Natural frequencies and damping ratios of Norway spruce (*Picea abies* (L.) Karst) growing on subalpine forested slopes

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Abstract We investigated the natural frequencies and damping ratios of 24 Norway spruce trees (*Picea abies* (L.) Karst) growing on subalpine forested slopes using swaying experiments. Trees were winched to the release position using a manually operated cable winch. Before release, the stem base rotation was approximately 2 degrees. All trees were tested in both fall-line and cross-slope direction if possible to reveal the difference between the two perpendicular directions. A new method based upon the Hilbert transformation for evaluating the damping ratio for trees was used. It enabled us to investigate the damping ratio for all excited modes of vibration in the tree structure during the swaying experiments and to study if velocity proportional damping (viscous damping) is a reasonable assumption when predicting the tree response to different external actions using mechanical models. No difference in first natural frequency and damping ratio was found between the fall-line and cross-slope direction for all tested trees. The first natural frequencies correlated best with the diameter at breast height and squared total tree height DBH/H^2 . For the damping ratio, no significant correlation with different tree characteristics could be found. A non-velocity proportional damping ratio was observed for two trees. This indicates that using a viscous damping for all

frequencies is an appropriate simplification when modelling the wind-tree interaction.

Keywords Hilbert transformation · Swaying experiments · Tree mechanics · Tree stability · Tree–wind interaction

Introduction

The natural frequencies and the damping characteristics of a tree are important parameters when modelling the wind–tree interaction. A tree absorbs wind energy at all its natural frequencies, however, most energy is absorbed at the tree's first natural frequency (Holbo et al. 1980; Mayer 1989; Peltola 1996). The relationship between the natural frequencies of the tree and the frequency of the acting wind provides guidelines on how to model the wind induced swaying of a tree (Wood 1996). A wide range of trees has been tested for their first natural frequencies (Gardiner 1989; Roodbaraky et al. 1994; Moore and Maguire 2004). Milne (1991) investigated Sitka spruce (*Picea sitchensis* (Bong.) Carr) trees growing on flat ground in Scottish plantation in two perpendicular directions without finding any difference in the natural frequencies. Baker (1997) tested urban lime trees (*Tilia × europea*) during both summer and winter conditions and found that the measured frequencies were larger in winter compared to the summer.

The damping of trees is grouped into internal and external damping (Hoag et al. 1971). Internal damping is due to energy dissipation in the root-soil system and internal friction within the wood material of the stem and the branch system (Milne 1991; Wood 1996; Moore and Maguire 2004). External damping is due to aerodynamic drag of the crown and due to collision with neighbouring

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trees (Milne 1991). When studying the contribution from each of these terms to the total damping, the ratio between damping due to contact with neighbouring trees, aerodynamic damping, and damping in the tree stem and root system is found to be ~50/40/10%, respectively (Milne 1991). However, this relationship might not be constant because the damping ratio is often non-linear and amplitude dependent (Moore and Maguire 2004). Furthermore, the aerodynamic damping also increases with wind speed (Wood 1996).

A damping model can describe energy dissipation in a dynamic system. For a natural system like a tree, this model is complex and difficult to define, as the dissipation mechanisms are not completely understood. It is therefore common to assume that the damping is proportional to the velocity, so-called viscous-damping. With this assumption, the damping coefficient can be determined with experimental methods like the logarithmic decrement, the resonant amplification method, and the half-power (bandwidth) method (Clough and Penzien 1993). The simplest and most common method to evaluate the damping is the logarithmic decrement. With this method, the damping is evaluated in the time-domain and calculated between two adjacent amplitude peaks, or averaged over several peaks in the measured signal. With the resonant amplification method and the half-power method, the damping is evaluated in the frequency-domain. Plotting the measured displacement against the frequency, a frequency–response curve is obtained. Outgoing from the peak in the frequency–response curve, the damping ratio can be estimated using the resonant amplification method. Instead of using the peak in the frequency–response curve, the half-power method uses the maximum value divided with the factor $\sqrt{2}$. For more information about the different methods, see Clough and Penzien (1993).

