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Velocity profile variations in granular flows with changing boundary conditions: insights from experiments

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We present results of detailed velocity profile measurements in a large series of granular flow experiments in a dam-break setup. The inclination angle, bead size, and roughness of the running surface were varied. In all experiments, the downstream velocity profiles changed continuously from the head to the tail of the avalanches. On rough running surfaces, an inflection point developed in the velocity profiles. These velocity profiles cannot be modeled by the large class of constitutive laws which relate the shear stress to a power law of the strain rate. The velocity profile shape factor increased from the head to the tail of the avalanches. Its maximum value grew with increasing roughness of the running surface. We conclude that flow features such as velocity profiles are strongly influenced by the boundary condition at the running surface, which depends on the ratio of bead size to the typical roughness length of the surface. Furthermore, we show that varying velocity profile shape factors inside gravitationally driven finite-mass flows give rise to an additional term in the depth-averaged momentum equation, which is normally solved in the simulation software of hazardous geophysical flows. We therefore encourage time dependent velocity profile measurements inside hazardous geophysical flows, to learn about the importance of this “new” term in the mathematical modeling of these flows. © 2013 AIP Publishing LLC. [<http://dx.doi.org/10.1063/1.4810973>]

I. INTRODUCTION

A special feature of granular flows is the coexistence of a liquid phase and a gaseous phase (see, for example, Refs. 1–3): the particles not only slide over each other like molecules in a liquid, but also exchange momentum through collisions, like molecules in a gas. This causes conceptual problems when looking for unique constitutive laws of granular flows, since the viscosity of a liquid decreases with increasing temperature (agitation of the molecules), while the viscosity of a gas increases. Because of these conceptual difficulties, extensive experimental investigation is necessary.

Much of the experimental work done on granular flows can be divided into two classes: (1) dam-break experiments, where a finite size of granular material is released from the rest and the evolution of the mass is tracked from initiation to deposition to test models’ predictions; (2) experiments where a steady flow of granular material is induced in different geometries⁴ and constitutive laws can be found by a systematic variation of the flow parameters.^{5–7}

An example of the first class is the extensive experimental work of Hutter *et al.* on different topographies^{8–11} to test the Savage-Hutter avalanche model. These experiments were mostly conducted on very smooth running surfaces (plexiglass), with rather large particles impeding a strongly sheared motion.

Pioneering work belonging to the second class of experiments was conducted by Savage in the late 1970s.⁶ He produced a steady flow of polystyrene beads on a laboratory chute, by controlling the

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flow rate with a partly opened trap door and measured velocity profiles on a rough running surface. He observed an inflection point in the velocity profile which designated the point of maximum shear to a position inside the flow. Pouliquen and Ancey found similar velocity profiles on comparable experimental setups.^{5,12}

Schaefer *et al.* tracked the evolution of the velocity field inside avalanches of glass beads (ballotini) with a digital high-speed camera.¹³ They observed the formation of an inflection point in the velocity profiles in the body of the granular avalanches when using very small ballotini (0.1 mm). With larger ballotini (1.4 mm), however, the shear rate was maximum on the running surface and decreased monotonously with distance from the ground. In this paper we present velocity profile measurements obtained on the same experimental facility, with a much wider range of experimental parameters, namely, three different roughnesses of the running surface, three different ballotini sizes and chute inclinations between 20° and 45°.

In Sec. II, we present the experimental setup and explain the experimental procedure and the data analysis method. In Sec. III, we present the experimental results and discuss them in Sec. IV. We calculate velocity profile shape factors and note a positive correlation between the roughness of the running surface and the maximum value of the shape factor. We find that the velocity profile shape factors increase from the head to the tail of the avalanches. The measured velocity profiles are compared to the predictions of the widely used power-law fluid rheology. Difficulties arise while modeling the velocity profiles of flows with a rough boundary condition. The inflection point in the velocity profile, typically observed in these flows, cannot be reproduced by applying the power-law rheology.

