Shortwave radiation parameterization scheme for subgrid topography

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[1] Topography is well known to alter the shortwave radiation balance at the surface. A detailed radiation balance is therefore required in mountainous terrain. In order to maintain the computational performance of large-scale models while at the same time increasing grid resolutions, subgrid parameterizations are gaining more importance. A complete radiation parameterization scheme for subgrid topography accounting for shading, limited sky view, and terrain reflections is presented. Each radiative flux is parameterized individually as a function of sky view factor, slope and sun elevation angle, and albedo. We validated the parameterization with domain-averaged values computed from a distributed radiation model which includes a detailed shortwave radiation balance. Furthermore, we quantify the individual topographic impacts on the shortwave radiation balance. Rather than using a limited set of real topographies we used a large ensemble of simulated topographies with a wide range of typical terrain characteristics to study all topographic influences on the radiation balance. To this end slopes and partial derivatives of seven real topographies from Switzerland and the United States were analyzed and Gaussian statistics were found to best approximate real topographies. Parameterized direct beam radiation presented previously compared well with modeled values over the entire range of slope angles. The approximation of multiple, anisotropic terrain reflections with single, isotropic terrain reflections was confirmed as long as domain-averaged values are considered. The validation of all parameterized radiative fluxes showed that it is indeed not necessary to compute subgrid fluxes in order to account for all topographic influences in large grid sizes.

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1. Introduction

[2] Incident radiation plays a major role in the surface energy balance computed from atmospheric and land surface models. It is well-known that topography alters incident shortwave (SW) radiation. Therefore, in mountainous regions a detailed radiation balance is required. Topographic influences are shading, limited sky view (expressed by the so-called sky view factor) and multiple, anisotropic terrain reflections.

[3] On small scales, radiative transfer models using various approaches (e.g., Spherical Harmonics Discrete Ordinate Method (SHDOM), Monte Carlo method (MC), Radiosity approach) and various degrees of sophistication are nowadays able to mimic all of these topographic influences [e.g., Evans, 1998; Chen *et al.*, 2006; Helbig *et al.*, 2009; Mayer, 2009]. For large-scale applications, such as meteorological

or hydrological runoff models, most of the so-called subgrid (or subscale) topographic influences are usually simplified or even neglected. However, with increased grid resolution topographic influences become increasingly important. Müller and Scherer [2005] and Senkova *et al.* [2007] therefore introduced topographic effects on surface radiation in the Nonhydrostatic Mesoscale Model (NMM) and in the High Resolution Limited Area Model (HIRLAM), respectively. Lai *et al.* [2010] studied topographic influences on SW radiation with various grid sizes for the topography of Taiwan. All these approaches require detailed preprocessing of subgrid topography to compute average radiation fluxes for the larger grid sizes. For resolved topography, Manners *et al.* [2012] introduced shading, limited sky view and isotropic longwave terrain emission in the Met Office Unified Model (MetUM). In general, the challenge of meteorological models is to maintain computational performance while at the same time increasing the modeling accuracy. This is why subgrid topographic treatment is so far often neglected.

[4] Parameterizations of subgrid solar radiative fluxes over mountainous regions enable a computationally efficient solution for large-scale models. Subgrid radiation fluxes do

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Table 1. Characteristics of Tested Digital Elevation Models From the United States and From the Alps in Switzerland^a

Label	Geographic Location	L (km)	Δx (m)	ζ (deg)	σ_{ζ} (deg)	F_{sky}	ξ (m)	σ (m)	L/ξ
Lone Mountain	Montana, USA	3×3	30	26	21	0.97	688	170	4.4
Blizzard Mountain	Idaho, USA	3×3	30	24	19	0.97	863	169	3.5
Grand Canyon	Arizona, USA	3×3	30	34	34	0.88	847	255	3.5
Parsenn	Grisons, Switzerland	2.5×2.5	25	25	21	0.94	561	145	4.5
Wannengrat	Grisons, Switzerland	5.2×2.4	25	27	22	0.93	856	208	4.4
Tschuggen	Grisons, Switzerland	4.5×4.5	25	25	20	0.93	969	230	4.6
Gothard	Uri, Switzerland	7.775×5.225	25	34	30	0.89	1666	484	3.9

^aFor U.S. models, see <http://ned.usgs.gov>. For models from the Alps, see <http://www.swisstopo.ch>. Given slope angles ζ , standard deviation σ_{ζ} and sky view factors F_{sky} are domain-averaged values; ζ is computed from first partial derivatives (equation (1)). Gaussian covariance parameters ξ and σ were obtained from fitting Gaussian covariances to the computed covariances of the real topographies. Two different grid sizes Δx occur.

not need to be explicitly computed and a vast preprocessing of subgrid topography characteristics is redundant. First parameterizations of solar radiative fluxes under cloudless sky were suggested by *Dubayah et al.* [1990], *Essery and Marks* [2007], and *Lee et al.* [2011]. *Dubayah et al.* [1990] presented a parameterization for domain-averaged and variance of direct solar beam radiation derived under the assumption of constant slope angles but with uniformly distributed azimuth angles. *Essery and Marks* [2007] scaled slope characteristics of real topographies by means of a Laplace distribution. They developed parameterizations for direct solar beam and diffuse sky radiation using shaded fractions and sky view factors which only depend on solar elevation and the standard deviation of the slope components along the solar azimuth. Results from these parameterizations compared well with statistics from their spatial simulations. However, given the extreme variability of real topographies it is questionable that parameterizations verified and partly based on radiation simulations of a limited ensemble of real topographies are widely applicable. Likewise, *Lee et al.* [2011] parameterized all solar fluxes by means of multiple linear regression equations based on MC simulations for various regions of the Sierra Nevada in the United States. While the parameterization works well for their test area it is again unclear how it performs in other mountainous regions.