None of the above mentioned methods are sufficient to obtain the continuous relationship between the damping and the amplitude of the oscillation. However, using the resonant amplification method and the half-power method, the damping ratio can be calculated for all excited modes (Clough and Penzien 1993). Using the Hilbert transformation, the continuous relationship between damping ratio and oscillation amplitude can be calculated as the damping ratio is evaluated between two adjacent points in the measured signal and not between the amplitude peaks (Agneni and Balis-Crema 1989; Johnson 1999). Furthermore, a signal recorded from a free vibration experiment (acceleration, velocity or displacement) contains in most cases the contribution of many different vibration modes excited by the applied initial deformations (Agneni and Balis-Crema 1989). By separating the measured signal into all excited modes using a frequency filter, the damping ratio for every mode can be investigated.

Norway spruce (*Picea abies* (L.) Karst) trees were selected in this study because it is the most common species in European subalpine forests (Ellenberg 1996). These trees are often found in protection forests and have a defence capacity against natural hazards such as rockfalls and avalanches (Bartelt and Stöckli 2001; Dorren and Berger 2006). The swaying experiment presented in this paper is the first experiment in a series of three, with the aim to finally investigate the complex interaction between a rock and a tree. The swaying experiments were followed by winching experiments to ~2.5° rotation at stem base (Jonsson et al. 2006), and finally full-scale rockfall experiments (Lundström et al. 2007b). A method based upon the Hilbert transformation was applied to 24 Norway spruce trees subjected to swaying experiments in the fall-line and the cross-slope direction in order to investigate both the damping ratio and whether the damping ratio is proportional to the velocity, the so called viscous-damping.

Materials and methods

Experimental sites and overview of the swaying experiments

Three different experimental sites were used to perform the swaying experiments in subalpine forests near Davos, Switzerland, during the vegetation periods of 2003 and 2004 (Table 1). The soil at all experimental sites is podzolic brown earth (well graded gravel with silt and sand) with a high proportion of stones. The surroundings of Davos is characterised by an alpine topography with peaks reaching 3100 m. The dominant wind direction in the valley is north to south with the strongest winds coming from north (Meteoschweiz 2004). The magnitude of the strongest wind is approximately 20 m/s and the average wind calculated for every 30-min period is approximately 2 m/s. Both values evaluated between 1998 and 2006 (Unpublished data SLF).

Davos has a mean annual precipitation of 1,082 mm (January 74 mm, August 146 mm) and an annual mean temperature of 2.8°C (January –5.3°C, August 10.8°C) evaluated between 1961 and 1990 (Meteoschweiz 2004).

Table 1 Average slope angle, location, and exposition of the test sites

Test site	Slope angle	Latitude	Longitude	Exposition
Mattawald (1)	34°	9°50'24"	46°47'42"	NW
Brüchwald (2)	32°	9°48'12"	46°46'59"	SO
Seehornwald (3)	10°	9°51'7"	46°48'55"	SW

The trees on the three test sites were all mature trees with a mean diameter at breast height DBH ($\pm$ one standard error) 0.32 ± 0.14 , 0.40 ± 0.15 , and 0.26 ± 0.10 m, and dominant height of 33.0, 35.0, and 28.0 m, respectively. The stand density for respective test site was 450, 320, and 580 trees per hectare.

Twenty-four Norway spruce trees (*P. abies* (L.) Karst) growing on slopes in subalpine forests were tested in swaying experiments. The tested trees were selected so that the full-scale rockfall tests could be performed (Lundström et al. 2007b), this implied that some trees could not be tested by means of swaying experiments in both fall-line and cross-slope direction because of crown interaction with other trees. After the trees had fallen to the ground, either by the simulated rock or manually winched, tree characteristics, such as stem-diameter and stem-weight from the base to the tree top H , starting point of tree crown, length of the branches, and weight of the tree crown per meter were measured.

To get the tree into position before the swaying experiments, a steel cable was mounted at 75% of total tree height H (Fig. 1), which is approximately $15.6\text{--}25.9\text{ m} \pm 0.1\text{ m}$, depending upon tree height (Table 2). In order to mount these sensors on the tree stem, it was necessary to climb the tree (Fig. 1). When climbing the tree stem, only a minimum of branches was removed in order to maintain the tree crown as intact as possible and thus to minimize the influence on the natural frequencies and the damping ratios.

All trees were winched to the release position using a manually operated cable winch. The trees were pulled until $\sim 2^\circ$ stem base rotation. This corresponds to a horizontal deflection of the winching point of approximately 2 m. Two sensors mounted at 53 and 75% of H in the swaying direction (Fig. 1) recorded the acceleration after release.

