = \tau_{ce}$. Although producing reasonable rill patterns this method was not suitable for simulating the rill network in the Chicken Creek catchment, as rills can easily get as deep as several meters. In order to reduce the simulated rill depth we defined d_e as static parameter with a value of 0.01 m. The rill network length and especially the average rill depth are sensitive to changes in d_e . Decreasing d_e to 0.002 m reduced total rill network length by 28% and average rill depth by 78%. On the other hand, assuming a d_e of 0.02 m increased the total rill network length by 40% and the average rill depth by 67%.

[46] The second free parameter is the critical value for deposition, τ_{cd} . It has been argued by *Partheniades* [1977] and *Mimura* [1993] that erosion and deposition do not occur simultaneously. This means that the critical value for erosion τ_{ce} is equal to the critical value for deposition τ_{cd} . Thus, if the shear stress τ is higher than the critical value erosion takes place and for subcritical τ deposition occurs. This simple, binary approach of erosion and sedimentation has often been used [e.g., *Mercier and Delhez*, 2007] because of its straightforward implementation in models. However, other studies suggest erosion and deposition processes at the same time [*Rose et al.*, 1983; *Huang et al.*, 1999; *Hairsine et al.*, 2002] indicating that the critical value for deposition is larger than the critical value for erosion ($\tau_{cd} > \tau_{ce}$). For shear stresses between τ_{ce} and τ_{cd} erosion, as well as deposition occur. We tested several values of τ_{cd} up to two times the τ_{ce} value to assess the condition of coexisting erosion and deposition processes and found low variance in simulated average rill depth and total network length (<10%). Significant changes occur only for $\tau_{cd} > 2\tau_{ce}$. Thus, we consider it justified to use $\tau_{ce} = \tau_{cd} = 0.81$ Pa for the Chicken Creek catchment.

[47] The last free parameter in the SOCN model is the erodibility factor H , which accounts for a decrease of soil erodibility with progressing erosion at a cell. Assuming an

increasing density with soil depth due to compaction of the dumped sand body we chose a value of $H = 4\%$. Total network length, as well as average rill depth, is sensitive to changes of H . Choosing $H = 3\%$ increases total network length by 10% and average rill depth by 22%, whereas the selection of $H = 5\%$ decreases total network length by 6% and average rill density by 17%.

[48] In summary, the SOCN model uses three free parameters only two of which are sensitive to changes (elevation reduced by erosion and erodibility factor). The range of reasonable values for these two parameters is rather narrow. Further research on the magnitude of eroded material per rainfall event and of the change of critical shear stress with soil depth is needed to determine the two sensitive free parameters.

5.4.2. Parameters in the Manning-Based Model

[49] All free parameters of the MB model were derived from data or from empirical studies and therefore the possibility for varying parameters is limited. However, the particle diameter d should be further discussed as it is considered the most uncertain parameter and directly affects the erosion rate. We used a single grain size (0.4 mm diameter) representative for the texture class “middle sized sand,” which is the most abundant class occurring in the texture samples of the catchment. Decreasing d to the lower-particle-size boundary for sand (0.063 mm) has three effects: more sediment is eroded for a certain shear stress, sediment entrainment starts for lower shear stresses, and sediment is deposited at a reduced rate. Increasing d to the upper particle size boundary for sand (2 mm) has the opposite effects. The reduction of d to 0.063 mm increased the network length by 69%. The average rill depth increased slightly by 14%, mainly because of enhanced rill depths at the centerline of the catchment, where surface runoff concentrates (maximum simulated rill depth: 1.48 m). Choosing d equal to the upper particle diameter of 2 mm for sand reduced the rill network length by 74% and increased the average rill depth by 71%. The high simulated average rill depth is the result of the short, but relatively deep rill network with a maximum simulated rill depth of 0.72 m. A thorough investigation of the model behavior for polydisperse sands is left to future research.

5.5. Outlook

[50] In a next step, the two erosion rill models presented in this paper will be used in combination with a hydrological model to investigate the effects of erosion rill evolution in the Chicken Creek catchment on hydraulic response, such as surface runoff, infiltration, subsurface flow and catchment runoff. Due to the concentration of surface flow in evolving rills these variables are likely to change dramatically with increasing erosion rill density. Thus, the water balance of the catchment, the fluxes of water transported chemical compounds and the development of vegetation patterns in the initial phase of ecosystem development can only be understood by combining a rill evolution model with a hydrological model.