[5] The radiation balance in mountainous terrain can be envisaged as radiative transfer in an apparently random environment. In order to thoroughly study the influence of complex topography on the radiation balance a sufficiently large number of topographies covering a wide range of terrain characteristics (i.e., characteristic length and height scales of valleys/mountains) is required. Using a set of real topographies is difficult given the wide range of geomorphologies. We therefore base our study on a comprehensive ensemble of simulated topographies which are easier to characterize but still cover a wide range of characteristic scales. The scaling properties of topography have been addressed in many studies. The complexity emerges from the vast topographic variety resulting from erosional processes that might alter the original topography in opposing ways [e.g., *Klinkenberg and Goodchild*, 1992]. In previous studies real topographies were described using power law characteristics (i.e., fractal characteristics) [e.g., *Huang and Turcotte*, 1989; *Dietler and Zhang*, 1992; *Weissel et al.*, 1994; *Zhang et al.*, 1999; *Deems et al.*, 2006; *Abedini and Shaghaghian*, 2009] or exponential distributions [e.g., *Essery*, 2001, 2004; *Essery and Marks*, 2007]. However, in other studies real topographies could not be fitted with a

single distribution. For instance, *Klinkenberg and Goodchild* [1992] found power laws to provide a good fit for some but not all of their 55 topographies across the United States. *Vico and Porporato* [2009] found various distributions ranging from Gaussian to Laplace and Pearson type VII to describe the slopes of four different topographies across the United States whereas *Montgomery* [2001] allocated normal or exponential slope distributions to drainage areas in the United States. Here, we found Gaussian statistics to best approximate real topography slopes and chose Gaussian random fields (GRF) as stochastic topography model to solely investigate the topographic influence on radiative fluxes.

[6] A parameterization radiation scheme is proposed allowing us to study all influences of topography individually by applying the parameterization of the subgrid direct beam radiative fluxes of *Dubayah et al.* [1990] and the parameterization of terrain-reflected radiation based on the isotropic view factor approach. This scheme provides solar radiative fluxes for flat, horizontal grid sizes in large-scale models. Thus, even without actually resolving for the subgrid topography in large flat grid sizes, the parameterized fluxes account for all topographic influences of the underlying topography.

[7] In section 2 we will present a brief study containing a comparison of various theoretical distributions with the slope characteristics of diverse topographies from Switzerland and the United States. We then give a description of our ensemble of simulated topographies with a domain size comparable to the grid size of large-scale models and specify our method to derive and apply all radiative fluxes, e.g., for grid sizes in large-scale models that contain subgrid topography. In section 3 we will study the performance of the parameterizations in comparison to modeled domain-averaged radiative fluxes for the ensemble of simulated topographies. We will also present quantifications of the individual topographic impacts on the SW radiation balance. Furthermore, a previously presented parameterization of the sky view factor is validated by our modelings. Discussions and conclusions are given in section 4 with special emphasis on open questions.

2. Method

2.1. Characteristics of Real Topographies

[8] To describe the characteristics of real topographies we considered different candidate statistical models. Seven topographies from Switzerland and the United States covering a wide range of different geomorphologies, mean slope

Table 2. Root-Mean-Square Errors (RMSE) of Probability Density Function (pdf) Values of Mean Slope Angles ζ ; Slope Components $\partial_x z$; $\partial_y z$; and Semivariograms to Fitted Gaussian (G), Laplace (L), Exponential (E), or Power Law (P) Models^a

Topographies	RMSE								
	pdf(ζ)		pdf($\partial_x z$)		pdf($\partial_y z$)		Semivariogram		
	G	L	G	L	G	L	G	E	P
Lone Mountain	0.0035	0.0083	0.0990	0.0655	0.0951	0.2087	2276	4391	5866
Blizzard Mountain	0.0035	0.0065	0.0987	0.2538	0.2134	0.3716	614	1540	834
Grand Canyon	0.0025	0.0051	0.0716	0.0402	0.0704	0.0290	3739	2095	2093
Parsenn	0.0045	0.0090	0.0919	0.0642	0.1005	0.0412	611	749	864
Wannengrat	0.0032	0.0066	0.0507	0.1137	0.0723	0.0991	1368	2656	4176
Tschuggen	0.0037	0.0089	0.1369	0.2119	0.1625	0.1894	987	5099	7955
Gothard	0.0015	0.0053	0.0600	0.1207	0.0642	0.1128	-	-	-

^aThe values in bold indicate the respective lower RMSE.

angles ζ , sky view factors F_{sky} and domain sizes L were used (see Table 1 for more details). We fitted power law, exponential, Laplace and Gaussian models to three topographic descriptors, namely the probability density function (pdf) of slope angles ζ , the pdf of first partial derivatives (slope components) $\partial_x z$ and $\partial_y z$ in orthogonal directions and omnidirectional semivariograms. Slopes were computed from first partial derivatives via

$$\tan \zeta = \left[(\partial_x z)^2 + (\partial_y z)^2 \right]^{1/2}. \quad (1)$$