The sensors used for the swaying tests were of the type AMOS-ABM-25 (AMOS Sensoren & Meßtechnik, Ludwigshafen, Germany) featuring a range of 10 m/s^2 and a precision of 0.2 m/s^2 . The sampling rate for both sensors was 20 Hz.

Approach used for analysis of natural frequencies and damping ratios

A method to obtain the continuous relationship between the damping and the amplitude of the oscillation based upon the properties of the Hilbert transformation is proposed in Agneni and Balis-Crema (1989). An overview of this method is given below. The equation of motion for a linear single degree of freedom system can be written

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = P(t) \quad (1)$$

where $x(t)$ is the displacement and $\ddot{x}(t)$ and $\dot{x}(t)$ are the acceleration and velocity, respectively. The parameters m , c , and k describe the mass, damping, and the stiffness for the system. For free vibrations, no forces are present, i.e. the acting force $P(t)$ is zero (Bachmann and Ammann 1987). Assuming viscous damping, c can be expressed as

$$c = 2m\zeta\omega \quad (2)$$

where ζ is the damping ratio (the proportion of critical damping) and ω the angular frequency of the system expressed as

$$\omega = 2\pi f \quad (3)$$

The analytical solution to the equation of motion can then be expressed as

Fig. 1 a The real test-set up for the test in fall-line direction and the positions of the two sensors and the original stem configuration. In the lower right corner, the sensor used as well as how they were attached to the tree stem can be seen. **b** The schematic test set-up for the fall-line direction, $0.53H$ and $0.75H$ indicate the positions of the sensors, the pulling height was $0.75H$, H total tree height, β_s slope angle, δ_h horizontal deflection of the winching point before release, Φ stem base rotation

Table 2 Calculated first natural frequency f_1 , and corresponding damping ratio of the test trees for all three test sites in both fall-line and in cross-lope directions

Test- site	Tree no.	DBH (m)	H (m)	Fall-line			Cross-slope		
				f_1 (Hz)	$\xi_{1/0.53H}$ (%)	$\xi_{1/0.75H}$ (%)	f_1 (Hz)	$\xi_{1/0.53H}$ (%)	$\xi_{1/0.75H}$ (%)
1	1	0.40	31.3	0.20	8.62	8.46			
	2	0.39	34.1	0.14	4.95	5.03	0.16	4.43	4.37
	3	0.49	33.3	0.20	4.07	4.02	0.22	4.11	3.98
	5	0.44	30.0	0.18	6.82	6.75	0.22	5.59	5.59
	7	0.43	30.0	0.22	5.29	5.37			
	11	0.27	21.0	0.25	2.89	2.98	0.22	4.22	4.14
	12	0.23	21.3	0.25	4.50	4.69	0.27	4.30	4.57
2	1	0.35	25.6	0.22	6.06	5.79	0.21	7.65	7.50
	2	0.48	32.9	0.17	5.96	5.85	0.20	5.04	4.88
	3	0.44	29.9	0.20	6.92	6.77	0.22	4.32	4.25
	4	0.58	33.5	0.23	3.78	3.87			
	5	0.43	27.9	0.21			0.22	6.75	6.50
	6	0.22	20.7	0.29	6.94	6.86	0.29	9.41	8.82
	7	0.38	24.6	0.26	6.03	5.88	0.27	4.73	4.48
	8	0.35	25.4	0.27	5.18	5.71	0.30	5.21	5.28
	9	0.50	34.5	0.22	4.00	4.20	0.22	3.76	3.69
	10	0.51	32.0	0.25	3.80	3.79	0.25	3.66	3.56
3	1	0.32	26.7	0.27	4.78	4.37			
	4	0.37	27.0	0.24			0.24		
	5	0.29	27.0	0.18	4.42	4.33	0.19	3.44	3.44
	6	0.39	26.9	0.25	3.85	3.90	0.23	3.83	3.60
	7	0.33	24.5	0.19	4.47	4.45	0.19	4.30	4.26
	8	0.36	27.7				0.26	5.04	
	9	0.42	28.1	0.28	3.01	2.97			

