Debris-flow velocities and superelevation in a curved laboratory channel

Christian Scheidl, Brian W. McArdell, and Dieter Rickenmann

Abstract: The vortex equation is often used to estimate the front velocity of debris flows using the lateral slope of the flow surface through a channel bend of a given radius. Here we report on laboratory experiments evaluating the application of the vortex equation to channelized debris flows. Systematic laboratory experiments were conducted in a 8 m long laboratory flume with a roughened bed, semi-circular cross section (top width 17 cm), and two different bend radii (1.0 and 1.5 m) with a common bend angle of 60°, and two channel inclinations (15° and 20°). Four sediment mixtures were used with systematic variations in the amount of fine sediment. In the experiments, 12 kg of water-saturated debris were released in a dam-break fashion, and multiple experiments were conducted to verify the repeatability for a given sediment mixture. Data are available for 69 experimental releases at a channel inclination of 20° and 16 releases at an inclination of 15°. Flow velocity was determined with high-speed video, and flow depth and the lateral inclination of the flow surface (superelevation) were measured using laser sensors. In general, the results from an individual sediment mixture are repeatable. We found that the channel slope as well as centerline radius have a significant influence on the correction factor k used in the vortex equation. Relatively coarse-grained sediment mixtures have larger superelevation angles than finer-grained mixtures. We found a statistically significant relation between the correction factor and Froude number. Correction factors of $1 < k < 5$ were found for supercritical flow conditions. However, for subcritical flow conditions the correction factor shows a larger value as a function of the Froude number, which leads to an adaptation of the forced vortex formula considering active and passive earth pressures. Finally, based on our experimental results, we present a forced vortex equation for debris-flow velocity estimation without a correction factor.

Key words: debris flow, front velocity, superelevation, physical model, forced vortex equation.

Résumé : L'équation de vortex est souvent utilisée pour estimer la vitesse de front de coulées de débris à partir de l'inclinaison latérale de la surface d'écoulement dans un canal au rayon de courbure déterminé. Nous décrivons ici les expériences de laboratoire qui ont permis d'évaluer l'application de l'équation de vortex aux coulées de débris canalisées. Des expériences ont été réalisées de manière rigoureuse en laboratoire au moyen d'un canal expérimental de 8 m de long à fond rugueux et à section transversale semi-circulaire (17 cm de largeur au sommet) formant deux courbes de rayons différents (1,0 et 1,5 m) et comportant une pente de 60° et deux inclinaisons différentes (de 15° et 20°). On a utilisé quatre mélanges sédimentaires en faisant varier régulièrement la quantité de sédiments fins. Les expériences réalisées ont consisté à déverser d'un seul coup dans le canal 12 kg de débris saturés en eau et ont été répétés plusieurs fois afin de vérifier la répétitivité obtenue pour un mélange sédimentaire donné. On dispose des données de 69 expériences de déversement réalisées avec un degré d'inclinaison du canal de 20° et de 16 expériences effectuées avec un degré d'inclinaison de 15°. La vitesse d'écoulement a été déterminée à l'aide d'une caméra vidéo à grande vitesse et la profondeur de l'écoulement et l'inclinaison latérale de la surface d'écoulement ont été mesurées au moyen des capteurs laser. D'une manière générale, les résultats obtenus pour un mélange sédimentaire en particulier sont répétables. Nous avons constaté que le degré d'inclinaison du canal et les rayons de courbure influent beaucoup sur le facteur de correction k utilisé dans l'équation de vortex. Par ailleurs, les angles d'inclinaison latérale sont plus élevés dans le cas des mélanges sédimentaires à grains relativement gros que dans celui des mélanges à grains plus petits. Nous avons en outre découvert une corrélation importante d'un point de vue statistique entre le facteur de correction et le nombre de Froude. Des valeurs du facteur de correction k supérieures à 1 et inférieures à 5 ont également été observées dans le cas d'écoulements supercritiques. Cependant, dans ce dernier cas, le facteur de correction prend une valeur élevée, fonction du nombre de Froude, ce qui implique une adaptation de l'équation de vortex forcé qui tienne compte des pressions active et passive des terres. Enfin, en nous basant sur les résultats de nos expériences, nous présentons une équation de vortex forcé permettant de calculer la vitesse d'écoulement de débris sans utiliser de facteur de correction. [Traduit par la Rédaction]

Mots-clés : coulée de débris, vitesse de front de coulée, inclinaison latérale, modèle physique, équation de vortex forcé.

Received 4 March 2014. Accepted 28 July 2014.

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A correction was made to the e-First version of this paper on 27 January 2015 prior to the final issue publication. The current online and print versions are identical and both contain the correction.

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Introduction

Gravitational mass-movements like debris flows endanger human settlements all over the world, especially in densely populated mountainous regions, where people are challenged to find a balance between spatial development and potential hazards. This approach, integral risk management, requires fundamental knowledge of the expected natural hazardous processes, and their spatial extent and intensities. The estimation of flow parameters like discharge, flow velocity or flow height are considered to be essential (Guzzetti 2000; Lin et al. 2002; Glade and Crozier 2005; Fuchs et al. 2008; Jakob and Friele 2010). Estimates of debris-flow characteristics are often based on post-event field investigations, and may be related to a certain cross section in the channelized transit zone of a torrent channel. If no measurements or observations are available, only flow marks on banks after a debris-flow event allow for the estimation of the parameters (such as flow velocity, flow area or flow height). Normally only the flow area at a given cross section can be directly measured in the field. Flow velocity, in particular, is an important parameter for designing mitigation structures (Armanini 1997; Bugnion et al. 2012; Scheidl et al. 2013), as well as for simulating the runoff on the fan (Revellino et al. 2004; Chen et al. 2007; Hürlimann et al. 2008). Costa (1984), Rickenmann (1999), and Prochaska et al. (2008) give a sound overview of methods used to estimate debris-flow velocities. In general, these methods are based on empirical or theoretical concepts, and should be verified by field observations of mean or front velocity (Zhang 1993; Berti et al. 1999; Suwa and Yamakoshi 2000; Marchi et al. 2002; Hürlimann et al. 2003, 2013), or by measured velocity profiles in laboratory flows (Kaitna et al. 2014).

