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Key Points:

- We empirically investigate the synchronization between snowfall and avalanches
- We quantify the entropy associated with avalanche prediction from snowfall
- We assess slope effects on avalanche return period and detachment rate

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Quantifying snowfall and avalanche release synchronization: A case study

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Abstract We quantify the synchronization between snowfall and natural avalanches in relation to terrain properties at the detachment zone. We analyze field statistics of 549 avalanche events in terms of slope, aspect, timing, coordinate, and release area, identified by a georeferencing procedure applied on terrestrial photography. The information from the digital pictures, together with associated meteorological data, provides us with the input needed for model calibration, namely, the magnitude of snowfall, the snow compaction rate, and the timing of precipitation and of avalanche events. Synchronization between snowfall and avalanches is established for different slope categories. We obtain an average probability of release after a snow event of 30% and 16% for the high- and low-slope categories (average slope 44° and 36°, respectively). Using the notion of information entropy, we quantify the uncertainty in predicting avalanche occurrence from a snow event. The steeper slopes correspond to a larger entropy in avalanche prediction. Further, the presented method allows us to establish the return period of avalanches without requiring a long series of data. When considering events regardless of their release depth, the avalanches had a return period of 48 days (higher slopes) and 88 days (lower slopes). Finally, we determine the average daily detachment rate as a function of snow depth and the return period of avalanches as a function of the release depth.

1. Introduction

Modeling the release of snow avalanches still represents a significant challenge due to difficulties in the collection of comprehensive field data [Schweizer *et al.*, 2003]. Practitioners normally rely on their experience or on simple rules [Schaerer and McClung, 2006] to estimate the probability of avalanche release following snow precipitation. It is well known, however, that there are numerous factors that can be linked to avalanche release. In particular, snowpack stability is influenced by a large set of physical parameters related to meteorological conditions such as snowfall frequency and magnitude, terrain topography (e.g., slope, aspect, and elevation), and metamorphism [Hopfinger, 1983].

As remarked by Eckert *et al.* [2008], while deterministic approaches are commonly used for avalanche forecasting, stochastic approaches are unavoidable when investigating avalanche activity over longer time periods or for hazard estimation and mapping purposes [Cappabianca *et al.*, 2008; Barbolini and Savi, 2001; Schweizer *et al.*, 2009]. It is, however, difficult to obtain sufficiently long data series (the records used by Smith and McClung [1997] or Föhn [1975] are illustrative in this regard) for model calibration in the context of snow avalanches, even when considering basic statistics such as the distribution of the return time of avalanche events. In addition, the inherent uncertainty in physical avalanche models [Eckert *et al.*, 2008] on top of the uncertainty on the data makes the use of statistical models less frequent in snow hydrology [Ancey *et al.*, 2004] than for conventional hydrological risks. The combined uncertainty makes it difficult to distinguish artifacts in the modeling or fitting procedure from physically meaningful results. This can, for example, be seen in Castebrunet *et al.* [2012] when the authors discuss opposite possible explanations (involving changes in snowpack stability) for the influence of the snow depth anomaly on avalanche statistics.

Our research focuses on the return time of small frequent avalanches and on quantifying to which extent the avalanches in a study site can be statistically related to the preceding snow event. This requires a stochastic approach that links natural releases to snow precipitation. In this direction Smith and McClung [1997] studied the return period of frequent (3 to 21 avalanches per year and per path) avalanches for 43 avalanche paths in British Columbia over a 24 year period. Even earlier, Föhn [1975] performed a similar analysis by describing larger infrequent events (typical return period of 30 years, Swiss Alps) for a much

longer record (420 years), together with a record of relatively small events in the Austrian Alps (typical return period of 1 year). In both studies, the distribution of the return time of avalanches is modeled as a Poisson process with constant rate and in *Föhn* [1975] an approach based on the Gumbel distribution [*Gumbel*, 1958] is used to obtain the return time of events with given magnitude. The present work can be seen as an extension of *Smith and McClung* [1997] and *Föhn* [1975], where we use a state-dependent ("marked") Poisson process coupled to the equation for the evolution of the snow depth (deterministic and stochastic components) instead of the constant rate Poisson process approach to model the return time of avalanches. For a general introduction to marked Poisson processes, see, for example, *Daly and Porporato* [2007, and references therein].

To assess the synchronization between avalanches and snowfall, we use the method proposed by *Perona et al.* [2012], who tried to address issues about the stochasticity driving natural avalanches by proposing a simple state-dependent stochastic model from a restricted set of physical parameters. They modeled the evolution of the snow depth according to snowfall, snowpack compaction/melting, and avalanche detachment. For the sake of self-consistency, the specifics of the model are briefly reviewed in section 2.

The work by *Perona et al.* [2012] was essentially theoretical and focused on the mathematical interplay between two superposed Poisson processes (snowfall and avalanches) randomly forcing a deterministic trajectory (snow depth reduction due to compaction and melting). As mentioned above, a critical point for calibrating stochastic avalanche models is to obtain a sufficient data set for the computation of statistical distributions. Specifically, either a very long record of natural avalanche releases at one location or an appropriate methodology to aggregate events occurring on several zones are needed. The purpose of this work is to use an experimental statistics of 549 avalanche events and determine the synchronization between avalanches and snowfall from the calibration of the model. Note that due to the incomplete knowledge on snowpack properties, we use a stochastic description for both precipitation and mechanical failure. While most model parameters can directly be obtained from meteorological data, one parameter quantifying the influence of the local topography and terrain on the state-dependent detachment rate needs to be calibrated using a statistics of natural avalanche releases. We carefully discuss the choice of appropriate statistics for comparison of a point process model [*Perona et al.*, 2012] to spatially distributed observations of avalanche events. We obtain the missing parameter by maximizing the likelihood of the field statistics regarding the value of the topography/terrain parameter [*Aldrich*, 1997]. Sorting the events according to the slope at the detachment zone allows us to highlight the influence of the slope on the detachment rate. We can then interpret the field statistics in terms of snowfall-avalanche synchronization and use the results by *Perona et al.* [2012] in order to determine the information entropy associated with the prediction of an avalanche from snowfall for the type of slopes we consider. Information entropy was first used in the context of avalanches by *LaChapelle* [1980], although the discussion remained on a qualitative level in this paper. As an interesting further application, the method allows us to obtain the return period of the avalanche events given the return period of snowfall. This application is particularly interesting for a general characterization of the avalanches occurring in a specific site. The association of a release volume to a given return period is indeed a key parameter for hazard-mapping procedures [*Harbitz et al.*, 2001; *Anczy et al.*, 2003; *Hopfinger*, 1983; *Eckert et al.*, 2007], together with the runout distance [*Keylock et al.*, 1999] and runout elevation [*Eckert et al.*, 2013].