$\xi_{1/0.53H}$ and $\xi_{1/0.75H}$ are the damping ratios calculated for the sensors at 53 and 75% of the total tree height, respectively. An empty cell indicates that the tree was not tested in this direction, or that no value was obtained during the evaluation of the test results
DBH, Diameter at breast height; H, total tree height

$$x(t) = Ae^{-\xi\omega t} \sin(\omega t + \psi). \quad (4)$$

A tree is a complex mechanical system with many degrees of freedoms. For such a system, the final motion can be approximated as a sum of all excited modes (Agneni and Balis-Crema 1989). This implies that for a measured acceleration $a(t)$, from a swaying experiment, that contains M excited modes and assumed viscous damping, the acceleration can be expressed as

$$a(t) = \sum_{n=1}^M A^n e^{-\xi^n \omega_d^n t} \sin(\omega_d^n t + \psi^n). \quad (5)$$

where A^n is the amplitude, ψ^n the phase shift, ξ^n the damping ratio (i.e. the ratio between the actual and the critical damping), and ω_d^n the damped angular frequency for mode n . Every periodic signal can be written as a series of sine and cosine terms (Clough and Penzien 1993). When the series is defined only by a number of finite points N , the measured signal can be represented by the finite series, at $t = t_k$, $k = 1, \dots, N$ the measured signal can be written as

$$f(t_k) = \frac{a_0}{2} + \sum_{n=1}^{N/2} \left(a_n \cos\left(\frac{2\pi nk}{N}\right) + b_n \sin\left(\frac{2\pi nk}{N}\right) \right). \quad (6)$$

The result of the Hilbert transformation is the original signal plus a complex part (the original signal phase shifted 90°). Rewriting Eq. (6) with the assumption that the acceleration is zero at the beginning of the test, and that the tree is oscillating around the z axis (Fig. 1), the following expression is obtained

$$h(t_k) = \sum_{n=1}^{N/2} \left[\left(a_n \cos\left(\frac{2\pi nk}{N}\right) + b_n \sin\left(\frac{2\pi nk}{N}\right) \right) + i \left(a_n \cos\left(\frac{2\pi nk}{N} + \frac{\pi}{2}\right) + b_n \sin\left(\frac{2\pi nk}{N} + \frac{\pi}{2}\right) \right) \right]. \quad (7)$$

Making use of basic trigonometric relationships, Eq. (7) can for a specific time step be written as

Fig. 2 **a** The recorded acceleration plotted in the time domain. **b** The recorded acceleration transferred to the frequency domain using the Fourier transformation shows the calculated power spectra and the two excited frequencies. **c** The envelope was fitted using the least square method in a log plot. The slope of the fitted line divided by the angular frequency ω_d^n , was defined as the damping ratio ζ^n

$$h(t_k) = \sum_{n=1}^N \left[\left(a_n \cos\left(\frac{2\pi nk}{N}\right) + b_n \sin\left(\frac{2\pi nk}{N}\right) \right) + i \left(-a_n \sin\left(\frac{2\pi nk}{N}\right) + b_n \cos\left(\frac{2\pi nk}{N}\right) \right) \right] \quad (8)$$

Calculating the amplitude of the signal given by Eq. (8) for every time step, the following expression is obtained

$$|h(t_k)| = \sqrt{a_n^2 + b_n^2} \quad (9)$$

Thus, performing this operation for all time steps, the envelope of the signal is obtained. To investigate the properties of only one frequency at a time, the measured signal is filtered to obtain the frequency of interest. Every frequency found in the signal can then be expressed as

$$a(t) = A^n e^{-\zeta^n \omega_d^n t} \sin(\omega_d^n t + \psi^n) \quad (10)$$

Calculating the envelope for every frequency that was found during the analysis and plotting the relationship in a logarithm plot, the slope of the curve will be the variation in time of $\zeta^n \omega_d^n$. If the damping is linear, a first order polynomial will be sufficient to fit this curve (Johnson 1999). In this study, the envelope was fitted using a regression analysis with a first order polynomial. Note that if the envelope cannot be approximated using a first order polynomial, the damping is non-linear. This can mean that the damping is either dependent on the amplitude of the oscillation or not viscous at all. The main steps needed to obtain an estimation of the damping ratios ζ^n and the

natural frequencies f_n through Hilbert transformation can be summarized as follows.