II. METHODS

A. Setup

The granular avalanche experiments were performed on a 0.5 m wide, 5 m long laboratory chute that could be inclined between 0° and 45° to the horizontal (Fig. 1). The chute bottom and one lateral side wall were made of wood, whereas the other side-wall was made of wood in the upper part and glass in the lower part. Running surfaces with different roughnesses were glued on the chute's bottom in the different experimental setups (Fig. 1(a)). A smooth running surface was created with

FIG. 1. (a) Frontal view of the chute, sandpaper is glued on the chute; (b) the laboratory chute in action: the hatch has been opened manually and the ballotini flow down the chute. Strobes illuminate for the high-speed cameras; (c) image of the flow of a medium ballotini avalanche recorded by the high-speed camera; computed velocity field (arrows) and horizontally averaged grey level of the image (mid-grey—red curve) are superimposed.

TABLE I. Material properties of the ballotini.

	Small ballotini	Medium ballotini	Large ballotini
Name of the product	2227 spheriglass	1416 spheriglass	1400 starlitebead
Size specification (μm)	90% between 75 and 150	85% between 425 and 590	92.5% between 1180 and 2000
Density (kg/m^3)	2400	2400	2400
Bulk density (kg/m^3)	1410	1496	1590
Angle of repose ($^\circ\text{deg}$)	(24 ± 1)	(24.5 ± 1)	(25 ± 1)
Roundness (% of perfect spherical)	92	92	92
Number of particles	6.7×10^{10}	4.4×10^8	1.7×10^7
Flow dimensions (particle diameters)	$5 \times 10^4 \times 5000 \times 300$	$10^4 \times 1000 \times 60$	$3600 \times 360 \times 21$

drawing paper, a rougher surface with an average particle diameter of $100 \mu\text{m}$ was created with sand paper ISO/FEPA Grit designation P150, and finally a very rough surface with an average particle diameter of $764 \mu\text{m}$ with sand paper ISO/FEPA Grit designation P24. The three running surfaces will be referred to as smooth, rough, and very rough running surface in this paper. At the upper end of the chute, a hatch held the granular material in a 1 m long, 0.6 m high hopper.

Two high-speed cameras were installed in the lower part of the chute behind the glass side-wall. At the maximum frame rate of 1825 frames per second and an image resolution of 256×1024 pixels, the first camera's memory of 1 gigabyte allowed a recording time of two seconds. The second camera had an internal memory of only 512 megabytes and the acquired data were used to check velocity gradients in the downstream direction in parts of the avalanches. To obtain an image section of approximately 4 cm height and 1 cm width, we used a lens with 25 mm focus with an additional spacer of 5 mm. The image distortion due to the spacer was negligible. High-speed strobe lights, that were triggered by the shutter of the high-speed cameras provided the required illumination. The recording of the cameras was triggered by a force plate, which was installed flush on the basal surface at the same downstream location as the first high-speed camera.

B. Granular material

Glass beads (ballotini) of three different sizes were used as granular material. The mean diameters of the beads were approximately $d = 1.4 \text{ mm}$, $d = 0.5 \text{ mm}$, and $d = 0.1 \text{ mm}$ and the three bead sizes will be referred to as "large ballotini," "medium ballotini," and "small ballotini" in this paper. Density was the same for all ballotini sizes, but the resulting bulk properties differed considerably. For the detailed properties, see Table I.

C. Measurements

The measurement procedure was as follows: the hatch was closed and blocked with a pin; the hopper was filled manually with the granular material; the upper part of the chute was lifted until the desired inclination was reached; the pin was removed and the hatch was held manually, the sensors were initiated and the hatch was released manually. This order was necessary because the removal of the pin sometimes caused a shock-wave to travel through the chute which caused a non-zero signal on the force plate and thus triggered the measurements. A precise trigger signal shortly before the arrival of the avalanche front, however, was necessary because of the short recording time of the high-speed camera at the highest frame rate. Another problem that occurred with the small and medium ballotini was that some material escaped through small cracks between the hatch and the side-walls of the chute before the experiments. Although small adjustable brushes were mounted on the hatch to seal the hopper, material still escaped through small holes between the brushes. The brushes had to be adjusted before each experiment to avoid ballotini escaping through these holes.