We computed root-mean-square errors (RMSE) for all descriptors (see Table 2), the two-sample Kolmogorov-Smirnov test (K-S test) statistic values D for the pdf's (nonparametric method) and the squared correlation coefficients R^2 for the semivariograms (parametric method) (see Table 3). We used ordinary least squares (OLS) fitting for the semivariograms. In Table 2, values in bold indicate the lowest RMSE. The critical K-S test statistic value $D_{n,\alpha}$ for our tests was $D_{50,0.05} = 0.272$ with equal sample sizes of 50 and a significance level of 0.05. The null hypothesis of the fit with a distribution is thus accepted when $D < D_{50,0.05}$. In Table 3, D values in bold indicate a p value for the K-S test of at least 0.95. In the majority of cases, Gaussian forms fitted the topographic data best. The lowest RMSEs, the accepted null hypothesis with statistically significant D values ($D < D_{50,0.05}$ and p value ≥ 0.95) and the highest R^2 values were predominantly found for fits with Gaussian distributions (Tables 2 and 3). The simplest choice of

random fields which is consistent with these findings are GRF with Gaussian covariance which were accordingly chosen as simulated topographies.

2.2. Random Topographies

[9] Simulated topographies were generated from discretized isotropic GRF. While we showed that real topographies can be reasonably well approximated by Gaussian statistics, GRF also have the advantage that topography is reduced to only two relevant length scales. In addition, the first- and second-order moments, i.e., mean and covariance, entirely describe the statistical properties of GRF as simulated topographies (for an introduction to random fields, see, e.g., *Adler* [1981]). We generated a large ensemble of several thousand simulated topographies with fixed domain size $L = 3$ km, fixed grid size $\Delta x = 30$ m and a wide range of characteristic length scales (Table 4). Two characteristic length scales are defined [*Helbig et al.*, 2009]: a valley-to-peak elevation difference σ (height of mountains), which is the square root of the variance, and a lateral extension ξ (width of mountains), which is the correlation or scale length of topographic features. Thus, our ensemble of discrete topographies is obtained from vectors $z_I = z(x_I)$, $I = 1, 2, \dots, N$ drawn from a multivariate Gaussian distribution with covariance matrix

$$C_{IJ} = \overline{(z(x_I) - \bar{z})(z(x_J) - \bar{z})} = \sigma^2 e^{-|x_I - x_J|^2 / \xi^2}$$

and homogeneous mean $\bar{z} = 2000$ m independent of $I = 1, 2, \dots, N$. We denote ensemble averaging by an overbar

Table 3. Kolmogorov-Smirnov Test Statistics (D) of Probability Density Function Values (pdf) of Mean Slope Angles ζ and Slope Components $\partial_x z$, $\partial_y z$ and Squared Correlation Coefficients (R^2) of Semivariograms to Fitted Gaussian (G), Laplace (L), Exponential (E), or Power Law (P) Models^a

Topographies	D						R^2		
	pdf(ζ)		pdf($\partial_x z$)		pdf($\partial_y z$)		Semivariogram		
	G	L	G	L	G	L	G	E	P
Lone Mountain	0.082	0.143	0.184	0.163	0.102	0.143	0.974	0.904	0.829
Blizzard Mountain	0.102	0.122	0.102	0.204	0.102	0.204	0.998	0.986	0.996
Grand Canyon	0.102	0.184	0.510	0.306	0.449	0.245	0.981	0.994	0.994
Parsenn	0.082	0.143	0.225	0.122	0.367	0.163	0.994	0.992	0.989
Wannengrat	0.082	0.122	0.367	0.143	0.245	0.102	0.995	0.981	0.952
Tschuggen	0.143	0.143	0.225	0.163	0.225	0.143	0.998	0.949	0.876
Gothard	0.061	0.122	0.306	0.122	0.347	0.082	-	-	-

^aThe critical K-S test statistic value $D_{n,\alpha}$ is $D_{50,0.05} = 0.272$ with sample size of 50 and a significance level of 0.05. The computed D values in bold indicate a p value of at least 0.95. The R^2 values in bold indicate a $R^2 \geq 0.99$.

Table 4. Combinations of σ and ξ for Each of the Five Mean Slope Angles ζ , the Mean Standard Deviations σ_ζ , and the L/ξ Ratio^a

ξ (m)	ζ^b					L/ξ
	9°	17°	31°	42°	51°	
1000	88	176	364	577	840	3
900	79	159	328	520	755	3.3
800	71	141	291	462	671	3.8
700	62	123	255	404	587	4.3
600	53	106	218	346	504	5
500	44	88	182	289	420	6
400	35	71	146	231	336	7.5
300	27	53	109	173	252	10
200	18	35	73	116	168	15
100	9	18	36	58	84	30

^aFor ζ , see equation (1). For σ_ζ , see section 2.4. The corresponding σ values are in meters.