- The recorded acceleration was plotted in the time domain and the interesting part (IP) of the acceleration was selected for future analysis. The minimum requirement for IP was set to six swaying cycles but eight or more cycles were used in most cases (Fig. 2a).
- IP is transferred to the frequency domain using the Fourier transformation with the obtained power spectra showing all the frequencies f_n excited during the test (Fig. 2b).
- IP was filtered, using a first order Butterworth filter.
- Thereafter, the Hilbert transformation was used to obtain the envelope of IP as a function of time (Fig. 2c).
- Finally, the slope of the fitted line divided by ω_d^n is defined as the damping ratio ζ^n (Fig. 2c).

After estimating the first natural frequency f_n of every tree using the procedure outlined in the previous paragraphs, a statistical investigation using the commercial software S-plus® (Insightful Corporation, Seattle, Washington, USA) was carried out. Aim of the statistical investigation was to find the most meaningful correlation between f_n and other important mechanical and geometrical characteristics of the tree. A physical meaningful regression line with tree characteristics such as diameter at breast height *DBH*, total tree height *H*, crown base, and crown length, etc. was always searched, i.e., the intersection was forced to go through zero. For the regression analysis, the sum of squared errors R^2 was calculated, and the significance of the polynomial

Table 3 Calculated second natural frequency f_2 , and corresponding damping ratio in fall-line and cross-slope directions

Test site	Tree no.	Fall-line			Cross-slope		
		f_2 (Hz)	$\xi_{2/0.53H}$ (%)	$\xi_{2/0.75H}$ (%)	f_2 (Hz)	$\xi_{2/0.53H}$ (%)	$\xi_{2/0.75H}$ (%)
1	2	0.43	3.27	3.78	0.46	4.50	4.95
	7	0.72		2.74			
2	4	0.56	3.18	3.72			
	8	0.79	5.27	4.50	0.77	4.58	5.53

$\xi_{2/0.53H}$ and $\xi_{2/0.75H}$ are the damping ratios calculated for the sensors at 53 and 75% of the total tree height, respectively. An empty cell indicates that the tree was not tested in this direction, or that no value was obtained during the evaluation of the test results

Fig. 3 First natural frequency f_n for the 23 tests in fall-line direction. The straight line indicates the best regression line, $p_1 \times DBH/H^2$, $p_1 = 454.53$ and $R^2 = 0.31$

coefficients was controlled using the F test. All experiments in fall-line direction were evaluated together. To investigate the difference between first natural frequency and damping in the fall-line and the cross-slope direction, and in the damping between 53 and 75% of H , the Wilcoxon test for paired differences was used.

Results

For all the tested trees, both the natural frequencies and the damping ratio were calculated for every mode excited during the swaying experiment (Tables 2, 3). No differences could be found between natural frequencies in fall-line and cross-slope direction for trees growing at experimental site 2. For this reason, it was assumed that the natural frequencies in fall-line direction could be used to represent the natural frequencies for the tree structure.

Fig. 4 Measured acceleration from sensor at 75% of H for test in fall-line direction, tree 4, experimental site 2, separated into its two excited frequencies. The solid line represents the first natural frequency $f_1 = 0.23$ Hz, damping ratio $\xi_{1/0.75H} = 3.18\%$, the dashed line the second natural frequency $f_2 = 0.56$ Hz, damping ratio $\xi_{2/0.75H} = 3.78\%$

The first natural frequency was best explained by the relationship $p_1 \times DBH/H^2$, $p_1 = 454.53$, $P < 0.001$ with a $R^2 = 0.31$ (Fig. 3). No significant correlation was found between damping ratio and tree characteristics such as DBH , H , crown base, and crown length, etc.

For four trees, more than one frequency was excited and observed (Table 3). The amplitudes of the two separated oscillations were different, with the amplitude for the second oscillation always smaller (Fig. 4).

The statistical analysis showed that the damping ratio at 75% of H was slightly higher than at 53% of H . No statistical difference in damping ratio was found between fall-line and cross-slope direction. A non-linear damping was observed for two trees. For these trees, the damping ratio was not calculated. However, the low amount of observed trees showing a non-linear damping ratio indicate that using a velocity proportional damping (viscous damping) when modelling tree structural damping is a appropriate assumption. Comparing the damping ratio between the first and the second mode of vibration ($n = 3$) in fall-line direction using data from the sensor mounted at $0.53H$ showed that the damping ratio were of the same order of magnitude.