The paper is organized as follows: in section 2 we briefly review the theoretical model and the concept of prediction entropy introduced by *Perona et al.* [2012]. In section 3 the hillside site and the methodology for data collection are presented, followed by model calibration. Finally, we discuss our results (section 4) and present our conclusions (section 5).

2. Modeling Approach

2.1. Stochastic Model

The stochastic model by *Perona et al.* [2012], the calibration of which is the core of the present work, attempts to capture the interplay between snowfall and avalanches using a minimal set of physical parameters. It also introduces the notion of prediction entropy in order to quantify the uncertainty in avalanche prediction from a single snow event. The avalanches are modeled as a point process, more specifically by the Langevin-type equation that represents the evolution of snow depth (a single-state variable h) distribution. Snow depth varies according to three processes. Following *Perona et al.* [2007], positive jumps generated by precipitation are modeled by an instantaneous marked Poisson process N^s with constant rate

Figure 1. Evolution of the snow depth at the ground $h(t)$ for different values of the terrain parameter ν_1 and of the detachment probability after a snowfall P_a : (a) $\nu_1 \approx 0 \text{ d}^{-1} \text{ cm}^{-1}$ and $P_a = 0$. (b) $\nu_1 = 2 \cdot 10^{-3} \text{ d}^{-1} \text{ cm}^{-1}$, and $P_a = 0.5$. (c) $\nu_1 = 1 \cdot 10^{-1} \text{ d}^{-1} \text{ cm}^{-1}$ and $P_a = 1$. For the three panels, the average return period of snowfall, the compaction rate of the snow mantle, and the average magnitude of snow events are fixed at $\bar{\tau}_p = 14$ days, $\rho_1 = 0.0335 \text{ day}^{-1}$, and $\alpha = 48$ cm, respectively.

$\lambda = 1/\tau_p$ and τ_p the average return time of the precipitation events. We take an exponential distribution for the magnitude y of the precipitation events

$$p(y) = \gamma e^{-\gamma y}. \quad (1)$$

The variable γ is defined in relation to the average snowfall magnitude α as $\gamma = \frac{1}{\alpha}$. Decrease in the depth of the snow layer is modeled as an exponential deterministic decay with state-dependent rate $\rho(h) = \rho_0 + \rho_1 h$, a deterministic drift assumed to be positive. Avalanches result in negative jumps, expressed by an instantaneous state-dependent Poisson process N^a with rate $\nu(h) = \nu_0 + \nu_1 h$ that resets the state variable $h(t)$ to zero: for simplicity we consider here only the situation with ground avalanches. Note that fluctuations of the temperature signal are one of the underlying forcings resulting in stochastic detachment. Both the deterministic decay rate $\rho(h)$ and the avalanche rate $\nu(h)$ include a constant part (ρ_0 and ν_0 , respectively) and a linear term (ρ_1 and ν_1 , respectively). In the following, we shall however assume $\rho_0 = 0$ and $\nu_0 = 0$, which describes well enough the compaction and release processes for our purposes (section 3.3). Collecting the three processes, one obtains the stochastic differential equation for the evolution of the snow depth with time

$$\frac{dh}{dt} = -\rho(h) + \sum_{i=1}^{N^s(t)} y_i \delta(t - t_i^s) - \sum_{j=1}^{N^a(t)} h(t) \delta(t - t_j^a). \quad (2)$$

In equation (2) $\delta(\cdot)$ indicates the Dirac delta distribution [Dirac, 1958; Landau and Lifshitz, 1980]. The times t_i^s and t_j^a are the instants of occurrence of the i th and j th events of the two Poisson processes $N^s(t)$ and $N^a(t)$, respectively.

For the sake of avoiding overparameterization, we compute statistics averaged over the whole winter and use a model which does not take seasonality into account. We also do not distinguish between different types of avalanches. Those working hypotheses will be discussed at the end of section 3.2. In this regard, when calibrating the model, we aggregate empirical statistics from succeeding winters. While λ , α and ρ can be measured directly from meteorological data (snow depth), the terrain parameter ν_1 aggregates the influence of several factors such as the aspect, the roughness of terrain or the fluctuations of the temperature signal. Physically, it has the dimension of an inverse length multiplied by an inverse time, such that one obtains the average daily probability of release when multiplying ν_1 by the depth of the snow layer.

In Figure 1, we present the evolution of snow depth taken from example simulations of the process studied by Perona *et al.* [2012]. We represent three possible outcomes of the process for different degrees of synchronization between snowfall and avalanches. Figures 1a–1c correspond to identical snowfall frequency and magnitude distribution as well as identical compaction and melting rates, while the state-dependent avalanche release rate increases from Figure 1a to Figure 1c. With the exception of the avalanche release rate, the parameters used in Figure 1 correspond to the values obtained for our study site. The increase of the release rate leads to an increase in the probability P_a of observing an avalanche event following a snow event. This probability is related to the synchronization between avalanches and the preceding snowfall. No detachment is observed in Figure 1a (minimal synchronization, $P_a = 0$) and the trajectory is typical of snow depth observed dynamics [Perona *et al.*, 2007]. In Figure 1b (partial synchronization, $P_a = 0.5$), detachment occurs after half of the snowfalls and is usually preceded by a compaction period. In Figure 1c ($P_a = 1$), avalanches and snowfall are perfectly synchronized: detachment is observed after each snowfall. As shown in the next subsection, the probability P_a can naturally be associated with an entropy (see Shannon [1948] and Appendix A) for avalanche prediction from a single snow event, which is maximal for the intermediate situation represented in Figure 1b.