^bFor σ_ζ , mean values are 0.11 for 9°; 0.22 for 17°; 0.46 for 31°; 0.72 for 42°; and 1.05 for 51°.

and all length scales are measured in meters. From the definition of the covariance given above it follows that the squared mean slope $\tan(\zeta)^2$ of a simulated topography equals $(2\sigma/\xi)^2$ [Adler, 1981]. Since the mean slope angle depends only on the ratio of σ and ξ it is possible to vary both scales while leaving the mean slope constant. This is interesting insofar as a fixed domain size L is used and thus the influence of varying L/ξ can be estimated for fixed slope. For each mean slope we were therefore able to create a variety of topographies. Selected values of σ, ξ combinations in our ensemble of simulated topographies cover a range of realistic mean slopes (see Table 4). We aimed at satisfying the condition $\Delta x \ll \xi \ll L$, which ensures reliable estimates of terrain-reflected radiative fluxes with minimal influences of grid and domain size [Helbig et al., 2009]. Note that we deliberately selected low values of domain sizes L similar to the smallest grid sizes of meteorological grids. To estimate the necessary number of realizations to obtain ‘stable’ results we analyzed the absolute error of the covariance parameter estimation with varying numbers of realizations. We found that with 200 realizations the maximum error of the steepest slope angle (51°) for all σ, ξ combinations was only 7% (not shown). For each combination σ and ξ we therefore generated 200 random topographies resulting in an ensemble of 10000 topographies. We note that all radiation results given below for one combination σ, ξ actually represent an ensemble average over 200 realizations.

2.3. Model Setup

[10] To model distributed radiation over our ensemble of topographies we used a detailed SW radiation balance model [Helbig et al., 2009, 2010] which is implemented in a modular alpine surface process model Alpine3D (cf. Lehning et al. [2004, 2006] for an overview of Alpine3D). We note that the radiation balance model applies the radiosity approach, well known in computer graphics, to compute multiple, anisotropic terrain reflections in complex terrain [Helbig et al., 2009]. In order to study the influence of topography, sun elevation angles and surface albedos, we used constant solar radiative fluxes consisting of direct beam radiation $S_{b,s}$ and diffuse sky radiation $S_{d,s}$ on an

unobstructed, horizontal surface. This does not imply a loss of generality since the radiosity equation [Helbig et al., 2009] is linear in the source terms S_b, S_d . Variations in incoming radiation are therefore implicitly included. Distributed direct beam radiation is obtained by accounting for sun incidence angles θ and shading: $S_{b,I} = S_{b,s} \cos\theta_I \chi_{\text{sun},I}$ with $\chi_{\text{sun},I}$ as the indicator function for sun or shadow for each grid size. We used a constant value for $S_{b,s}$ of 1000 W m⁻². Distributed diffuse sky radiation is obtained by accounting for limited sky view: $S_{d,I} = S_{d,s} F_{I,\text{sky}}$ with $F_{I,\text{sky}}$ as the sky view factor for patch A_I which is computed from the anisotropic terrain view factor sum (for computational details cf. Helbig et al. [2009]). We chose a constant value for $S_{d,s}$ of 150 W m⁻². Finally, we chose four sun elevation angles θ_e of 8°, 30°, 60° and 90°, a constant sun azimuth angle and homogeneous surface albedo values $\alpha = 0.1, 0.2, \dots 0.9$.

2.4. Parameterized Radiative Fluxes

[11] Global outgoing radiation S_g^{out} is composed of parameterized direct beam S_b , diffuse sky S_d and terrain-reflected S_t radiation accounting for subgrid topographic influences. To derive an equation for S_g^{out} we use the concept of the radiosity method, on which the present SW radiation balance model is based [Helbig et al., 2009]. Note that the exact expression of S_g^{out} derived below cannot be related to the effective albedo given by Helbig et al. [2009] which had a different scope. The radiosity approach applied to three-dimensional radiative transfer in complex topography describes radiation redistribution between Lambertian emitter/reflector surface patches [see, e.g., Goral et al., 1984]. The sky is considered a surface of zero reflectivity. As for each of the $I = 1, 2, \dots N$ inclined surface patches A_I of the decomposed topography surface, we have to solve the radiosity equation for each of the $J = 1, 2, \dots N$ patches A_J of the hypothetical, flat sky surface expanded over the subgrid topography. The sky can be considered as an opening; that is, incident S_b and S_d are transmitted. The outgoing global radiation of the flat sky surface therefore equals the outgoing global radiation S_g^{out} from a large flat grid size. Applying the characteristics of the sky in the radiosity equation [cf. Helbig et al., 2009, equation 9], domain-averaged outgoing global radiation $S_{g,m}^{\text{out}}$ of the sky (or the large grid size) is obtained from

$$S_{g,m}^{\text{out}} = \frac{1}{N} \sum_{J=1}^N \sum_{I=1}^N \alpha_I S_{g,m,I} F_{JI} \chi_{JI} \quad (2)$$

with the so-called view factors F_{JI} , expressing the fraction of the reflected global radiation ($\alpha_I S_{g,m,I}$) of A_I that is received by A_J , and the mutual visibility χ_{JI} as the indicator function for mutual visibility between two surface patches A_J, A_I . When we apply the reciprocity relation $F_{JI} = F_{IJ} A_I / A_J$ for view factors (for details of view factors see, e.g., Goral et al. [1984]), use $\chi_{JI} = \chi_{IJ}$, $A_J = \Delta x^2$ and $A_I = \Delta x^2 / \cos \zeta_I$ we arrive at

$$S_{g,m}^{\text{out}} = \frac{1}{N} \sum_{I=1}^N \frac{\alpha_I S_{g,m,I}}{\cos \zeta_I} \sum_{J=1}^N F_{IJ} \chi_{IJ} \quad (3)$$