D. Data analysis

Two-dimensional velocity fields were extracted from the images of the high-speed camera using a pattern matching algorithm.¹³ An important feature of the pattern matching was a correlation-map-averaging over 10 time steps which improved the quality of the data significantly and reduced temporal resolution of the velocity field to 1/182 s. The pattern matching algorithm was able to detect displacements with sub-pixel precision, which led to an uncertainty of the individual velocity measurements lower than 0.07 m/s. The accuracy of the velocity values, however, strongly depended on the image contrast, which varied from experiment to experiment, and also between the different parts of one single flow. An estimate of the uncertainty of a single velocity value can be obtained by the scatter of the velocity values around the theoretically continuous velocity profile function. Velocity profiles were obtained from the two-dimensional velocity fields by taking the median of the downstream component of velocity vectors at the same distance from the ground. This allowed outliers to be filtered out. The vertical resolution of the velocity field was 0.16 mm for the correlation window size of 8×64 pixels used in the algorithm. An example of the velocity field is shown in an image acquired by the high-speed camera in Fig. 1(c).

III. RESULTS

In the experiments, three external parameters, chute inclination, ballotini bead size and running surface roughness, were varied. This is why a large number of experiments (nearly 50) were conducted.

A. Dependency of velocity profiles on the running surface and the bead size

In Fig. 2 we present the evolution of the velocity profiles from the head to the tail of three medium ballotini avalanches on different running surfaces. In Fig. 3 we present the evolution of the velocity profiles of a large ballotini avalanche and a small ballotini avalanche both on the rough running surface. The inclination angle in these experiments was fixed at 35° . Regarding these experiments we can make some statements about the influence of the running surface and the bead size on the flows:

- Slip velocities depended on the ratio of roughness of the running surface to bead size. Small ratios (e.g., medium ballotini on the smooth surface, or large ballotini on the rough running surface) produced significant slip velocities (Figs. 2(a) and 3(b)). High ratios prohibited slip of the particles. Since the surface velocities did not show a strong dependency on the ratio of roughness of the running surface to bead size, high ratios produced higher average shear rates.
- When the slip velocity was zero (or near zero), the lower part of the velocity profile developed a positive curvature, which produced an inflection point in the velocity profile (Figs. 2(c) and 3(a)).
- In the experiments with the medium ballotini and the large ballotini, particularly on the smoother running surfaces, a dilute phase was observed at the head and the tail of the avalanches. Here, the particles were flying through the air and interacting through binary collisions instead of sliding past each other as they did in the dense part of the avalanches. In these dilute parts, the velocity profiles were rather uniform and are more scattered than in the dense part of the flow (t1, t6, t7 in Fig. 2(a) and t1, t6 in Fig. 3(b)).

A flow transition behind the front of the avalanche could also be observed in the small ballotini avalanches and the medium ballotini avalanches on the very rough running surface (Figs. 2(c) and 3(a)). Here we observe that the shape of the velocity profile changes from convex to concave behind the front. Also, in these flows, some beads are flying through the air before the dense part of the avalanche arrives. The denser part that follows (corresponding to the first curve (t1) in Figs. 2(c) and 3(a)) is a transition part, where the flow has not completely settled yet, and the friction with the ground is not high enough to prevent the ballotini to slip.

FIG. 2. The evolution of the velocity profile for three medium ballotini avalanches at a 35° inclination on the three different running surfaces. (a) smooth running surface, (b) rough running surface, (c) very rough running surface. t1 to t7 refer to different times in the high-speed movie. Profiles are plotted every 0.25 s on the smooth running surface and 0.33 s on the rough and the very rough running surface.

B. Dependency of velocity profiles on the inclination angle

To illustrate the effect of different inclination angles, in Fig. 4 we present velocity profile plots of two medium ballotini avalanches on the rough running surface, at inclinations of 30° and 40°. Comparing these plots with Fig. 2(b) we infer the following:

- Surface velocities increase significantly with increasing inclination angle.

FIG. 3. The evolution of the velocity profile for a small ballotini (a) and a large ballotini (b) avalanche at a 35° inclination on the rough running surfaces. t1 to t7 refer to different times in the high-speed movie. Profiles are plotted every 0.33 s.

FIG. 4. Evolution of the velocity profiles for two medium ballotini avalanches at two different inclination angles: (a) 30° and (b) 40° .

- The transition between the velocity profiles is faster at higher chute inclinations.
- The shape of the velocity profiles is similar at different inclination angles on the same running surface.

The last statement, however, was not true at small inclinations on the very rough running surface, because the avalanches became very slow and the beads were creeping rather than flowing down the incline.