Using a Master Equation approach—see, for example, Cox and Miller [1965], Gardiner [2004], Perona *et al.* [2009], and Crouzy and Perona [2012] for applications to environmental sciences—one can rewrite equation (2) as an equation for the probability density function (denoted “PDF” hereafter) of the snow depth $p(h, t)$:

$$\frac{\partial}{\partial t} p^c(h, t) = \frac{\partial}{\partial h} [(\rho_0 + \rho_1 h) p^c(h, t)] - \lambda p^c(h, t) + \lambda \int_0^h \gamma e^{-\gamma(h-u)} p^c(u, t) du + \lambda \gamma e^{-\gamma h} p_0^{\text{at}}(t) - (\nu_0 + \nu_1 h) p^c(h, t). \quad (3)$$

Equation (3) comprises a continuous part or density $p^c(h, t)$ and an atom $p_0^{\text{at}}(t)$ at $h = 0$, with a dynamics coupled to the dynamics of the continuous part

$$\frac{d}{dt} p_0^{\text{at}}(t) = \int_0^\infty (\nu_0 + \nu_1 h) p^c(h, t) dh - \lambda p_0^{\text{at}}(t) + [(\rho_0 + \rho_1 h) p^c(h, t)]_{h=0}. \quad (4)$$

It is necessary to introduce the atom $p_0^{\text{at}}(t)$ in order to account for the finite probability mass of observing the system at the state $h = 0$.

In addition to the time-dependent probability distribution of the snow depth, the PDF $p_\tau(\tau)$ of the interevent time τ between avalanches provides an insight into the behavior of the system [Daly and Porporato, 2006]. It is computed as a rate of loss of probability mass from the continuous part of the distribution. Note that, technically, transitions from the atom to the continuous part have to be subtracted in order to single out jumps from the continuous part to the atom. The exact PDF can be obtained according to the procedure described in Perona *et al.* [2012]. As a result, one gets

$$p_\tau(\tau) = \frac{\lambda e^{-\frac{\lambda \tau \nu_1}{\gamma \rho_1 + \nu_1}} (-1 + e^{\rho_1 \tau}) \nu_1 \left(\frac{\gamma \rho_1}{\gamma \rho_1 + \nu_1 - e^{-\rho_1 \tau} \nu_1} \right)^{-\frac{\gamma \lambda}{\gamma \rho_1 + \nu_1}}}{-\nu_1 + e^{\rho_1 \tau} (\gamma \rho_1 + \nu_1)}. \quad (5)$$

In Figure 2, we use this formula to show the PDF of the interevent time between avalanches with varying terrain parameter ν_1 (Figure 2a) and compaction rate ρ_1 (Figure 2b). It appears that increasing ν_1 leads to a decrease in the variance coupled to a shift of the mode toward shorter interevent times, with the PDF of avalanche return time that tends to follow the one of the marked Poisson process of snowfall. For increasing ρ_1 the variance increases and the mode shifts toward shorter return times.

For the synchronization analysis presented in the next subsection, it is useful to compute the average avalanche return period $\bar{\tau}$ and the average frequency of avalanches $\bar{\nu} = \frac{1}{\bar{\tau}}$

$$\bar{\tau} = \int_0^\infty \tau p_\tau(\tau) d\tau. \quad (6)$$

The integral can be computed analytically. However, since the resulting expression is cumbersome, we prefer to present it in Appendix B.

To summarize the review of the model, the parameters needed for calibration are as follows: $\gamma = \frac{1}{\alpha}$ accounting for the snowfall average magnitude α , λ the rate of snow events, ρ_1 the compaction rate, and ν_1 the terrain parameter (related to the slope). Since all the parameters except ν_1 can be measured directly,

Figure 2. Probability density function of the interevent time between avalanches for varying (a) terrain v_1 (with fixed $\rho_1 = 0.0335 \text{ day}^{-1}$) and (b) compaction ρ_1 (with fixed $v_1 = 0.002 \text{ cm}^{-1} \text{ d}^{-1}$) parameters. For both panels, the average return time of precipitation and the average magnitude of snow events are fixed at $\bar{\tau}_p = 14$ days and $\alpha = 48$ cm, respectively.

we shall determine v_1 in section 3 indirectly from the distribution of the timing of precipitation events and avalanches.

2.2. Snowfall-Avalanche Synchronization and Prediction Entropy

Assuming that only one avalanche can be triggered at the same location between two snow events, i.e., that avalanches remove the entire snow layer, the ratio between the average avalanche frequency and the average snowfall frequency gives the probability of having an avalanche following a snowfall

$$P_a = \frac{\bar{v}}{\lambda} = \frac{1}{\lambda \bar{\tau}}. \quad (7)$$

This quantity is also a measure of the degree of synchronization between avalanches and snowfall: $P_a = 1$ corresponds to perfectly synchronized processes occurring at equal frequency.

As classically done in information theory [Shannon, 1948; Cover and Thomas, 2006], we relate the probability P_a to an entropy that quantifies in our case the uncertainty in predicting an avalanche from a precipitation event for a given slope (the terrain parameter v_1 accounting for the characteristics of the slope),

$$H = -P_a \log_2 P_a - (1 - P_a) \log_2 (1 - P_a). \quad (8)$$

For a brief review on the concept of information entropy, see Appendix A. In total, one bit of information is necessary to describe the possible outcomes, for example, by assigning “0” to the absence of avalanche release and “1” to a release after the observation of snowfall. Consequently, the maximal possible value for the entropy is also $H = 1$ (often the unit “bit” is used together with the entropy value). This maximal entropy is reached for $P_a = 0.5$, a situation where the uncertainty on the outcome is complete. In both limits $P_a = 1$ and $P_a = 0$ the entropy is zero. Those two limits correspond to situations where avalanche detachment is either certain after each snowfall or conversely detachment can be excluded. Intermediate values of P_a give nonzero entropy and correspond to different degrees of uncertainty, between zero and the number of bits necessary to describe the ensemble of possible outcomes.

The model predicts a higher synchronization for steeper slopes (i.e., larger v_1), assuming that the triggering dominant factor is the charge increase following intense snowfall. Conversely, the interevent time between avalanches becomes large for very low slopes. For practical purposes, triggering under a critical angle value of around 30° can even be assumed to be nonexistent. For the “intermediate” slope category, more than one precipitation event can occur before detachment and the distribution of the interevent time between avalanches presents a significant variance. The entropy is maximal for these slopes, so that predicting the release of an avalanche from the observation of a snow event becomes uncertain. In the present work we quantify the level of uncertainty H and compute the probability of detachment after a snowfall P_a for a field situation.