By neglecting boundary effects we have $\sum_{J=1}^N F_{IJ} \chi_{IJ} = F_{I,\text{sky}}$. Domain-averaged outgoing global radiation from a large

Figure 1. Standard deviations of slope components σ_{ζ_x} , σ_{ζ_y} for various real topographies (Table 1). Here σ_{ζ_x} and σ_{ζ_y} are the averages over 5° azimuth angle intervals. The solid line is a circle with a radius of 0.8.

flat grid size (equation (3)) is then approximated by its individual domain averages leading to a parameterization for the outgoing global radiation S_g^{out} from a large flat grid size

$$S_g^{\text{out}} = \alpha(S_b + S_d + S_t) \frac{F_{\text{sky}}}{\cos \zeta} \quad (4)$$

with domain-averaged direct beam S_b , diffuse sky S_d and terrain-reflected radiation S_t , and domain-averaged sky view factors F_{sky} . Note that here and in the following, we omit the overbars when indicating domain-averaged variables and note also that radiative fluxes without subscript indicate parameterized quantities while we use the subscript ‘m’ for domain-averaged subgrid quantities simulated with the radiation balance model.

[12] Here, we aim to design a SW radiation parameterization scheme for a large grid size based on easily derivable terrain parameters, i.e., a scheme that does not require extensive preprocessing of subgrid topography. We apply the parameterization for domain-averaged direct beam radiation proposed by *Dubayah et al.* [1990]

$$S_b = S_{b,s} \cos \zeta \sin \theta_e. \quad (5)$$

Equation (5) was derived assuming that all subgrid topographic azimuth angles are uniformly distributed within a large grid size. As pointed out by *Essery and Marks* [2007], topography is statistically isotropic when slope components in orthogonal directions are uncorrelated. We found that for most of our real topography examples (see Table 1) the averaged standard deviations of slope components in orthogonal directions are roughly isotropic (Figure 1). Note that we used

$\sigma_{(\phi)}^2 = \sin^2 \phi \overline{\partial_x z^2} + \cos^2 \phi \overline{\partial_y z^2} - 2 \sin \phi \cos \phi \overline{\partial_x z \partial_y z}$ as standard deviation of mean slopes along azimuth ϕ [*Essery and Marks*, 2007]. From this the standard deviations in orthogonal directions result in $\sigma_{\zeta_x}(\phi = \pi/2) = \partial_x z$ and $\sigma_{\zeta_y}(\phi = 0) = \partial_y z$. We averaged standard deviations of slope components σ_{ζ_x} , σ_{ζ_y} over 5° azimuth angle intervals (Figure 1). We did not implement a different parameterization for direct beam radiation presented by *Essery and Marks* [2007] since it is based on average and standard deviation of horizon angles which are generally not available. Additionally, compared to *Dubayah et al.* [1990], the parameterization of *Essery and Marks* [2007] did not fit better to simulations of solar radiation over their four test topographies.

[13] Domain-averaged diffuse sky radiation is parameterized by

$$S_d = S_{d,s} F_{\text{sky}}. \quad (6)$$

[14] While the isotropic view factor parameterization only accounts for single, isotropic terrain reflections *Helbig et al.* [2009] showed for one surface albedo that similar domain-averaged values were obtained than with multiple, anisotropic terrain reflections. However, they found that maximum local differences reached remarkable values between 106 and 242 W m^{-2} . This demonstrates that multiple, anisotropic terrain reflections have indeed a considerable influence on local surfaces and might therefore lead to considerable surface temperature differences. However, since we here focus on domain-averaged values, as a first step, most of the actual terrain reflections are captured by simply considering single, isotropic terrain reflections using

$$S_t = \alpha(S_b + S_d)(1 - F_{\text{sky}}). \quad (7)$$

[15] To summarize, our parameterization scheme to compute solar radiation over mountainous terrain is based on the subgrid parameterization for domain-averaged direct beam radiation presented by *Dubayah et al.* [1990] (equation (5)), for diffuse sky radiation by the domain-averaged sky view factor F_{sky} (equation (6)) and for terrain-reflected radiation by assuming single, isotropic reflections (equation (7)). This way, the entire radiation parameterization scheme is solely based on sun elevation angle, domain-averaged slope angle, surface albedo and sky view factor. Aside from the sky view factor these parameters are in general easily obtained. The subgrid sky view factors can be computed using different approaches with varying accuracy and complexity [*Helbig et al.*, 2009].

3. Results

[16] We evaluated the parameterization of direct beam radiation S_b (equation (5)) with modeled domain-averaged direct beam radiation $S_{b,m}$ for our ensemble of simulated topographies. The results show that the parameterization agreed well with modeled data except for steeper topographies (mean slope angles $\zeta > 31^\circ$) in combination with a low sun elevation angle ($\theta_e = 8^\circ$) (Figure 2). The largest normalized difference between $S_{b,m}$ and S_b of 36% was for domain-averaged slope angles of 51° and $\theta_e = 8^\circ$. In Figure 2, $S_{b,m}$ represents averages over ten combinations of σ and ξ for each mean slope angle (see Table 4). If not

Figure 2. Differences between modeled $S_{b,m}$ and parameterized S_b (equation (5)) direct beam radiation as a function of the four sun elevation angles θ_e . Different symbols indicate mean slope angles. Direct beam radiation is normalized by $S_{b,m}$.

stated otherwise, in the following our results are presented as averages over the ten combinations of σ and ξ for each slope angle. Since we used a constant diffuse sky radiation value at the surface ($S_{d,s}$), parameterized (equation (6)) and modeled diffuse sky radiation were identical (not shown).