A frequently applied method to back-calculate the velocity of a debris-flow event is based on superelevation information from deposited material (Mizuyama and Uehara 1981; Iverson et al. 1994; Bulmer et al. 2002; Bertolo and Wiczorek 2005). Superelevation can be observed in curved channels, where the flow height along the inner curve is lower than the flow height along the outer curve, caused by the centrifugal acceleration of the flow. This semi-empirical method is described as a vortex or forced vortex method (Apmann 1973; Costa 1984; Hungr et al. 1984; Chow 1988; McClung 2001; Prochaska et al. 2008). This approach assumes a constant radius, determined from the center line of the channel bend.

Based on worldwide data from debris-flow events as well as laboratory results, Rickenmann (1999) analysed flow-resistance relations to estimate the velocity of the frontal part of debris flows. When applying those approaches to debris-flow events he found a similar scatter of velocity estimates compared to estimates of clear water flows. This vortex approach, originally developed for pure water flows, might therefore also be applied to debris-flow events, although the theoretical background (regarding rheological characteristics of debris flows) has not yet been clarified.

Only a few attempts have been made to analyse the vortex phenomena for debris flows. Observations from debris-flow monitoring stations, like at Lattenbach (Arai et al. 2013), Illgraben (Rickenmann et al. 2001; Hürlimann et al. 2003), the Moscardo torrent (Marchi et al. 2002) or Chalk Cliffs (McCoy et al. 2011), lack superelevation data. Mizuyama and Uehara (1981) as well as Ikeya and Uehara (1982) first analysed the application of the forced vortex equation for debris-flow velocity estimates, based on the results of small-scale laboratory experiments. In large-scale experiments at the United States Geological Survey (USGS) debris-flow flume, Iverson et al. (1994) compared measured velocities in channel curves with radii (R) of 4 m and 10 m with estimates using the vortex equation (eq. (7) with $k=1$). They found that the vortex approach underestimates maximum speed by almost 30%, and that assumptions of the effective channel width (B) greatly influenced their results, yielding less biased estimates when using B as the observed channel width (defined by the flow

traces) instead of the spacing between the channel side walls. However, the short-radius (4 m) curve produced flow behavior that was intermediate between superelevation and runoff. The long-radius (10 m) curve produced classical superelevation (R.M. Iverson, personal communication, 2015).

In this article we analyze debris-flow velocities obtained in small-scale experiments in a curved laboratory channel. The main objective of this work is to study the influence of channel radius and material properties on the applicability of the forced vortex equation to debris flows. After an overview of vortex equations to estimate debris-flow velocities, a detailed description of the laboratory channel and the experimental conditions are given. Measured velocities and superelevation data for all experiments are presented and discussed.

Background

Velocity estimation based on the forced vortex approach

For Newtonian fluids, a characteristic of open channel flow in a bend with a radius R is a deformation of the free surface due to the acting centrifugal force, F_c , with a given mass, m

$$(1) \quad F_c = m \frac{v^2}{R}$$

where v is the flow velocity.

Specifying the declination of the free surface by the superelevation angle β , eq. (6) takes the general form, valid for all cross-sectional shapes,

$$(2) \quad v = (R_c g \tan \beta)^{0.5}$$

where R_c is the centerline radius of the bend, g is the acceleration due to gravity, and β is related to the superelevation $\Delta h = y_2 - y_1$ (where y_2 and y_1 are the depths at the inner and outer boundaries of the channel, respectively) and the observed surface width of the flow B (Fig. 1).

$$(3) \quad \tan \beta = \frac{\Delta h}{B}$$

Combining eqs. (2) and (3) gives the most commonly used form, described by Henderson (1992) and Chow (1988), among others, for clear water flows

$$(4) \quad v = \left(\frac{R_c g}{B} \Delta h \right)^{0.5}$$

Assuming that v is uniform across the channel and R_c exceeds the channel width, a consideration of horizontal forces in the cross section requires a balance of centrifugal force and hydrostatic force (per unit length of channel)

$$(5) \quad m \frac{v^2}{R_c} = \frac{1}{2} \rho g (y_2^2 - y_1^2)$$

where ρ is the fluid density. Equation (5) can be rewritten for a known cross section

$$(6) \quad v = \left[\frac{R_c g}{2A} (y_2^2 - y_1^2) \right]^{0.5}$$

where A denotes the wetted cross-section area.

Fig. 1. Definition of the geomorphometric parameters for the forced vortex equations. Flow direction is toward the reader.

For rectangular cross sections with $A = (B/2)(y_2 + y_1)$, eq. (6) gives identical results compared to eq. (4). For the assumption of a triangular cross section, with a reduced flow area and different horizontal hydrostatic forces, velocity estimations are smaller by a factor of $\sqrt{2}$ for our experimental conditions, compared to the rectangular cross section assumption. However, the influence of the cross section shape for the derivation of the forced vortex equation based on a force balance approach, is not yet fully clarified and is beyond the scope of this publication.