Another way to look at the synchronization between snowfall and avalanches is to introduce a measure of the correlation between the intensity of snowfall preceding the avalanche and the release depth. Since we restrict ourselves to ground avalanches, we can simply compute the cross correlation between last

Figure 3. Study site: (a) Hillside of the Vallée de la Sionne (Valais, Switzerland), with the location of the meteorological station, the camera position, and field of view (green area). (b) Avalanche event locations on the hillside (center of the detachment zone). The underlying map is colored according to slope values.

precipitation magnitude and total snowpack depth before the avalanche. In *Perona et al.* [2012] it has been shown that the model reproduces the fact that the release depth tends to become more correlated to preceding precipitation as the slope increases. The entropy is correspondingly maximal for intermediate slopes, where both stochastic and deterministic processes are important to determine the resulting snow depth trajectory (e.g., Figure 1b).

In section 3 we discuss the influence of the compaction rate ρ_1 and of the terrain parameter v_1 on the synchronization and the prediction entropy. In particular, we will see that an increasing value of the compaction rate shifts the maximal entropy toward higher slopes (in the sense of a larger v_1). This means that for low slopes compaction reduces the uncertainty in predicting (the absence of) avalanches following a snowfall. Hence, compaction can delay or even prevent release in that case. Conversely, for high slopes, an increase in compaction rate raises the uncertainty (within a given range).

3. Model Calibration

3.1. Study Site

For model calibration, a study site of the Swiss Institute for Snow and Avalanche Research in Davos (denoted “SLF” hereafter) was chosen (Figure 3a, see also *Steinkogler et al.* [2014] for a recent study performed on this site). This hillside is situated in the Vallée de la Sionne in Valais, Switzerland. The elevation varies between 1600 and 2600 m, and the general aspect is about 120° . A camera installed by SLF (Mobotix D12 with an L32 lens) took a picture every 30 min from 2007 to April 2013, generating thousands of pictures. The pictures following an avalanche event and the preceding ones were kept to obtain the date of detachment. This allowed us to extract 610 avalanche events (Figure 3b). Out of those events, we kept the 549 events with available meteorological data and a detachment zone that could be clearly identified. We excluded events for which the scope of the camera remained completely in the clouds several days. For calibration, we used the IMIS weather station from SLF of Vallée de la Sionne-Donin du Jour. This station is situated at an elevation of 2390 m and located at the WGS84 coordinates $7^\circ 21' 59.681''$ and $46^\circ 19' 14.688''$.

3.2. Georeferencing

In order to obtain the topographic information needed to characterize avalanche events, a useful method is georeferencing oblique photographs [*Bozzini et al.*, 2011; *Burrough and McDonell*, 1998; *Corripio*, 2004]. Following the method by *Corripio* [2004], a viewing transformation followed by a perspective projection was applied to a digital elevation model (DEM) to match the two-dimensional pixels of the picture with points in the three-dimensional space. The input needed for georeferencing the digital images consists of the following: (1) terrestrial photography, (2) a digital elevation model, (3) the focal length of the camera, (4) the coordinates of a series of ground control points, (5) the coordinates of the center of the picture (target), (6) the position of the camera, (7) the sensor resolution, (8) the sensor dimension, and (9) the image distortion (optional, to improve accuracy close to the edges).

Using the method by Corripio [2004], we derived the visible portion of the DEM (viewing window) from the angular field of view

$$\phi = \arctan \frac{h_s}{2f} \quad (9)$$

with f the focal length of the camera and h_s the vertical dimension of the sensor. The viewshed of the camera was generated in Arc/INFO in order to obtain the points visible by the camera on the DEM within the viewing angle. This step removed all the surfaces hidden from the camera field of view in order to prevent errors resulting from mapping picture pixels to nonvisible DEM cells.

The points of the DEM were mapped to points in the camera coordinate system by a viewing transformation. The origin was set at the camera position by a translation, then translated coordinates were rotated in the viewing reference system by another transformation matrix [Corripio, 2004]. Finally, the points in the three-dimensional space were mapped to a two-dimensional space within the viewing window by a perspective projection as shown in Figure 4a. The viewing transformation equation can be written as

$$P_c = M_r \cdot M_t \cdot P_w \quad (10)$$

with P_c the coordinates of the point in the camera coordinate system, P_w a point in the Swiss coordinate system,

$$M_t = \begin{pmatrix} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & 1 & -C_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad M_r = \begin{pmatrix} U_x & U_y & U_z & 0 \\ V_x & V_y & V_z & 0 \\ N_x & N_y & N_z & 0 \\ 0 & 0 & \frac{1}{f} & 1 \end{pmatrix} \quad (11)$$

the translation matrix ($\mathbf{C}$ is the camera position) and the rotation matrix ($\mathbf{U}$, $\mathbf{V}$, $\mathbf{N}$ form a set of orthogonal unit vectors defining the reference system of the camera). The use of four vectors and four matrices allows to account for the focal length f of the camera in the scaling for the perspective projection (see Corripio [2004] for details).

If high precision is required, ground control points (denoted "GCPs" hereafter) can be acquired by differential GPS. However, for our purposes (1 pixel in the image corresponding to 30 cm to 1.5 m, depending on the distance), it was sufficient to estimate them on topographic maps. Since the target coordinates were not precisely known, GCPs allowed a better picture orientation and were used to evaluate the suitability of the match between the perspective projection of the DEM and the picture (see the details of the match in Figure 4b).

Once the referenced picture had been generated (Figure 4c), the information on the detachment zone was extracted by manual polygon selection for each avalanche, as shown on Figure 4d. From these polygons, values of slope, aspect, elevation, and coordinates were derived using an interactive script written in R [R Core Team, 2013] and based on the R packages "raster" [Hijmans, 2014], "sp" [Pebesma and Bivand, 2005; Bivand et al., 2013], and "insol," this last is an implementation of the algorithms derived by Corripio [2003]. Note that the whole distributions of the aspect and slope within the polygon are available (Figure 4e).

The timing of the event was assessed from the photos with a resolution of 1 day and linked to topographic data. Trying to go below this 1 day resolution is difficult due to the presence of nonexploitable pictures (night or clouds). A single brief cloudless window during the day is sufficient to identify all the releases which occurred previously that day or the preceding night. We collected the statistics of the interevent time between precipitation and release; therefore, only releases during completely foggy days (without precipitation) would result in an uncertain determination of the interevent time.