[17] We compared modeled terrain-reflected radiation $S_{t,m}$ to parameterized S_t (equation (7)) as a function of homogeneous surface albedos α for different slope and sun elevation angles (Figure 3). The results confirm that single, isotropic terrain reflections are a reasonable approximation for multiple, anisotropic terrain-reflected radiation when domain-averaged values are compared. We use $S_{b,m} + S_{d,m}$ in equation (7) instead of the parameterized radiation from the sky to solely estimate the error arising from the approximation of terrain reflections with isotropic terrain reflections. Parameterized and modeled terrain-reflected radiation agreed very well for domain-averaged slope angles of 9° to 31° . For the extremely steep slope angles of 42° and 51° the agreement was poorer, especially for larger surface albedos.

Figure 3. Differences between modeled $S_{t,m}$ and parameterized S_t (equation (7)) terrain-reflected radiation as a function of surface albedos α . Different colors indicate sun elevation angles, and different symbols mean slope angles. Terrain-reflected radiation is normalized by $S_{b,m} + S_{d,m}$.

Figure 4. Differences between modeled $S_{g,m}$ and parameterized S_g global radiation as a function of surface albedos α . The domain-averaged sky view factor $F_{sky,m}$ is used in S_g . Different colors indicate sun elevation angles, and different symbols mean slope angles. Global radiation is normalized by $S_{g,m}$.

The largest normalized difference of 12% was for mean slopes of 51° and $\alpha = 0.9$.

[18] When computing the global radiation S_g , overall, the parameterization agreed well with modeled domain-averaged global radiation values $S_{g,m}$ for different mean surface albedos α (Figure 4). We found the largest normalized differences of 20% for mean slopes of 51° , $\alpha = 0.1$ and $\theta_e = 8^\circ$. S_g was generally larger than $S_{g,m}$, except for $\theta_e = 8^\circ$.

[19] Overall, parameterized outgoing global radiation S_g^{out} for a large grid size (equation (4)) is overestimated when computing the outgoing global radiation as $S_{g,\perp}^{out} = \alpha(S_{b,s}\sin\theta_e + S_{d,s})$ which does not parameterize subgrid topography (Figure 5). The largest difference between both outgoing global radiative fluxes was for mean slopes of 51° , $\alpha = 0.1$ and $\theta_e = 90^\circ$. Differences were generally lower for larger mean surface albedos and lower sun elevation angles.

Figure 5. Parameterized S_g^{out} (equation (4)) global radiation is shown as a function of surface albedos α . The domain-averaged sky view factor $F_{sky,m}$ is used in S_g^{out} . Different colors indicate sun elevation angles, and different symbols mean slope angles. Parameterized global radiation is normalized by $S_{g,\perp}^{out} = \alpha(S_{b,s}\sin\theta_e + S_{d,s})$.

Figure 6. Modeled sky view factors $F_{\text{sky},m}$ are shown as a function of standard deviations of mean slope angles σ_{ζ} for all simulated topographies. The dashed line denotes the power law fit suggested by *Essery and Marks* [2007], and the solid line is a power law fit with $1 - 0.23\sigma_{\zeta}^{1.11}$. Additionally, sky view factors of our seven real topographies, as given in Table 1, are depicted as a function of σ_{ζ} .

[20] For our parameterized radiation scheme (see equations (4)–(7)) the sky view factor is the only quantity which is computed from subgrid topography. We therefore investigated the sky view factor parameterization proposed by *Essery and Marks* [2007]. *Essery and Marks* [2007] scaled domain-averaged sky view factors for their group of real topographies with standard deviation of slopes and found the following power law: $F_{\text{sky}} = 1 - 0.57\sigma_{\zeta}^{5/3}$ to provide a good fit. Recall that *Essery and Marks* [2007] used $\sigma_{(\phi)}^2 = \sin^2\phi\overline{\partial_x z^2} + \cos^2\phi\overline{\partial_y z^2} - 2\sin\phi\cos\phi\overline{\partial_x z\partial_y z}$ as standard deviation of mean slopes along azimuth ϕ . Our modeled domain-averaged sky view factors $F_{\text{sky},m}$ agreed well with the power law parameterization for mean slope angles of 9° and 17° but did not agree well for slopes of 31° , 42° , and 51° (Figure 6 and Table 4). An empirical power law fit given by $F_{\text{sky}} = 1 - 0.23\sigma_{\zeta}^{1.11}$ showed good agreement even for steep slopes. Note that in Figure 6, $F_{\text{sky},m}$ is not ensemble averaged but rather depicted for each of the 10000 topographies to demonstrate the variability. Computed domain-averaged sky view factors for our seven real topographies (see Table 1) compared much better to our empirical power law fit than that of *Essery and Marks* [2007] (Figure 6).