Forced vortex equation adapted to debris flows

Equation (4) was modified and applied to debris flows by introducing a correction factor k to estimate debris-flow velocities (e.g., Hungr et al. 1984; Chen 1987; Bulmer et al. 2002; Prochaska et al. 2008)

$$(7) \quad v_{df} = \left(\frac{R_c g^*}{Bk} \Delta h \right)^{0.5}$$

For application to mountain slopes, as it is considered for eq. (7), it is necessary to correct for the downslope component of the gravity vector. Therefore, the slope normal component of gravity (g^*) is used, when considering the pressure in the force balance on a plane normal to the flow direction (McClung 2001). Johnson and Rodine (1984) consider g^* to be relevant for channel slopes exceeding approximately 15° . They further point out two assumptions in the derivation of eqs. (4) and (7). One is that the surface in the cross section is normal to the direction of acceleration to which the debris (or water) flows. The other is that if the correction factor $k = 1$, eq. (7) equals eq. (2) for clear water flows. Then the debris is assumed to be a perfect fluid (e.g., Muñoz-Salinas et al. 2007), which is approximately valid for flows in large channels for flowing debris with low shear strength (e.g., lahars). If θ_c is the angle of the channel, $g^* = g \cos \theta_c$. Transforming eq. (7), the correction factor (k) can then be expressed as

$$(8) \quad k = \frac{R_c g \cos \theta_c \Delta h}{B v_{df}^2}$$

Procter (2012) relates k to the vertical velocity distribution within the flow, and therefore to the Reynolds number (R_c) and

the momentum correction coefficient (κ_m). Whilst R_c describes the degree of turbulence in the flow, κ_m accounts for the viscous drag forces making the velocity lower near the solid boundaries than at a distance from them. For a uniform velocity profile, as assumed in turbulence flows, the momentum correction coefficient has a value of unity (≈ 1). The further the flow departs from uniform, the greater the value of the coefficient becomes (> 1) (Henderson 1992). Accounting for the viscosity and the boundary effects in bends for debris flows, it is therefore reasonable to assume that the correction factor k can only be ≥ 1 .

However, several studies comparing experimental or observed superelevation data with estimated velocities suggest a wide range of values for the correction factor k . Hungr et al. (1984) recommend $k = 2.5$ to calculate the velocity from superelevation data because Mizuyama and Uehara (1981) as well as Ikeya and Uehara (1982) showed with experiments and detailed field observations that the actual superelevation Δh may be 2.5–5 times larger than predicted with eq. (7) and $k = 1$. Based on observed superelevation data, Bulmer et al. (2002) estimated the flow velocity of the Generals Slide debris-flow event (Virginia, USA) and compared the results with observed velocity values. They found that debris-flow velocities are best described with $2 < k < 10$. Chen (1987) reported that k may be as large as 10, and that its value may depend on bend geometry and debris-flow material properties.

VanDine (1996) relates the correction factor k not only to viscosity, but also to vertical sorting that exists for coarse-grained debris mixtures, where k varies between 1 and 5.

Consideration of earth pressure

The derivation of the vortex eqs. (4) and (7) assumes a hydrostatic approach, where the bed-normal stress (σ_n) equals the bed-lateral stress (σ_l) of the flowing mass. However, the hydrostatic pressure distribution may be unrealistic when dealing with the flow of granular material that has internal strength due to its frictional nature (Savage and Hutter 1989). We therefore hypothesize that much of the observed variability in the correction factor for debris flows can be described by nonhydrostatic pressure distributions in granular material.

Savage and Hutter (1989), Hungr (1995), Iverson (1997), Iverson and Denlinger (2001), and others applied Rankine earth pressure theory to describe the constitutive stress evolution of debris flows

by introducing a proportional ratio ($K_{a/p}$) between the bed-lateral stress (σ_l) and bed-normal stress (σ_n).

$$(9) \quad \sigma_l = K_{a/p} \sigma_n$$

For the hydrostatic approach, $K_{a/p} = 1$ and the forces acting at the inner and outer sides of the curve equal the weight of the overlaying fluid. If we consider earth pressure, $K_{a/p}$ has different values depending on whether the flow is actively extending (active earth pressure, K_a) or passively compressing (passive earth pressure, K_p) (Iverson 1997).

A first approach, accounting for the internal strength of flowing granular material around curved channels bends, was derived by McClung (2001) for snow avalanches. His vortex equation considers a passive earth pressure coefficient describing the compression of material due to the centrifugal force at the outer side of the curve. If we treat the experiments as single-phase flows we can extend eq. (6) to consider earth pressure effects, and thereby define the velocity $v_{df_{ep}}$ by assuming an active earth pressure ($\rho g y_1 K_a$) acting at the inner curve and a passive earth pressure ($\rho g y_2 K_p$) acting at the outer curve

$$(10) \quad v_{df_{ep}} = \left[\frac{R_c g}{2A} (K_a y_1^2 - K_p y_2^2) \right]^{0.5}$$

With a superelevation $\Delta h = y_2 - y_1$ and assuming a rectangular cross section of width B , eq. (10) can be rewritten as

$$(11) \quad v_{df_{ep}} = \left\{ \frac{R_c g^*}{B} \Delta h \left[K_p + (K_p - K_a) \left(\frac{y_2^2}{y_2^2 - y_1^2} - 1 \right) \right] \right\}^{0.5}$$

or

$$(12) \quad v_{df_{ep}} = \left(\frac{R_c g^*}{B} \Delta h \frac{1}{k_{ep}} \right)^{0.5}$$

introducing k_{ep} , a correction factor based on the earth pressure, as well as uninundated flow heights on the inner and outer sides of the curve

$$(13) \quad k_{ep} = \left[K_p + (K_p - K_a) \left(\frac{y_2^2}{y_2^2 - y_1^2} - 1 \right) \right]^{-1}$$

It should be noted that eqs. (10)–(13) are based on a force balance approach and on the assumption of a rectangular cross section.