More problematic is the situation of steep slopes: reduced accumulation before triggering results in releases which can be hidden by subsequent snowfall. However, those potential hidden releases do not bias model calibration. The slope classes we used for calibration consist of events with well-defined slope. A better tracking of the avalanches on steep slopes would simply increase this data set, allowing eventually to add one more slope category. This hypothetical "very steep slope" category would not be relevant for practical purposes: it does not take an elaborate statistical analysis to obtain the synchronization value $P_a \sim 1$ for

Figure 4. Georeferencing process and data extraction: (a) Perspective projection of the DEM (red dots) over the picture. (b) Details of the matching at the edges of a picture. (c) Georeferenced image containing the DEM information and the picture pixels. (d) Topographic data extraction for two avalanche events by polygon selection. (e) Slope and aspect histograms from the detachment zone of an avalanche. (f) Maximal versus average slope values for the polygons corresponding to the detachment zones of each of the 549 avalanche events.

those slopes. Intermediate slopes, such as the one we analyze, are characterized by a significant uncertainty in avalanche release, the quantification of which is a purpose of the paper.

The maximal slope value over the detachment zone polygon was selected after a smoothing of the DEM, performed in order to prevent local extreme slope values. The smoothing was applied by averaging over

Figure 5. Characterization of the observed avalanche events: (a) Delay between detachment and preceding snowfall versus detachment slope. The gray scale gives the snow height before the event. (b) Histograms of the delay between avalanche and preceding snowfall. (c) Histograms of the maximal slope values at the detachment zone.

a seven-cell-wide (i.e., 3.5 m) radius filter (we used the open-source solution SAGA GIS) passed through the DEM. Figure 4f shows the average versus the maximal slope values for all detachment zone polygons after the smoothing. From this figure, it is clear that the average slope over the whole zone does not account for the presence of steeper regions where the release can be initiated.

As a first analysis, we computed the correlation between the slope and the delay between the avalanche events and the preceding snowfall. Despite the size of the data set (549 usable avalanche events over a 6 year record), one sees in Figure 5a that correlation is not readily apparent. This results from the fact that avalanche detachment depends on additional factors. Indeed, as seen in section 2, the complete history of snowfall and avalanches at a given location, together with the stochastic forcing by the temperature determines the probability of detachment. In order to calibrate our model we cannot simply use this correlation and therefore need to reproduce the whole probability distribution of delays between avalanches and the last snowfall.

In Figure 5 we see that the slope distribution has most events concentrated within the range between 35 and 48°. As already mentioned, accumulation is difficult to observe on high slopes due to the immediacy of the release. Furthermore, the distribution has a marked mode for slopes around 40°. This overrepresentation of the modal slopes, associated with the lack of extreme values, makes the analysis of the slope influence on synchronization difficult. In order to select statistically significant subsets we therefore compared only two categories of slope values around the mode of the distribution.

The presence of a certain degree of subjectivity in the determination of the detachment zone is mitigated by the fact that we did not use the area in the calibration of the stochastic model. Since we characterized the release using the maximal slope value taken over the detachment zone, a variation in the drawing of the release polygon only had consequences if a steeper region was captured. However, in order to give to the reader an idea on the type of avalanche events we are considering, we present in Figure 6a the statistics of the release area, which is indeed directly influenced by the subjectivity in the determination of the detachment zone and needs therefore to be considered only for indicative purposes. One readily observes that the distribution of the area is unimodal, with around 60% of the events associated with an area under 100 m². The distribution of the release month is relatively flat, with the exception of the rare events in November and May. Around one quarter of the events occurred during the warmer months of April and May. In addition to the compilation of those statistics we looked for a trend in the detachment area over the winter to highlight possible changes in the nature of the avalanches. Remarkably, we obtained a Pearson correlation coefficient between the month of release and the area of the zone of -0.032 , indicating no significant correlation. It appears therefore that one can observe various release areas at any month of

Figure 6. Characterization of the observed avalanche events in terms of (a) detachment zone area and of (b) release month.

the winter, a fact that is specific to our study slope. This led us to associate the return period with the release depth and not with the release area.

3.3. Meteorological Data

Data from the meteorological station (for its location regarding the study site, see Figure 3) was used in order to estimate the average precipitation magnitude $\alpha = \frac{1}{\gamma}$ and frequency λ . The station provided us with snow height data with a time resolution of 30 min. In addition, recording of the evolution of the snow height between snow events allowed to compute the compaction rate ρ_1 . Figure 7 illustrates the estimation of the meteorological parameters and of the snow compaction rate.

Meteorological data was cross checked with visual information from the pictures: increases in the snow signal of the meteorological station resulting from measurement issues could be excluded. Only significant snowfall (exceeding 10 cm) was kept. As a result, we obtained an average return time of snowfall from the pictures of $\tau_p^{\text{observed}} = 14.0$ days close to the raw result from the meteorological station $\tau_p^{\text{station}} = 14.36$ days. In the computation we aggregated snowfalls separated by less than 10 h.

The snow depth decrease rate ρ_1 was calculated as an exponential decrease over a long period of decay in snow depth values between two snow events (Figure 7). Thirty snow depth decrease periods (the snow depth decreases from h_0 to h_1 over a time t) were used to estimate the average compaction rate as,

$$\rho_1 = \frac{1}{t} \log \frac{h_0}{h_1} \quad (12)$$

Figure 7. Snow height evolution and assessment of the model parameters: return time of snowfall τ_p , snowfall magnitude α , and compaction rate ρ_1 .

resulting in $\rho_1 = 0.0335 \text{ day}^{-1}$. For our analysis and in the absence of detailed local snow data allowing to single out the altitude and exposition dependence, we kept for simplicity the compaction rate $\rho_1 = 0.0335 \text{ day}^{-1}$ constant, as computed at our meteorological station.