(a)

(b)

(c)

(d)

Figure 7. Modeled global $S_{g,m}$, direct beam $S_{b,m}$, diffuse sky $S_{d,m}$, and terrain-reflected $S_{t,m}$ radiation as a function of surface albedos α . (a) $S_{g,m}$ is shown. (b) $S_{b,m}$, (c) $S_{d,m}$, and (d) $S_{t,m}$ are normalized by $S_{g,m}$. Different colors indicate sun elevation angles, and different symbols mean slope angles (Figure 7d).

[21] Shading and limited sky view are generally assumed to be the dominant topographic influences on decreased outgoing radiation. In order to quantify each of the fractions we compared modeled direct beam $S_{b,m}$, diffuse sky $S_{d,m}$ and terrain-reflected radiation $S_{t,m}$ to modeled global radiation $S_{g,m}$ for different surface albedos α (Figure 7). Overall, $S_{g,m}$ strongly increased with sun elevation angles θ_e , decreased with increasing mean slope angles and showed little dependence on surface albedos (Figure 7a). For mean slope angles of 9° , the contribution to $S_{g,m}$, stemming from direct beam radiation, increased by about 40% when the sun elevation angle increased from 8° to 90° (Figure 7b). On the other hand, the contribution of diffuse sky radiation decreased with increasing sun elevation angle by about the same fraction (Figure 7c). In contrast, terrain-reflected radiation strongly increased with increasing mean slope angles and surface albedos while it showed little dependence on sun elevation angles. We found negligible $S_{t,m}$ fractions (below 1%) for mean slope angles of 9° , while we observed largest fractions of 9% for slope angles of 51° and $\alpha = 0.9$ (Figure 7d).

4. Discussion and Conclusion

[22] We developed a subgrid parameterization scheme for shortwave (SW) radiative fluxes and topography accounting for shading, limited sky view and terrain reflections. The scheme has been validated by a detailed SW radiation balance model applying the radiosity approach to three-dimensional radiative transfer in complex terrain [Helbig *et al.*, 2009, 2010]. The parameterization scheme provides a computationally efficient solution to account for subgrid topographic influences in meteorological or hydrological and climate models on large flat, horizontal grid sizes that do not resolve the underlying topography.

[23] To obtain the global radiation which is effectively reflected by a large flat grid size containing but not resolving for subgrid topography, each of the radiative fluxes, namely direct beam, diffuse sky and terrain-reflected radiation is parameterized individually. The direct beam radiation flux is parameterized as proposed by Dubayah *et al.* [1990] (see equation (5)). Diffuse sky radiation is parameterized by the domain-averaged sky view factor (see equation (6)). Terrain-reflected radiation is parameterized based on the isotropic view factor approach, i.e., single, isotropic terrain reflections (see equation (7)). For the entire parameterization scheme only sun elevation angle, domain-averaged slope angle, surface albedo and sky view factor are required.

[24] To study all influences of topography we used a large ensemble of simulated topographies covering a wide range of terrain characteristics. According to a review presented by Gao and Xia [1996], topographies are traditionally scale-dependent where their fractal nature depends on the scale. They summarize that even though power laws can be applied to certain types of topographies at certain scales, other models should be considered in conjunction with fractal analysis. Our choice of topographies was motivated by the analysis of seven diverse topographies from Switzerland and the United States which suggest that Gaussian random fields (GRF) are well suited to mimic real topography slopes (Tables 1–3). The fact that domain-averaged sky view factors of the seven topographies were within the scatterplot of our simulated topographies further supported

the use of GRF (Figure 6). We compiled a large ensemble of several thousand topographies with five mean slope angles, fixed domain size of 3 km, fixed grid size of 30 m and a wide range of the two characteristic length scales, namely a valley-to-peak elevation difference (height of mountains) and a lateral extension (width of mountains) (Table 4). Given the vast variety of geomorphologies [e.g., Klinkenberg and Goodchild, 1992] and the differing results from previous studies that scaled topographic properties, we believe that our ensemble of simulated topographies covers most of the real topographies at the scales relevant for large-scale models. Even though real topographies will seldom have slope angles of 42° or 51° for domain sizes of several kilometers (see mean slope angles for real topographies in Table 1) we also included such steep topographies in our study to investigate the limitations of our parameterization scheme.

[25] Domain-averaged radiation was computed using a detailed SW radiation balance model. Overall, parameterized radiative fluxes compared well with modeled radiation over our ensemble of topographies. The direct beam radiation parameterization by Dubayah *et al.* [1990] (equation (5)) was robust over the entire range of slopes investigated (Figure 2), i.e., even for slopes steeper than those for which it was developed. It was derived for large grid sizes with uniformly distributed subgrid topographic azimuth angles. For our real topographies (Table 1) we found that the standard deviations of slope components in orthogonal directions are indeed roughly isotropic (Figure 1). While the parameterization might be incorrect for highly anisotropic topographies, a careful choice of the location and extension of large grid sizes could provide a temporary solution. Overall, for mean slope angles of 31° or lower the parameterization only led to minor errors for all sun elevation angles. Even for larger, less realistic mean slope angles of 42° and 51° with larger sun elevation angles the parameterization worked well. Parameterized (equation (6)) and modeled diffuse sky radiation were identical since we used a constant diffuse sky radiation value at the surface $S_{d,s}$. Finally, the approximation of single, isotropic terrain reflections for multiple, anisotropic terrain reflections was justified as long as domain-averaged radiation is compared (Figure 3). This confirmed results presented by Helbig *et al.* [2009], albeit for a wider range of mean slopes and surface albedos. For our steepest topographies single, isotropic terrain reflections were less adequate. However, since domain-averaged slope angles of 42° or 51° rarely occur for domain sizes of several kilometers, we do not believe that this limitation prevents a wide application of the isotropic view factor approach. The underestimation of direct beam radiation for low sun elevation angles consequently led to underestimated parameterized global radiation for sun elevations of 8° (Figure 4). However, overall the parameterization scheme was accurate for the more common mean slope angle range of 9° to 31° .