Methods

Experimental device

The experiments were conducted at the Swiss Federal Institute WSL, Birmensdorf. The channel has a reservoir at the start, an acceleration section, and a semicircular cross section flume measurement section (Fig. 2). The physical model used in this study assumes geometric similarity for a typical debris-flow reach, which we defined, based on documentations of superelevation data of debris-flow events by Johnson and Rodine (1984), Jackson et al. (1987), Jordan (1994), Jakob et al. (1997), and Bertolo and Wiczorek (2005).

The reservoir is outfitted with a door which can be rapidly opened to initiate a dam-break-like release of the sediment mixture. The reservoir empties into a 1.0 m long rectangular cross section acceleration section, which smoothly transitions to the main flume. The flume section consists of a flexible plastic half-

pipe with a diameter of 0.17 m mounted on a wooden supporting construction. To account for a uniform channel slope, the supporting construction can be displaced in all spatial directions. The flume section is about 8 m long and consists of an upper bend with a radius of $R_u = 1.5$ m and a lower bend with a radius of $R_l = 1.0$ m. The aperture angle, δ , of both bend sections was held constant for all experiments at 60° . The upper bend starts approximately 2.0 m from the end of the acceleration section and is followed by a 2.5 m long straight flume segment before the start of the lower bend. The surface of the flume is covered with 40-grit silicon carbide sandpaper providing a uniform basal friction layer. The slope of the flume along the channel centerline was held constant at 20° or 15° .

Experimental debris mixtures

The experimental sediment or debris mixtures are similar in composition to those used for the impact experiments described in Scheidl et al. (2013). To establish the variability of the debris-flow process, four different realistic debris mixtures (grain-size distributions) were defined. These mixtures are based on combinations of loam (0.0002–0.1 mm) and crushed stone (0.1–11 mm), mixed with water. The variability of the grain-size distributions found in nature is quite large (Fig. 3; VAW 1992), and a comprehensive analysis of all sediment mixtures is beyond the scope of this study. The sediment mixtures used in our experiments are roughly comparable to sediment mixtures used by others (Major 1997; D'Agostino et al. 2010; Scheidl et al. 2013; Fig. 3).

The total bulk mass for each experiment was kept constant at 12 kg (Table 1). The maximum grain diameter of 11 mm was defined considering the maximum potential flow depth within the flume (≈ 90 mm).

Mixtures A, B, and C are composed of 60% sediment and 40% water by volume including fine particles (loam) that consist of 26% clay, 60% silt, and 14% sand. In contrast, mixture D consists only of noncohesive material. Here, we substituted the finer particles with additional water to achieve a constant mass. The clay fraction counteracts phase separation so no segregation of fluid and solid phases was observed for mixtures A, B, and C. Some phase separation (described below) was observed for mixture D.

Velocity and superelevation data

Identical measuring devices were installed in the upper and lower curvatures to document superelevation and front velocity. To constrain the discharge for comparison with velocity and depth measurements, a load cell measuring weight at 1 kHz was mounted at the outlet of the flume; however, the results are not described herein.

In each bend section, a high-speed camera (monochrome, 120 frames per second, 800×600 pixels) records the passage of debris flows within at a certain section of the flume. For post-processing all videos were overlaid with a grid, situated upstream of the laser devices (Fig. 4). A mean velocity of each experimental debris flow (v_{obs}) was then estimated using the travel distance of the debris-flow front and the video image sequence.

Three laser sensors centered on a channel cross section were situated at 20° from the downstream end of each bend (Fig. 4) to measure the flow depth to determine the superelevation. They were installed normal to the channel centerline with a lateral separation of <0.9 mm (Fig. 5). The sensors were sampled at 1 kHz and have a precision of ± 0.2 mm, which is smaller than the median grain size of all sediment mixtures reported in Table 1. By assuming a linear flow surface in the bends, we approximated the superelevation angle of the experiments (β) with a best fit straight line defined by the flow height measurements in the upper and lower bend sections (Fig. 5). The relevant superelevation for further analysis (Δh) is then derived from the superelevation angle estimated at the moment of the maximum cross section area. Prior to this setup we tried several different methods. Because

Fig. 2. (a) Top view of flume construction with laser devices, mounted at the upper and lower bends, with a channel slope of 20°. (b) Reservoir (door closed) and acceleration section.

Fig. 3. Ternary plot of grain-size distributions of source debris used in this study compared with selected grain-size distributions of debris-flow events in Switzerland (VAW 1992), mixtures used for large-scale experiments (Major 1997), as well as laboratory mixtures used by Scheidl et al. (2013) and D’Agostino et al. (2010).

Table 1. Characteristics of the experimental mixtures used for this study.

Mixture	d_{30} (mm)	d_{50} (mm)	d_{90} (mm)	W (vol.%)	ρ_b (kg/m ³)	Total mass (kg)
A (fine)	0.1	0.8	4.9	0.4	1484	12
B (fine)	0.2	1.2	5.9	0.4	1484	12
C (fine)	0.4	1.8	6.9	0.4	1484	12
D (coarse)	0.6	3.6	6.8	0.5	1399	12

Note: d_{30} , d_{50} , and d_{90} , soil particle diameter at which 30%, 50%, and 90% of the mass of a soil specimen is finer, respectively; W (vol. fraction), water content in percent per volume; ρ_b , bulk density.

superelevation of clear water flows varies with angular distance in the bend due to acceleration of the fluid in entering and leaving the curve and of the varying curvature of streamlines through the bend (Apmann 1973), we first evaluated several different methods to observe superelevation within a larger region of the bend section. First we tested the high speed cameras, used for velocity

estimation, but the superelevation (Δh) could not be measured accurately because of the low resolution of the video images. Subsequent tests with a 3D camera (PMD[vision] CamCube 3.0) showed that the device seems to be unsuitable to record images of distinct concave obstacles like the semi-circular shape of the flume. After additional tests we observed that the maximum superelevation occurred near the end of each bend section and at the flow front. This observation is confirmed by the results of (Mizuyama and Uehara 1981) for similar bend radii.