3.4. Maximum Likelihood Estimation

As seen in the previous subsection, three model parameters (ρ_1 , α , and $\lambda = 1/\tau_p$) could directly be obtained from meteorological data. In order to determine the remaining model parameter v_1 aggregating the influence of slope and terrain characteristics, we calibrated our model using an empirical statistics related to the timing of avalanches and snowfall. Statistics from the model were generated

Figure 8. Maximum likelihood estimation of the terrain parameter v_1 for (a) slopes lower than 40° , (b) all the avalanches events of our data set, and (c) the slopes above 40° . The likelihood parameter is given by $\ln(L)$ with L as defined in equation (13).

by performing Monte Carlo simulations for a range of values of v_1 . A maximum likelihood estimator was then computed to select the value of the terrain parameter (Figure 8). For general information on the procedure, see Aldrich [1997], Kottegoda and Rosso [2008], and for an application of the method, Steude [2011]. The idea is to compute the likelihood of obtaining a set of n empirical values $\{\tau_i^e\}$ as

$$L = \prod_{i=1}^n f(\{\tau_i^e\}; v_1), \quad (13)$$

with $f(\tau_i^e; v_1)$ the probability of observing the i th empirical value for the model parameter value v_1 . The estimated parameter was chosen in order to maximize (varying v_1) the joint probability L of observing the interevent times $\{\tau_i^e\}$, conditional to the model parameter value v_1 . For practical purposes, one usually works with the logarithm of this quantity in order to obtain additive contributions from the n values of the empirical data set.

The choice of an appropriate empirical quantity τ^e is essential for model calibration. There is an intrinsic difficulty originating for the difference between the model assumption (section 2) of a point process, which does not directly translate to the field situation with avalanche detachments occurring over the whole mountain slope. This can be problematic when comparing quantities like the interevent time between avalanches (introduced in equation (5)). The presence of multiple releases results in an underestimation of the interevent time using field data when computing the time between events occurring at different locations. In order to resolve the issue one could in principle identify and delimit precisely over the mountain face detachment zones corresponding to releases always occurring at the same position (as could be the case in a gully).

This remains, however, difficult due to factors like the wind leading to slightly different positions of the detachment zones. The difficulty is illustrated in Figure 4d where two avalanche detachments separated by only 20 m can be observed. The interevent time $\tau = 2$ days between the two releases should not be considered, even if the two events are close in space, because there was no snowfall between the two avalanches. In order to solve the issue, we chose instead the time between the avalanche and the preceding precipitation introduced in Figure 5, which we denote τ^e hereafter. Since precipitation events occur globally over the

Table 1. Summary of Main Characteristics of the Different Categories of Slopes

Category	Average Slope	Synchronization (Pa)	Terrain Parameter (v_1)	Number of Events
> 40°	43.6°	0.30	$5.8 \cdot 10^{-4}$ (d ⁻¹ cm ⁻¹)	157
< 40°	35.7°	0.16	$2.0 \cdot 10^{-4}$ (d ⁻¹ cm ⁻¹)	392
Total	38.0°	0.20	$2.8 \cdot 10^{-4}$ (d ⁻¹ cm ⁻¹)	549

whole mountain face, the issue of aggregating different point processes is not present in this case, under the assumption that meteorological and terrain characteristics do not vary significantly. Furthermore, the aggregation of events occurring over the whole mountain face increases the size of the empirical set $\{\tau_i^e\}$ used for calibration. While the statistics we chose for calibration allows us in principle to aggregate events occurring at different points (provided slope values are similar), a change in terrain properties (e.g., roughness or concavity) may still result in different release rates for a given slope value.

The estimation of the terrain parameter accounting for the whole distribution of τ^e had to be implemented because the correlation between the snowfall-avalanche delay and the slope was partly hidden by the state dependency of the detachment process (Figure 5). In other words, due to the reduced accumulation on higher slopes, one may counterintuitively observe *punctually* a longer interevent time between avalanches on higher slopes, depending on the details of the sequence of snowfall.

The results of the maximum likelihood estimation are presented in Figure 8. The simulation of the model needs to be much longer than the period of observation of the avalanches. We indeed have to simulate a distribution from the model for delays up to 34 days (the longest empirical delay). The least probable delay observed has a probability of around 1/500, so on average one needs 500 trials to get this delay (recall that we have 549 avalanche events in our data set). There is on average one snow event every 14 days, and 10 to 30% of those events result in an avalanche (depending on the slope). Finally, in order to get the distribution with a sufficient precision, we need around 1000 avalanche events pro delay. Multiplying those factors gives the order of magnitude estimate 10^9 days. More precisely, we confirmed that this duration is sufficient by repeating the maximum likelihood estimation (MLE) and verifying that the estimated parameter remained invariant for the resolution we chose. It appears from Figure 8 that the estimation produces a clear maximum. In addition, the relative smoothness of the curves suggests that the collection of empirical events is sufficiently large.

As shown in Figure 8 and summarized in Table 1, we performed this estimation for three different subsets of the data: the whole data set and events with maximal slope over the detachment zone below and above 40°. We chose the maximal slope to categorize the detachment zones since we obtained stronger correlation with this choice between slope value and slope parameter v_1 than when taking the average slope over the detachment zone. We refrained separating the data set into smaller subsets in order to obtain significant likelihood estimates. We see in Figure 8 that the values are ordered according to the average slope of the subsets considered ($v_1^{<40^\circ} < v_1^{\text{tot}} < v_1^{>40^\circ}$).

Moreover, this ordering can be considered statistically significant: by computing the overlap between the upper and lower likelihood curves in Figure 8, one can reject the null hypothesis $v_1^{<40^\circ} > v_1^{>40^\circ}$ with a p value of 0.036. Correspondingly, we can estimate the relative uncertainty on the likelihood estimation from the curvature of the likelihood curve (for large samples the MLE distribution tends to become normal)

$$\frac{\sigma_{v_1}^{\text{tot}}}{v_1^{\text{tot}}} \approx 32\%. \quad (14)$$

From equation (7) and the analytical formula for the avalanche return period $\bar{\tau}$ presented in Appendix B, we can compute the synchronization Pa from the calibrated model. As a result, the estimations for the terrain parameter correspond to synchronization values $Pa^{<40^\circ} = 0.16$, $Pa^{\text{tot}} = 0.20$, and $Pa^{>40^\circ} = 0.30$ yielding a significant entropy in avalanche prediction from a single snow event. The maximal possible information entropy $H = 1$ is indeed for $Pa = 0.5$. In Figure 9 the entropy corresponding to various values of the terrain parameter can be found. Since the three subsets have a synchronization lower than 0.5, lower slopes show a lower entropy than higher slopes. The synchronization values reflect a partly desynchronized regime, meaning that there is often more than one snowfall between two avalanches at the same location and