[26] Even though the parameterization of Dubayah *et al.* [1990] does not explicitly account for horizon and self shading, it parameterized direct beam radiation for large grid sizes very well. This becomes clear when comparing the parameterization (equation (5)) to the commonly used cosine of the sun incidence angle $\cos \theta = \cos \zeta \sin \theta_e + \sin \zeta \cos \theta_e \cos(\phi_{\text{sun}} - \phi)$. Note that since we used isotropic GRFs as simulated topographies, it was logical to

use a constant sun azimuth angle ϕ_{sun} for our validations. For low to intermediate domain-averaged slope angles the mean cosine of the sun incidence angle ($\cos \theta$) roughly equals the parameterization ($\sin \zeta \approx 0$) which explains the good agreement between parameterized and modeled direct beam radiation for these slope angles and all sun elevation angles. For large sun elevation angles $\cos \theta$ again roughly equals the parameterization ($\cos \theta_e \approx 0$) which is why the parameterization also worked well for larger mean slope angles with larger sun elevation angles. We found that the parameterization of *Dubayah et al.* [1990] performed less well for our steepest topographies with mean slope angles of 42° and 51° combined with the lowest sun elevation angle of 8° . In those cases the parameterization provided only small direct beam radiation values ($\cos \zeta \approx 0$ and $\sin \theta_e \approx 0$) whereas the second term of $\cos \theta$ increased ($\sin \zeta \approx 1$ and $\cos \theta_e \approx 1$) leading to a larger modeled direct beam radiation. As a result, for the steepest topographies at very low sun elevation angles parameterized direct beam radiation was underestimated. In all, not accounting for horizon shading in the parameterization only led to minor differences compared to modeled direct beam radiation.

[27] A parameterization of the influences of subgrid topography on the radiation balance is essential. Compared to the parameterized outgoing global radiation, which accounts for subgrid topography, overestimations of up to 20% were found for the lowest mean surface albedo when the impacts of subgrid topography are not accounted for (Figure 5). Less trapping of radiation by topography was observed with increasing mean surface albedos. Of note, by deriving the radiosity equation for the hypothetical, flat sky surface (equations (2)–(4)), i.e., a large flat grid size, we have implicitly shown that it is not sufficient to solely account for the extra surface of the inclined surface patches projected on a horizontal grid box area. In order to correctly describe the outgoing global radiation of a large flat grid size over subgrid topography, applicable for large-scale models, the mean radiative flux of the subgrid topography (or the parameterized flux) must also be scaled with the sky view factor, expressing what is actually reflected back to the sky.

[28] Finally, we quantified the contribution of the three subgrid topographic influences on decreased outgoing global radiation. As expected, shading and limited sky view contributed much more than terrain-reflected radiation. Note that when we refer to shading, we actually also include changing sun incidence angles. For mean slopes below 31° direct beam and diffuse sky radiation together contributed 97% to 100% to the global radiation depending on surface albedos (Figures 7b and 7c). However, for steeper topographies terrain reflections accounted for up to 9% depending on surface albedos (Figure 7d). Shading and limited sky view alter direct beam and diffuse sky radiation by up to 40% depending on sun elevation angles. For low sun elevation angles direct beam and diffuse sky radiation are roughly equal; that is, shading considerably reduces direct beam radiation such that limited sky view for diffuse sky radiation becomes equally important. In contrast, for large sun elevation angles, direct beam radiation accounts for almost 90% of global radiation. Of note, for the largest surface albedos, large sun elevation angles and the largest mean slopes, the

contribution from diffuse sky and terrain-reflected radiation are roughly the same (Figures 7c and 7d).

[29] In summary, the thorough validation of the parameterization scheme on our ensemble of simulated topographies showed that it is not necessary to compute subgrid radiation fluxes in order to account for subgrid topographic influences in large-scale models. It is clear that the application of this radiation parameterization scheme is limited when domain-averaged sky view factors are not available. There are different approaches with varying accuracies to compute sky view factors. However, these require vast pre-processing of subgrid topographies. We could not confirm the power law relationship of sky view factors with variance of mean slope angles proposed by *Essery and Marks* [2007] over the entire range of topographies (Figure 6). In the future we therefore intend to study the influence of topographic parameters on the sky view factor. Furthermore, the influence of inhomogeneous surface albedos on the assumption of single, isotropic terrain reflections in the terrain-reflected radiation parameterization should be addressed. Spatially inhomogeneous albedo distributions are relevant for snow ablation scenarios in subgrid topography where either mountain peaks are snow-covered and valleys are snow-free or where the entire domain is covered by patchy snow-covered and snow-free patterns.

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