Superelevation (Δh) and the horizontal flow width (B) were calculated using eq. (3). Assuming predominant inertial and gravitational forces, we further classified our experiments by the Froude number (F_r), which is based on the observed flow velocity. For each experiment, F_r was determined by relating v_{obs} to the cross section average flow height h

(14)
$$F_r = \frac{v_{obs}}{\sqrt{gh}}$$

Fig. 4. Images from high-speed cameras showing scale and laser devices installed near the end of each bend: (a) upper bend section; (b) lower bend section with balance construction.

Fig. 5. Illustration of how geomorphometric parameters for forced vortex eqs. (6), (2), and (4) are estimated for this study. Direction of flow is toward the reader. (a) Top-view situation. (b) Situation of the laser measurement device (sketch) and parameters used for eqs. (4), (7), and (8). Stars schematically show flow heights measured by laser devices L1, L2, and L3, situated in the upper bend, as well as L6, L7, and L8 situated in the lower bend section. IP, inner point of best-fit straight line — approximating a linear flow surface. (c) Parameters used in eqs. (6), (10), (11), (12), (14), and (15).

with

$$(15) \quad h = 2r \sin^2\left(\frac{\alpha}{4}\right)$$

where α is the angle of center related to flow depth (Fig. 5).

Results

More than 150 tests were performed to acquire at least 10 repetitions for each identical setup (i.e., the same channel inclination, mixture setup, and bend radius). Unfortunately, limited numbers of useful results were observed with a slope inclination of 15°. No experiments could be performed at 15° for mixtures C and D, which might be caused by the higher internal friction angles compared to mixtures A and B. Experiments with mixtures A and B could be conducted at 15°, but only negligibly small superelevation ($\Delta h < 0.05$ mm) were observed at the upper bend section ($R_u = 1.5$ m). Here the internal resistant stress of the flowing mass seems

to exceed the acting centrifugal force. Due to the spatial limitations of the laboratory no experiments could be conducted at steeper channel slopes.

In total, 85 experiments could be used for further analyses. Sixty-nine experiments were conducted using the maximum channel inclination of 20°. The other 16 experiments could be performed at a channel inclination of 15°. In total, we performed 32 experiments for the 1.5 m bend radius and 53 experiments for the 1.0 m bend radius.

For the first step, we back-calculated the correction factor (k^*) for each experiment based on eq. (8) and substituting v_{df} with v_{obs}

$$(16) \quad k^* = \frac{R_c g \cos \theta_c \Delta h}{B v_{obs}^2}$$

Table 2 summarizes the mean values and standard deviations of observed velocities (v_{obs}), gradients of the best fit superelevation

Table 2. Mean values and standard deviations of selected parameters, estimated for different experimental mixtures at 20° and 15° slope inclinations.

θ_c (°)	Mixture	R_c (m)	v_{obs} (m/s)	$\tan\beta$	A ($\times 10^{-4}$ m ²)	k^*	F_r	N
20°	A	1.0	0.81±0.20	0.28±0.09	63±15	4.3±1.1	1.1±0.3	11
		1.5	1.49±0.19	0.33±0.05	59±7	2.1±0.6	2.1±0.3	8
	B	1.0	1.00±0.19	0.34±0.08	63±10	3.3±1.4	1.4±0.3	9
		1.5	1.67±0.17	0.31±0.07	60±13	1.5±0.3	2.3±0.3	8
	C	1.0	0.88±0.18	0.26±0.06	77±7	3.1±1.0	1.1±0.3	9
		1.5	1.71±0.17	0.30±0.05	67±7	1.4±0.1	2.3±0.4	7
	D	1.0	1.38±0.12	0.44±0.01	57±2	2.1±0.3	2.0±0.2	9
		1.5	2.12±0.16	0.43±0.02	57±3	1.3±0.2	3.0±0.2	8
15°	A	1.0	0.47±0.09	0.19±0.03	48±11	8.9±3.8	0.7±0.1	7
		1.5	1.11	0.13	38	1.6	1.8	1
	B	1.0	0.58±0.05	0.19±0.06	57±6	5.6±1.7	0.8±0.1	8

Note: N , number of repetitive experiments.

Fig. 6. Boxplots of observed correction factors grouped by (a) flume slope and (b) bend radii.

Fig. 7. (a) Boxplot of pairwise comparison between correction factor and mixtures. (b) Results of Tukey's range test — only confidence interval of mixture comparison C-A and D-A shows a deviance in their mean levels from 0 (indicated by dashed line).

angle ($\tan\beta$), back-calculated correction factors (k^*), and Froude numbers (F_r) for all experiments with different mixtures, conducted at both 20° and 15° slopes.

Post-hoc analysis

Our results show a wide range for correction factor k^* , as proposed in other studies. Figures 6a, 6b, and 7 show boxplots of k^*

grouped by flume slope, centerline radii ($R_{u/l}$), and debris mixture. In Fig. 8 we compare the gradient of the superelevation angle ($\tan\beta$) observed at the maximum cross section area of the flow for different mixtures.

The flume slope θ_c as well as centerline radii, $R_{u/l}$, are correlated with the correction factor k^* (Figs. 6a, 6b), suggesting that these factors are worth further investigation. A one-sided Student's

Fig. 8. (a) Boxplot of pairwise comparison between gradient of superelevation angle and mixtures. (b) Results of Tukey's range test — only confidence interval of mixtures paired with D shows a deviance in their mean levels from 0 (indicated by dashed line).

t-test with a confidence interval (CI) of 95% confirms the hypotheses that the correction factors observed in the lower bend section ($R_l = 1.0$ m) are significantly greater than correction factors observed in the upper bend section ($R_u = 1.5$ m). Similarly, the correction factor tends to decrease with a steeper channel slope of the flume.