Figure 9. (a) Synchronization and (b) entropy values as a function of the terrain parameter for three different compaction rates ρ_1 . The different curves correspond to different compaction rates: $\rho_1^s = 0.0335 \text{ day}^{-1}$ (longer dashes, the value obtained in our field study), $4\rho_1^s$ (shorter dashes), and $\rho_1^s/4$ (solid line). The average return time of precipitation and the average magnitude of snow events are fixed at $\bar{\tau}_p = 14$ days and $\alpha = 48$ cm, respectively. On two curves the value corresponding to the maximum likelihood estimation for the given compaction rate is highlighted (circle and cross).

that the return period of snow events is significantly shorter than the one of avalanches. However, when considering the return period of events of *given magnitude* (i.e., for the avalanches: a given detachment depth and for the precipitation: a given accumulation of fresh snow), one may observe a crossover. From the model, we can compute the return period of events with a given magnitude (formula not reproduced here, see *Perona et al. [2012]*). Using the corresponding formula, we find that above a certain critical value $\tilde{h}$, the return period of avalanche events is smaller than the return period of snow events. When comparing the two slope categories, one obtains from the model with the calibrated values $\tilde{h}^{>40^\circ} = 101$ cm, smaller than $\tilde{h}^{<40^\circ} = 142$ cm. This suggests that the critical magnitude for the crossover between the return period of snow events and avalanches

decreases with increasing slope. The decrease can be explained given that larger accumulation is required on lower slopes for triggering events. The frequency of snowfall with magnitude comparable to those large accumulations is in turn very small.

To discuss possible effects of a varying compaction rate ρ_1 (or of an uncertainty in ρ_1), we further analyze in Figure 9 the influence of the compaction rate on the calibration procedure. A limitation of the approach is indeed the use of a constant ρ_1 for calibration of the model. One sees that the value of the terrain parameter v_1 obtained from the MLE for our empirical events increases with a decreasing compaction rate ρ_1 . This reflects the fact that a lower compaction rate for fixed slope allows for delayed releases, and thus a higher state-dependent release rate is necessary for reproducing the empirical statistics. The effect on the entropy (Figure 9b) differs whether one considers high or low slopes (high and low v_1 , respectively). For low slopes an increase in ρ_1 leads to a decrease in the entropy (by completely precluding the release). Conversely, an increase in ρ_1 can in principle lead to an increase in the entropy for higher slopes (making the process less synchronized). One also notices that the effect on the estimated v_1 is not symmetrical when increasing or decreasing the compaction rate from the values obtained at our meteorological station. On Figure 9, we do not give the estimated value of v_1 for the increased compaction rate: with this compaction rate we cannot reproduce the empirical statistics for realistic values of v_1 (the minimum lies below $10^{-5} \text{ day}^{-1} \text{ cm}^{-1}$).

4. Discussion

We performed a simple statistical analysis on the synchronization between snowfall and avalanches and on the return period of small frequent avalanches in a direction unexplored by previous research. We found that in our study site avalanches occur between two successive snow events in 16% (slopes below 40°) and 30% (slopes above 40°) of the cases. Those values yield an information entropy of 0.64 and 0.88, respectively, to be compared to the maximal information entropy $H_{\max} = 1$. When considering that the average return time of snow events at our study site is 14.4 days, one obtains an interevent time between avalanches of 88 days (slopes below 40°) and 48 days (slopes above 40°). Due to the gradually increasing role of accumulation as the slope decreases, the probability of having a large release depth can be higher than the probability of having an equally large snow event. Beyond the overall synchronization, we computed the critical value $\tilde{h}$ (detachment depth or given accumulation of fresh snow) above which the return period of avalanche events is smaller than the return period of snow events. In the case where the snow depth h is known, our

results provide us with a more precise estimate for the detachment probability than the synchronization value. From the estimations of the terrain parameter ν_1 summarized in Table 1, we can indeed obtain the average daily detachment probability for a known depth of the snow layer as $\nu_1 h$. As a possible follow-up study, the extension of our analysis to other study sites could give an insight into the influence of a change in terrain characteristics (e.g., aspect and roughness) on the daily detachment probability via the terrain parameter ν_1 .

Even if the problem of the synchronization that we tackled is simple in its formulation, finding and exploiting a sufficient data set to quantify the synchronization is not straightforward. It is difficult to assign events to a very precise detachment zone and in turn difficult to compute the interevent time between avalanches for well-defined zones. Furthermore, looking at a single zone, several decades of data collection would be necessary in order to obtain a significant data set (549 avalanche events in this study). Note that the slight differences in the position of the successive releases (Figure 4d) also prevented us from directly estimating the prediction entropy from the data. As a first approach we naturally tried to divide the study slope in order to isolate the different releases. From the resulting tentative statistics of the return time of avalanches we computed the slope-dependent snowfall-avalanche synchronization and the prediction entropy. However, we could not find a partition of the slope where the estimated entropy remained independent upon (slight) change of the domain size. This led us to infer the entropy value from the calibrated parametric model using the fact that the calibration does not rely on singling out exactly the detachment zones. Technically, this is only possible as long as the statistics used for calibration do not involve the comparison of the occurrence time of two local events (for example, the interevent time between two successive avalanches).

In order to maximize the number of avalanche events exploitable for computing the synchronization between snowfall and avalanches, we made use of a theoretical model by *Perona et al.* [2012]. By choosing an appropriate statistics (delay between the avalanche and the preceding snowfall), we were able to compare the point model to an empirical statistics with multiple detachment zones, taking advantage in the process of the increased data set. In this regard, we showed that, in general, it is necessary to compare the interevent time between events occurring globally over the whole study slope (snowfall in our case) and local events (avalanche detachment).