C. Depth averaged velocity and flow depth

In Fig. 5, we compare the evolution of the flow depth and the depth averaged velocity v_m for the three medium ballotini avalanches discussed already in Fig. 2. In every frame of the high-speed camera the flow depth was determined by a threshold value of the horizontally averaged grey level, which decreased rapidly above the dense part of the flow (Fig. 1(c)). At the measurement location, the maximum flow depth was lower for the rougher running surfaces (Fig. 5(a)). This is due to the lower slip velocities on the rougher running surfaces. The velocity difference between the particles on the top of the flow and the particles next to the running surface is higher. The faster particles on the top overtake the particles near the ground, which causes the stretching of the avalanches in downstream direction. Towards the end of the flows on the smooth running surface, a dilute phase can be identified, where the values of the flow depth are more scattered. At the very end, the value of the flow depth corresponds to a reflection from the white drawing paper, which is visible because most of the particles had already passed the measurement location.

FIG. 5. Evolution of the flow depth (a) and the depth averaged velocity (b) in the medium ballotini avalanches when flowing on different running surfaces.

FIG. 6. Depth averaged downstream velocity v_m , slip velocity v_s and surface velocity v_t for two medium ballotini avalanches with the same experimental parameters: 30° inclination, rough running surface.

The depth averaged velocity was higher on the smoother running surfaces (Fig. 5(b)), mainly due to the higher slip velocities. The depth averaged velocity decreased much faster on the smooth running surface.

D. Repeatability of the experiments

An important criterion for the significance of single experiments is their repeatability when keeping experimental parameters unchanged. In the large series of experiments, single experiments were repeated. In Fig. 6 we present the evolution of the flow depth and three velocity times series (slip velocity, top velocity, and depth averaged velocity), extracted from the velocity profiles, of two medium ballotini experiments conducted on two consecutive days at an inclination of 30° on the rough running surface. A good repeatability of the evolution of the flow depth and a very good repeatability of velocity measurements are observed. At the head of the avalanche, where the scatter of the data is high, the difference between the two runs is more pronounced. Here, the flow was diluted and it is more difficult to determine the flow depth. Small variations due to slightly different initial conditions, such as the filling patterns of the reservoir, can be noticed. The first avalanche exhibited a higher maximum flow depth value and the form of the maximum is more peaked than in the second avalanche. This is why the bend in the depth averaged downstream velocity is more pronounced at $t = 1$ s, when the surface velocity starts to decrease as well (Fig. 6).

IV. DISCUSSION

A. Velocity profile shape factors

Velocity profiles can be studied systematically by calculating the velocity profile shape factor

$$\alpha(x, t) = \frac{\int_0^{h(x,t)} u^2(x, y, t) dy}{\left(\int_0^{h(x,t)} u(x, y, t) dy \right)^2} = \frac{\bar{u}^2}{\bar{u}^2}, \quad (1)$$

where $h(x, t)$ is the flow depth, $u(x, y, t)$ is the velocity in the downstream (x -)direction and y is the flow-normal coordinate as defined in Fig. 7. For a uniform velocity profile (plug-flow), α is one and four thirds for a linear profile. We can calculate the shape factors directly from the measured velocity profiles. In Fig. 8, we present the velocity profile shape factors for the three medium ballotini avalanches already discussed in Sec. III A (Fig. 2).

On all three running surfaces, the shape factors of the medium ballotini avalanches increased in the first second. The increase was much stronger on the very rough running surface, where the shape factor reached 1.4 at the end of the measurement. On the smooth running surface, the maximum value was 1.05 and the shape factor decreased at the end of the avalanche where the flow was dilute and

FIG. 7. Definition of the framework: observation of the avalanches through a side-wall of the chute reduces the problem to two dimensions.

the velocity profile uniform. On the rough running surface, the maximum value was approximately 1.16 and decreased only slightly in the second part of the flow. Thus, the roughness of the running surface correlated positively with the maximum of the velocity profile shape factor.