A clear influence of the mixtures (A, B, C, and D) on the back-calculated correction factor k^* is not a priori evident when looking at the boxplots in Fig. 7; however, a Tukey's range test (CI = 95%) shows a significant difference between k^* for mixture A, with the highest amount of fine particles (loam), compared to mixtures C and D, which contain less fine material. This would confirm the assumption that the correction factor is related to viscosity; however, no differences appear when comparing the granular mixture D with the experimental mixtures B and C, which both contain a lower amount of fine particles than mixture A (cf. Fig. 3 and Table 1). A significant difference between the finer mixtures A, B, and C, and the granular mixture D can be shown when compared to the measured superelevation angles ($\tan\beta$). A small trend can be observed of increasing superelevation angles with decreasing fine material (Fig. 8). This is again confirmed by a Tukey test of multiple comparisons of means with a confidence interval of 95%, showing that the granular mixture D differs markedly from all other mixtures.

Experimental findings

Apparently the correction factor, k^* , in the forced vortex equation depends to some extent on the geometric boundaries of the bend, and on the material composition (texture, water content) or rheological characteristics of the debris-flow mixture. If debris-flow velocities are estimated by means of the vortex equation, it is expected that similar flow conditions (slope, curvature, mixtures) should result in similar velocity estimates (eq. (7)) or back-calculated correction factors, k^* (eq. (11)). However, a nonlinear relationship between calculated correction factors, k^* , and observed Froude numbers, F_r , for all 85 debris-flow experiments is apparent (Fig. 9). For supercritical flow conditions and increasing Froude numbers, the correction factors of our experiments converge to $k^* \rightarrow 1$. For subcritical flow and decreasing Froude numbers, back-calculated correction factors of our experiments increase exponentially and result in larger deviations from the proposed regression model.

The regression model in Fig. 9 is based on a robust nonlinear least square function to achieve equal variance of the dependent variable (k^*) across the data and an approximately symmetric dis-

tribution of the residuals of the fitted curve versus k^* . The error bars in Fig. 9 show that the back-calculated correction factor k^* can be quite feasibly explained by the regression model for the supercritical flow regime (Froude > 1). However, the residual plot within Fig. 9 also shows increasing heteroscedasticity of experiments with Froude numbers < 1 , indicating that more uncertainty is present in the power law function of the regression model. Heteroscedasticity of the residuals gives the least information about where the true regression line is. Because the variance of the residuals varies significantly in the subcritical regime, we assume a change in flow behaviour — yielding to the assumption that nonhydrostatic pressure distribution might be the dominant stress condition of the bulk mixture acting in bends.

We therefore tested an adaption of the forced vortex formula by considering active and passive earth pressure, including a correction factor k_{ep} as proposed in eq. (13). Following this approach, the active earth pressure of the flow (K_a) acts on the inner side of the channel curve, whereas the passive earth pressure of the flow (K_p), which might exceed the hydrostatic pressure, acts on the outer side of the curve. For this situation, as indicated by eq. (13), the passive earth pressure coefficient K_p has a considerable influence on the calculation of the correction factor k_{ep} . Exact values of K_a and K_p are difficult to specify for our experiments. We therefore assumed that K_a and K_p may vary in a range which was assumed by Hungr (1995) for numerical 1-d modelling of debris-flow propagation. He proposed that the two coefficients may range between the Rankine active value of 0.2 and passive value of 5.0. We further assumed that the passive earth pressure coefficient (K_p) might vary as a function of the Froude number for each experiment, similar to the power law relation in Fig. 9, and we assume the active earth pressure coefficient to have a constant value of $K_a = 0.5$. The calculation of K_p is then defined by

$$(17) \quad K_p = c_p F_r^{-1.19}$$

with a constant proportionality factor, c_p . Based on eq. (17), Table 3 lists passive earth pressure mean coefficients and their standard deviations with varying proportionality factors (c_p). Figure 10 compares estimated k_{ep} factors, based on eq. (13) and the presumed c_p values listed in Table 3, with back-calculated k^* factors, by means of eq. (16). The trend curves are again based on a robust nonlinear regression fit. Although k_{ep} values are estimated on the assumption of a rectangular cross section, Fig. 10 shows a reasonable

Fig. 9. Relation between back-calculated correction factors, k^* , and Froude numbers, F_r , for all experiments of this study (empty circles) as well as documented debris-flow events from Bulmer et al. (2002) with $F_r < 4$ (diamonds). Furthermore, mean values and standard deviations are indicated for all experimental mixtures (A–D), slopes (20° , 15°), and centerline radii (1.0 and 1.5 m) (cf. Table 2). Horizontal dashed line shows a constant relation of $k^* = 1$ with the Froude number as expected for clear water flows. Inset plot shows residuals versus predicted values for k^* . Heteroscedasticity increases for experiments with Froude numbers < 1 .

Table 3. Mean values and standard deviations of earth pressure coefficients based on eq. (17) with c_p values ranging from 2 to 5.

c_p values	$\overline{K_p}$
2	1.46 ± 0.87
3	2.19 ± 1.30
4	2.92 ± 1.74
5	3.65 ± 2.17

agreement between estimated k_{ep} factors and back-calculated k^* factors for c_p values ranging between 4 and 5. Regarding Table 3, these c_p values are in accordance with K_p values as assumed by Hungr (1995).