6. Conclusions

[51] Both of the presented models were successful in simulating the initiation and further evolution of the main

erosion rills emerging in the initial development phase of the Chicken Creek catchment. In steeper parts of the catchment and in zones of flow convergence the simulation of flow paths across the initial topography seems to be a reasonable indicator to predict the location of erosion rills. However, neither the dynamic change of topography and thus flow paths, nor the information on rill depth are represented in the simulation of flow paths. Therefore, the consideration of erosion and deposition and the successive adaptation of the surface topography due to these processes additionally had to be taken into account in the models.

[52] Regarding the questions asked in the introduction, this work demonstrated that the location of the main surface erosion rills could be predicted by both of the model approaches. The measured main rills in the catchment developed during the first year and the lengthways rill evolution was much less pronounced thereafter and a steady state was reached in 2010. Both of the models succeeded in reproducing the observed convergence of the rill network length to a steady state. The simulations further showed that erosion and deposition processes forced the surface flow paths to change gradually during rill evolution. However, these changes affected the flow paths only locally. The initial topography of the catchment governed the general direction of water flow during the entire rill development and thus was the key factor in determining the location of the rills.

Appendix A: Derivation of the Erosion Rate from Shear Stress in the Manning-Based Model

[53] The shear stress τ is a function of slope and flow depth and is related to the dimensionless Shields number τ^* [Shields, 1936] in the following way

$$\tau^* = \frac{\tau}{g(\rho_s - \rho_f)d}, \quad (A1)$$

where ρ_s is density of sediment particles (here 2650 kg m⁻³) and d is the mean particle diameter (0.4 mm in our case). The Shields number is used to compute the dimensionless bed load transport [Meyer-Peter and Müller, 1948]:

$$\Phi^* = 8(\tau^* - \tau_{ce}^*)^{3/2} \quad \text{For} \quad \tau^* > \tau_{ce}^*, \quad (A2)$$

[54] τ_{ce}^* is the dimensionless critical shear stress for erosion and was empirically derived by Parker *et al.* [1979] as 0.03. Einstein [1950] related the dimensionless bed load transport to the volumetric sediment discharge per unit channel width q_s :

$$q_s = \Phi^* d \left[\left(\frac{\rho_s}{\rho_f} - 1 \right) g d \right]^{\frac{1}{2}}. \quad (A3)$$

[55] Volumetric sediment discharge Q_s is then calculated as

$$Q_s = q_s b \quad (A4)$$

with b corresponding to the channel width (length of a grid cell). Finally, the erosion rate e is computed as

$$e = \frac{Q_s}{a} \quad (A5)$$

with a being the area of a model cell.

Notation

a	grid cell surface [m ²];
A_f	catchment area [m ²];
A	contributing area [m ²];
A_{crit}	critical contributing area [m ²];
b	channel width [m];
D_f	drainage density [m ⁻¹];
d	particle diameter [m];
d_d	deposited material [m];
d_e	erosion parameter [m];
d_f	flow depth [m];
d_s	sink distance of particles [m];
e	erosion rate [m s ⁻¹];
g	gravitational acceleration [m s ⁻²];
H	Erodibility Factor [-];
I	Iteration Step [-];
l	distance between cells [m];
L_f	total flowpath length [m];
L_i	flowpath length between lake and cell i [m];
L_p	relative flowpath length [-];
L_n	relative flow network length [-];
L_r	total rill length [km];
n	neighbor of cell in direction of steepest slope;
N	number of flow paths [-];
m	Manning coefficient [-];
P	precipitation [mm];
Q	runoff [m ³ s ⁻¹];
Q_s	volumetric sediment discharge [m ³ s ⁻¹];
q_s	volumetric sediment discharge per unit channel width [m ² s ⁻¹];
r	particle radius [m];
S_w	water surface slope [-];
S	steepest slope between cell and its neighborhood [-];
s	length of grid cell [m];
s_{oct}	length of octagonal grid cell [m];
t	time step [-];
T	day number with rainfall;
v	water flow velocity [ms ⁻¹];
v_s	sink velocity [ms ⁻¹];
Φ^*	dimensionless bed load transport [-];
η	fluid viscosity [Ns m ⁻²];
ρ_f	fluid density [kg m ⁻³];
ρ_s	soil density [kg m ⁻³];
τ	shear stress [Pa];
τ_{cd}	critical shear stress for deposition [Pa];
τ_{ce}	critical shear stress for erosion [Pa];
τ^*	Shields number [-];
τ_{ce}^*	dimensionless critical shear stress for erosion [-];
ω	Strahler order [-].

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