The strong variation in the shape factor in the experiments is an important observation. Since we are measuring in an Eulerian frame, this variation suggests that the shape factor changes from the front to the tail of the avalanches. In the depth averaged modeling of hazardous granular geophysical flows, the shape factor is assumed to be constant and is usually set to one.^{14–19} While a slight (constant) increase in the shape factor did not significantly change the results of the simulations,^{18,19} a varying shape factor could produce more changes. In this case, the depth averaged momentum balance in the downstream direction is

$$\frac{\partial}{\partial t}(h\rho\bar{u}) + h\rho\bar{u}^2 \frac{\partial}{\partial x}(\alpha) + \alpha \frac{\partial}{\partial x}(h\rho\bar{u}^2) = \frac{\partial}{\partial x}\sigma_{xx} - \tau_{xy}|_{y=0} + h\rho g \sin \theta, \quad (2)$$

where ρ denotes the density of the flow, σ_{xx} the first diagonal element, and τ_{xy} the off-diagonal element of the stress tensor in the 2-dimensional framework defined in Fig. 7, g the gravitational constant, θ the inclination, and the overbar denotes depth averaged quantities. The “new” term, which contains the variation of the shape factor inside the avalanche (second term on the left-hand side), could be important in fast flows with changing velocity profiles such as snow avalanches.²⁰ For our experiments, on average, this term was one order of magnitude lower than the driving force term.

FIG. 8. Velocity profile shape factors of three medium ballotini avalanches flowing on different running surfaces at 35° inclination.

B. Power-law fluid

One striking result of our measurements is how the shape of the velocity profiles changed from convex to concave, when the boundary condition changed from smooth to rough. In a steady, fully developed flow, the velocity profile is completely determined by the constitutive law, which connects the shear rates to the shear forces inside the material. If the shape of the velocity profile changes in such a configuration, this would immediately imply that the rheology of the material changes. Here we study the predictions of a very general constitutive law, the Power-law fluid. As the name suggests, it connects stresses to shear rates by a power law

$$\tau_{xy} = k \left(\frac{\partial u}{\partial y} \right)^n. \quad (3)$$

Here, τ_{xy} is the off-diagonal element of the stress tensor in the 2-dimensional framework defined in Fig. 7, and k is a proportionality factor whose dimensions depend on n . For $n = 1$, the Newtonian case is recovered, and k denotes the kinematic viscosity. $n = 2$ corresponds to the Bagnold law,²¹ which has been frequently applied to granular flows.^{4,22,23} The model comprises shear thinning fluids ($n < 1$) as well as shear thickening (dilatant) fluids ($n > 1$).

In the following we want to study how velocity profiles of a Power-law fluid would look like, in a steady state situation. As it has been evidenced in the results section, the flows of our experiments were clearly unsteady. On the other hand, to first order, the velocity profiles measured in our experiments should still be determined by the steady state velocity profile. In the steady case, the momentum balance in the downstream direction is

$$0 = \frac{\partial}{\partial y} \tau_{xy} + \rho g \sin \theta. \quad (4)$$

Integration yields

$$\tau_{xy}(y) = g \sin \theta \int_y^h \rho(y') dy' + \tau_{\text{surf}}, \quad (5)$$

where $\tau_{\text{surf}} = \tau_{xy}(h)$ denotes the stress at the free surface. Using (3), we obtain the first derivative of the downstream velocity profile

$$\frac{\partial}{\partial y} u = \left(\frac{g \sin \theta}{k} \int_y^h \rho(y') dy' + \frac{\tau_{\text{surf}}}{k} \right)^{1/n} \quad (6)$$

and the second derivative

$$\frac{\partial^2}{\partial y^2} u = -\frac{g \sin \theta}{nk} \rho \left(\frac{g \sin \theta}{k} \int_y^h \rho(y') dy' + \frac{\tau_{\text{surf}}}{k} \right)^{\frac{1}{n}-1}. \quad (7)$$

Note that, if the first derivative of the velocity profile is positive (as observed in all the experiments), the second derivative is strictly negative. This means that the shear rate is maximum on the ground, and decreases monotonously with distance from the ground: the velocity profiles have a negative curvature. Velocity profiles with a positive curvature or even an inflection point, like the ones measured on rough running surfaces, cannot be described by the power-law rheology.

Velocity profiles with a positive curvature have also been measured in steady-state situations.^{5,6,12} The incompatibility of these experiments with the power-law fluid rheology, and especially with the Bagnold rheology ($n = 2$), is important to stress.