Discussion

Scaling consideration

All experimental runs are based on kinematic similarity to prototype conditions of a real scale debris flow, realized by the model’s channel inclination, bed roughness, and defined sediment mixtures. Full dynamic similarity of all forces acting in nature and the model, however, could not be achieved for our experiments, and is not possible by using the same fluid with the same viscosity (Iverson 1997). This is a crucial point in modelling debris-flow phenomena in a small-scale approach, because dimensional analyses of debris flows consider the similarity of fluid-slurry dynamic viscosity, among other variables, significant to the debris-flow phenomenon (Davies 1994; Iverson 1997). Data on

Fig. 10. Relation between estimated correction factors k_{ep} with plausible c_p values (Table 3), and back-calculated correction factors k^* for all experiments.

Fig. 11. View of upper bend section showing flow front of three experimental debris flows: (a) mixture A, (b) mixture B, and (c) mixture C. Dashed line indicates a schematically drawn phreatic line between unsaturated and saturated part of flow.

viscosities of debris-flow events vary substantially in nature and have yet to be verified (e.g., [Tecca et al. 2003](#); [Cui et al. 2005](#)). Also, data of superlevation of debris-flow events with independently measured flow velocities (not based on a vortex approach) are rarely available. However, the field data of [Bulmer et al. \(2002\)](#) suggests that the flume results can be applied to the field ([Fig. 9](#)), despite the fact that the experiments do not fully reproduce dynamic similarity.

Qualitative assessment of the experiments

As indicated in the section titled “Scaling consideration”, full similarity between nature and model could not be achieved by the experimental setup. However, if measured flow velocities as well as flow heights of the experimental debris flows are scaled by Froude similarity, they are well in accordance with observed debris-flow events in nature. We further observed some typical phenomenological characteristics of real debris-flow events for our experimental debris flows.

All four sediment mixtures produced one single wave, whose front velocity was measured and considered for further analyses. We also identified a typical longitudinal debris-flow profile, as described for instance by [Pierson \(1986\)](#), for all experimental runs. It is characterized by an unsaturated boulder front and suspended particles in the fully liquefied tail of the flow. For experiments of mixtures A, B, and C the phreatic line, delineating the unsaturated front from the saturated tail (e.g., [Scotton and Deganutti 1997](#)), could be visually identified, showing longitudinal sorting with an accumulation of large particles at the front of the flow for the experimental mixtures ([Fig. 11](#)).

We also observed a difference between the surface velocity of the unsaturated front, which was approximately equal to the front velocity of the experimental debris flow, and the surface velocity of the liquefied tail. This difference in surface velocities became remarkable as soon as the experimental debris flow entered the bend section. Here the resistance to flow seems to increase due to shape drag ([Morvan et al. 2008](#)), which first affected the unsaturated debris-flow front. At this situation the liquefied tail showed higher inertia and appeared to push the frontal part of the flowing mass. However, the liquefied tail never overtook the unsaturated frontal part. Observa-

tions suggest that the surface velocities of the front and the liquefied tail become similar at the downstream end of the curved section.

The theoretical energy loss of water flow in a bend can be described after [Hager and Schleiss \(2009\)](#) with a coefficient ξ applied to the velocity head as

$$(18) \quad \xi = \frac{2\sqrt{2} \sin \delta / 2}{(1 + 2R_c/B)^2}$$

Although estimated ξ values for all experiments are very small, on average 0.0051 ± 0.0019 , our data show a significant decrease in the front velocity within the first part of the curved flume sections ([Fig. 12](#)). Considering ϕ , the ratio between superlevation and velocity head,

$$(19) \quad \phi = \frac{\Delta h}{2} \left(\frac{v^2}{2g} \right)$$

we get ϕ values >1 — exceeding the principal limit of 1 for pure water flow in bends — for experiments with $F_r \lesssim 1$. This suggests that a large part of the fluid energy might be transferred to super-elevation for experiments with $F_r \lesssim 1$.

Unfortunately, the loss of flow energy within the bend reach could not be measured accurately over the entire bend section because of the low resolution of the video images and the small observation area. The data in [Fig. 12](#) is therefore based on particle tracking analyses from eight experiments (one for each mixture setup and bend radii) for a channel inclination of 20° . Furthermore, the velocities could only be analyzed to the onset of the curved flume section (i.e., the first 10–70 cm of the curve where the flow enters the bend). Although velocity tends to decrease linearly with distance along the curve ([Fig. 12](#)), video images show that the front velocity reaches a new constant near the end of the bend where maximum discharge, and therefore relevant super-elevation, is observed.

Fig. 12. Indication of evolution of mean velocities of experimental debris flows along onset of curved flume sections for the two bend radii. Data consists of experiments from all four mixtures at 20° channel inclination.

Critical flow conditions and earth pressure considerations

The vortex equation is only applicable as long as the flow does not produce cross-wave disturbance patterns within the bend section, where a Froude number of 1 might be a threshold (Knapp 1951). Experimental results are available for supercritical free surface flow in channel bends in civil engineering waste water hydraulics literature (Reinauer and Hager 1997; Giudice et al. 2000; Gisonni and Hager 2002) to relate the maximum flow depth along the outer bend wall to the minimum flow depth along the inner bend wall, including their relative positions. The relative radii of curvature (channel width-to-bend radius ratio) of such hydraulic experiments (1/3–1/20) are in a similar range to the experimental setup of this study (1/6 and 1/9); however, the debris-flow mixture is different from clear water due to viscous and frictional forces (i.e., shows a larger “stiffness” of the mixture as compared to water), which is likely to reduce secondary flow (in the lateral direction) or spiral flow. For our experiments, cross wave disturbance patterns were only detected in the fully liquefied tails with $F_r > 1$, where the surface velocity exceeded the front velocity of the flow. Maximum discharges and corresponding maximum flow heights of the experimental debris flows were always observed within the frontal part of the flow. For this reason, we assume that the modified vortex approach to estimate debris-flow velocities can also be used for supercritical flow conditions. Additional research would be necessary to characterize the approach velocity and standing wave properties in tight channel bends to be able to compare the applicability of this body of hydraulics literature to interpret superelevation data, as it is beyond the scope of this paper.