Our approach is essentially limited by the model hypotheses [*Perona et al.*, 2012]: we do not discriminate between different types of avalanches and disregard the possibility of having releases involving only a superficial layer of the snow mantle. The generalization of our approach to ground avalanches is in principle possible using the stochastic framework developed by *Perona et al.* [2012]. In the modeling approach, the detailed history of the deposited snow layers should then be included in the master equation and a probability of transition to past layers introduced. This would, however, require further data for the calibration process in the form of an on-site measurement of the release depth, which, as a consequence, would probably be practicable only for a significantly reduced data set. The forcing by the temperature is included in the state-dependent release rate $\nu(h)$, which does not take in the present form seasonality into account. Clearly, the approach is justified as long as short-term temperature fluctuations drive the release. Precipitation in Valais presents as well a trend over the winter: the average monthly amount of precipitation drops of around one third between December and March. While the results presented in this work provide the synchronization between snowfall and avalanches averaged over the whole winter season, the general temperature and precipitation trends are likely to change the synchronicity over the winter (as examples of possible seasonal effects, see *Keylock* [2003] or *Castebrunet et al.* [2012]). In this direction a possible extension of our approach would be to discriminate between the different types of avalanches (this would as well require on-site monitoring throughout the season) and/or introduce seasonality explicitly in the model by considering a time-dependent detachment rate $\nu(h, t)$, snowfall rate $\lambda(t)$, and average snowfall magnitude $\gamma(t)$. The model assumption of a point process also deserves some discussion. From the modeling point of view, the assumption reflects the established approach for practitioners of using the slope and aspect as principal terrain predictors for avalanche release, with other variables such as terrain concavity/convexity coming in second line. From the point of view of model calibration, the assignment of

a local slope to the detachment zone is not straightforward. Our choice of using the maximal slope over the detachment polygon instead of the average slope was motivated by the stronger correlation obtained with the release rate.

5. Conclusion

Determining the frequency of small avalanches and the synchronization between snowfall and natural avalanches in relation to terrain properties are basic but fundamental problems, which have remained rarely investigated due to difficulties such as the collection of a sufficient number of events to build a statistics or the absence of well-defined avalanche paths. To solve those issues, we presented an extensive avalanche data set and a stochastic model for the coupled snowfall-avalanche dynamics. As mentioned by *Eckert et al.* [2008], statistical methods are indeed unavoidable when investigating long-term avalanche activity. We calibrated the model by maximizing the likelihood of the field observations conditional to the value of a single free parameter. We then used the calibrated model to derive statistical properties, beyond descriptive statistics, of the avalanches in our study site. The return time of avalanche events (dependent or independent of the release depth) was determined together with the snowfall-avalanche synchronization and the prediction entropy for the given slopes. Apart from the purely academic interest in terms of process understanding, the method allows us to evaluate site-specific influences (e.g., slope, aspect, terrain properties, and precipitation regime) on the release probability after snowfall, in the spirit of the empirical rules used by practitioners. Our approach is complementary to deterministic approaches that are more suitable for evaluating the risk (short term) when detailed information, beyond the mere presence/absence of snowfall, is available. Yet it also allows us to go beyond the practical qualitative knowledge that for steeper slopes avalanche release correlates more strongly to precipitation.

While we focused in this work on a case study, the methodology we proposed can readily be applied to study other sites. From the modeling side its simplicity is reflected by the use of an analytical framework. Critically, the only data that are required for calibration can be obtained from standard devices: the time series of snow height, a digital elevation model, and terrestrial photography of the slope. Typically, a webcam from a ski resort would be sufficient for our purposes; since using the method by *Corripio* [2004], it is possible to easily correct for the misalignment of the camera. Leaving the field of snow physics, the calibration procedure we presented for a point model from a spatially distributed empirical statistics can in principle be applied to the modeling of other phenomena such as landslides.

Appendix A

The prediction entropy related to a discrete probability distribution is defined as

$$H = - \sum_i p_i \log_2 p_i. \quad (\text{A1})$$

It was shown by *Shannon* [1948] that it is the only way to define an appropriate measure of the uncertainty associated with a random variable, up to a multiplicative constant (or equivalently up to the choice of the basis of the logarithm). By “appropriate measure” it is meant that the following properties are satisfied:

1. When organizing the possible outcomes in subclasses, the entropy associated with the whole ensemble of outcomes is the sum of the entropy of the subclasses.
2. The entropy is maximal if all the outcomes are equiprobable. In this configuration, the uncertainty associated with the possible outcomes is maximal.
3. The entropy is a continuous function of the variables p_i , symmetric upon permutation of the numbering of the outcomes. In other words, the ordering of the outcomes should not play a role in the definition.

If a certain outcome i is associated with a probability $p_i = 1$, then the definition (A1) yields a value $H = 0$, reflecting the certainty of the outcome.

Appendix B

To obtain the frequency of avalanche events $\bar{\nu} = \frac{1}{\bar{\tau}}$ used for synchronization analysis, we can compute analytically from the model developed in section 2 the average avalanche return time, given the probability density function $p_{\tau}(\tau)$. As a result, we get

$$\bar{\tau} = \int \tau p_{\tau}(\tau) d\tau = \frac{1}{\lambda} e^{-\frac{\lambda \nu_1 t}{\nu_1 + \gamma \rho_1}} \frac{\gamma \rho_1}{\nu_1 - e^{-\rho_1 t} \nu_1 + \gamma \rho_1}^{-\frac{\gamma \lambda}{\nu_1 + \gamma \rho_1}} \left(1 - \frac{e^{\rho_1 t} (\nu_1 + \gamma \rho_1)}{\nu_1} \right)^{-\frac{\gamma \lambda}{\nu_1 + \gamma \rho_1}} \\ \left(-\lambda \left(1 - \frac{e^{\rho_1 t} (\nu_1 + \gamma \rho_1)}{\nu_1} \right)^{\frac{\gamma \lambda}{\nu_1 + \gamma \rho_1}} t + \frac{1}{\lambda - \rho_1^2} e^{\rho_1 t} \lambda^2 \right. \\ \left. H \left[\left(1 - \frac{\lambda}{\rho_1}, 1 - \frac{\lambda}{\rho_1}, 1 - \frac{\gamma \lambda}{\nu_1 + \gamma \rho_1} \right), \left(2 - \frac{\lambda}{\rho_1}, 2 - \frac{\lambda}{\rho_1} \right), \frac{e^{\rho_1 t} (\nu_1 + \gamma \rho_1)}{\nu_1} \right] \right. \\ \left. - H \left[\left(-\frac{\lambda}{\rho_1}, -\frac{\lambda}{\rho_1}, 1 - \frac{\gamma \lambda}{\nu_1 + \gamma \rho_1} \right), \left(1 - \frac{\lambda}{\rho_1}, 1 - \frac{\lambda}{\rho_1} \right), \frac{e^{\rho_1 t} (\nu_1 + \gamma \rho_1)}{\nu_1} \right] \right) \quad (B1)$$

with H the hypergeometric function, following the notation convention from Abramowitz and Stegun [1972].

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