Since nearly all velocity profile measurements in granular flows are conducted at the side-wall of the flow, one could argue that the positive curvature could be caused by effects of the side-wall. Following Jop *et al.*,¹² we now want to examine how a Coulomb type sidewall friction would change the shape of the velocity profiles. An extra term arises in the momentum balance in the x-direction,

$$\tau_{xy}(y) = \rho g \sin \theta (h - y) - \rho g \frac{\mu_w}{W} \cos \theta (h - y)^2 + \tau_{\text{surf}}. \quad (8)$$

Here, μ_w denotes the Coulomb friction coefficient between the beads and the sidewalls and W the width of the flow. For simplicity, we are assuming a constant bulk density ρ here. Using (3), we

obtain for the first derivative of the downstream velocity profile

$$\frac{\partial}{\partial y} u = \left(\frac{\rho g}{k} (\sin \theta (h - y) - \frac{\mu_w}{W} \cos \theta (h - y)^2) + \frac{\tau_{\text{surf}}}{k} \right)^{1/n} \quad (9)$$

and for the second derivative

$$\frac{\partial^2}{\partial y^2} u = \frac{\rho g}{kn} \left(2 \frac{\mu_w}{W} \cos \theta (h - y) - \sin \theta \right) \left(\frac{\rho g}{k} (\sin \theta (h - y) - \frac{\mu_w}{W} \cos \theta (h - y)^2) + \frac{\tau_{\text{surf}}}{k} \right)^{1/n-1}. \quad (10)$$

Again, keeping in mind that the first derivative is positive in all the experiments, we obtain the following condition for a positive curvature of the velocity profile:

$$y < h - \frac{W \tan \theta}{2\mu_w}. \quad (11)$$

Using realistic values for our experiments ($h < 4$ cm, $\tan \theta > 0.36$, $W = 50$ cm, $\mu_w < 0.5$) we recognize, that (11) is only fulfilled for negative values of y . As a conclusion, we can state that at least a Coulomb type side-wall friction cannot cause the inflection point or positive curvature in the velocity profile as observed in the experiments.

V. CONCLUSIONS

In this paper, results are presented from a large series of experiments with granular material of different bead sizes, on a chute with varying inclination angles and different running surfaces in a dam-break setup. A great variety of velocity profiles has been observed in the flows: velocity profiles were changing shapes when applying different running surfaces and also from the head to the tail within a single experiment. Performing a brief analysis of the measured velocity profiles, two important implications for the modeling of granular flows could be found.

Contrary to the results of experiments and simulations in different flow geometries of the GDR MiDi Group,⁴ we found that the shape of the velocity profile can vary strongly for one single flow geometry. This suggests that the velocity profiles depend rather on the imposed boundary conditions than on the geometry of the flows. In the flows on a smooth running surface, the shear force can be modeled as a power law of the shear rate. Here, the glass beads seem to slide over each other, similar to the motion of molecules in (non-Newtonian) liquids. On a rough running surface, this no longer seems to apply. On a running surface with a roughness comparable to the size of the particles, a different particle motion occurs, as an agitation of the particles is produced at the running surface, which is transferred between the particles through inelastic collisions. Depending on the coefficient of restitution of the collisions and the rate of collisions, this agitation will decay. It appears that all these mechanisms cannot be described by a constitutive law that connects the experienced shear force to the shear rate by a power law with a constant coefficient and exponent. Generally, the coefficient k and the exponent n might depend, for example, on the collisional properties of the beads and on the volume fraction of the granular material. These properties are not necessarily constant in space and time, and therefore may introduce additional terms in Eq. (6), which could allow for a positive curvature and an inflection point in the velocity profile.

Analyzing the evolution of the velocity profiles shape factor in our experiments, we found that the shape factor increased during the experiments. This increase was more pronounced on the rougher running surfaces. A varying velocity profiles shape factor gives rise to an additional term in the depth averaged equation of conservation of momentum in the downstream direction (2), which is normally solved in simulation software of hazardous gravitationally driven geophysical flows.^{16,17,19} The upscaling of our small-scale laboratory experiments to gravitationally driven geophysical flows is not straightforward at all. However, changing velocity profiles were also detected in real-scale geophysical flows,²⁰ and therefore it is possible that this term could also play an important role for the mathematical modeling of gravitationally driven geophysical flows. Time-resolved measurements of velocity profiles inside gravitationally driven geophysical flows are necessary to be able to judge the importance of this “new” term for the mathematical modeling of these flows.

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