Our results (Fig. 9) indicate that correction factors of $1 < k < 5$ might be appropriate for supercritical flow regimes. For subcritical flow conditions, however, the correction factor shows k values even larger than 5, also depending on the F_r number. Here, fluid stress evolution seems to be superposed by constitutive stress conditions of the bulk mixture, as an effect of rheological characteristics.

A simple theoretical framework to account for the constitutive stress within a debris flow is based on the earth pressure theory. Considering lateral earth pressure in the derivation of the vortex

approach results in the modified eq. (12), including the correction factor k_{ep} . This approach is based on a horizontal force balance and assumes a rectangular cross section. The determination of k_{ep} depends on the active (K_a) and passive (K_p) earth pressure coefficients. Several assumptions to evaluate K_a and K_p for the flow of granular materials have been proposed (Iverson 1997; Pudasaini and Hutter 2007; Hungr 2008). For the approach presented in this study, the passive earth pressure might be the main influencing factor for the estimation of the modified correction factor k_{ep} , because it acts on the outer side of the curve where the largest flow depths can be observed. For this reason, and because larger passive earth pressures might be expected at higher flow velocities, we calculate K_p based on the Froude number of each experimental debris flow (eq. (17)). Figure 10 shows plausible results when comparing the modified correction factor k_{ep} for a presumed range of c_p values, with back-calculated correction factors k^* . Because Hungr (1995) states that the Rankine passive coefficient varies between 1.0 and 5.0, we propose a c_p value of 5 to be convenient. Varying K_a values within a plausible range of 0.2–1, according to Hungr (1995), has only a minor influence on k_{ep} when using eq. (13). However, more data on observed superelevation heights and related but independently measured flow velocities are needed to evaluate the modified correction factor k_{ep} , especially for subcritical flow conditions.

A forced vortex equation for debris-flow velocity estimation without correction factor

Based on the power law relationship between the back-calculated correction factors k^* and the Froude numbers F_r for all experiments (Fig. 9)

$$(20) \quad \bar{k}^* = 4.4 F_r^{-1.19}$$

a modified vortex approach to estimate debris-flow velocity from superelevation can be obtained by substituting k in eq. (7) with $\bar{k}^*$ of eq. (20)

$$(21) \quad \frac{R_c g \cos \theta_c \Delta h}{B v_{df}^2} = 4.4 \left[\frac{v_{df}}{(gh)^{0.5}} \right]^{-1.19}$$

which further leads to the empirical equation

$$(22) \quad v_{df_e} = \left[\frac{R_c \Delta h (g \cos \theta_c)^{0.40}}{4.4 B h^{0.60}} \right]^{1.19}$$

An alternative form of eq. (22) can be written as

$$(23) \quad v_{df_e} = \left(\frac{R_c g \cos \theta_c \Delta h}{B} \right)^{0.5} \left[\left(\frac{R_c}{h} \right)^{0.73} \left(\frac{\Delta h}{B} \right)^{0.73} 4.4^{-1.23} \right]$$

Defining

$$(24) \quad C = \left(\frac{B h}{R_c \Delta h} \right)^{1.46} 4.4^{2.46}$$

eq. (23) can be transformed to

$$(25) \quad C = \frac{R_c g \cos \theta_c \Delta h}{B v_{df_e}^2}$$

The notation of eq. (25) is similar to eq. (8) and allows a comparison with Chen (1987), who reports that the correction factor k might be as large as 10 times B/R_c . For our experiments with a

Fig. 13. Scatterplot of observed and estimated flow velocities, based on eq. (22), for all experiments of this study. Dashed line indicates a perfect agreement $v_{\text{obs}} = v_{\text{df}_e}$.

centerline radius of $R_u = 1.5$ m and the mean values of $B = 0.15$ m, $h = 0.06$ m, and $\Delta h = 0.05$ m, the factor C (calculated with eq. (24)) is 1.7. The average value of C for experiments with a centerline radius of $R_l = 1.0$ m and mean values of $B = 0.15$ m, $h = 0.05$ m, and $\Delta h = 0.04$ m is calculated as 3.3. These values are in good agreement with the observations from Japanese flume studies reported by Chen (1987).

A comparison of observed velocities and those calculated with eq. (22) (Fig. 13) shows a slight overestimation of velocity for relatively slow flows (e.g., on the order of about 50% larger than observed for observed velocities of 1 m/s). However, the use of eq. (22) does not involve a priori estimation of a correction coefficient. In any case, more detailed field observations of debris flows, especially those at well instrumented debris-flow observation stations, are necessary to evaluate the general applicability of eq. (22).

Conclusion

When applying the vortex equation to estimate debris-flow velocity based on superelevation data, a correction factor k is introduced to account for the different constitutional characteristics of debris flows. In general, previous workers have assumed that the magnitude of the correction factor depends on the viscosity and vertical sorting of the sediment in the debris flow. We found that the channel slope as well as the centerline radius have a significant influence on the correction factor k used in the vortex equation. Relatively coarse-grained sediment mixtures have larger superelevation angles than finer-grained mixtures. We found a statistically significant relation between the correction factor and the Froude number. Correction factors of $1 < k < 5$ were found to be typical for supercritical flow conditions, while for subcritical flow conditions, the correction factor shows a larger value as a function of the Froude number. The general observed dependence of k on F_r leads to an adaptation of the forced vortex formula considering active and passive earth pressures. Using these results, we propose an adaption of the forced vortex formula by considering active and passive earth pressure. Based on our experimental results we present a forced vortex equation for debris-flow velocity estimation without a correction factor.

Acknowledgement

The study is part of the research project "Analysis of debris-flow velocities due to superelevation" funded by the Austrian Science Fund (FWF): J 3089-N13.

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