Discussion

Damping

Using the properties of the Hilbert transformation (Agneni and Balis-Crema 1989; Johnson 1999) to evaluate the

damping ratio showed results in accordance with known literature values, which are approximately 4–15% (Moore and Maguire 2004). However, the values presented in this paper are closer to the lower part of this range. The internal damping ratio is generally lower than 5% (Moore and Maguire 2004), thus this might indicate that the contribution from the external damping for our trees, represents a minor part of the total damping ratio. This seems logical because all the trees were selected to minimize the interaction with neighbouring trees.

A common simplification when evaluating the damping is to assume viscous damping, which is proportional to the velocity (Clough and Penzien 1993; Moore and Maguire 2004). However, the damping is in most cases dependent upon the amplitude of the oscillation (Clough and Penzien 1993; Moore and Maguire 2004). The structural damping is a combination of internal and external damping (Moore and Maguire 2004). Wood (1996) mentioned that the external damping is not constant, and that the aerodynamic damping increases with wind speed. The reason for this increase is because the stiffness and damping coefficients are displacement dependent in reality (Gardiner 1992). For all 24 trees evaluated in this investigation, a non-velocity proportional damping ratio is only observed twice. The reason for this finding could be that these trees interacted with neighbouring trees, even though we tried to minimize this influence. The damping ratio for the second mode of vibration is evaluated for four trees and the results show that the damping ratios for both the first and second mode of vibration are of the same order of magnitude (Tables 2, 3). These results indicate that using a viscous damping for all modes of vibration is an appropriate simplification when modelling the wind-tree interaction. The constant damping ratio might be due to the small amplitude in our swaying experiments and that no interaction with neighbouring trees occurred. Winching the tree to a stem base rotation of $\sim 2^\circ$, a horizontal deflection of the winching point ($0.75H$) of approximately 1–2 m is obtained (Fig. 1). We therefore suggest future swaying experiments with larger vibration amplitudes to be performed.

No significant correlation could be found between tree structural parameters such as *DBH* and tree height and the damping ratio for the tree. The same has been observed in White et al. (1976) as well as in Moore and Maguire (2004). Furthermore, Milne (1991) investigated the difference in damping ratio in two perpendicular directions of *Picea sitchensis* (Bong.) Carr trees. He found a difference originating from asymmetric crown shape and contact with neighbouring trees. Our results could not verify his findings, as the crown shapes of the trees in our investigation were quite symmetric.

Natural frequency

No difference was found in first natural frequencies between fall-line and cross-slope direction. This is in accordance with earlier observations of Milne (1991), who investigated the first natural frequencies in two perpendicular directions of Sitka spruce (*Picea sitchensis* (Bong)) trees growing on flat ground in a Scottish plantation. Although the root architecture is asymmetric on slopes (Di Iorio et al. 2005, and own unpublished data), the first natural frequencies do not differ in the respective directions. This indicates that root architecture does not have a major influence on the first natural frequency. However, to what extent the form and size of the root-soil plate influences the frequencies cannot be revealed with the data available. Its influence must be further investigated to determine its importance.

The first natural frequency for trees growing in subalpine terrain is best related to squared total tree height H and diameter at breast height *DBH* (Fig. 3). This relationship is also suggested by Moore and Maguire (2004). The differences between the three experimental sites for both the natural frequencies and the damping ratio were not statistically analysed, as the variance within the experimental sites were larger than for all experimental sites combined. During these tests, a weak correlation $R^2 = 0.31$ was found. There might be several explanations for this. The investigated regression model is theoretically derived in Gardiner (1992). This relationship assumes the ratio between the wood density and Young's modulus of elasticity to be approximately constant over the tree height. However, investigations on Norway spruce trees have shown that density increases and that Young's modulus of elasticity decreases with increasing tree height (Lundström et al. 2007a). Furthermore, an increased soil moisture context can cause an increased f_n of 10% (Mayer 1985). Consider that the trees were tested under different weather conditions and during two successive years, this effect cannot be neglected. Unfortunately, the effect could not be quantified because no measurements of the soil moisture were carried out.

During this study, the main findings were (1) that the wind-tree interaction can be modelled using viscous damping for all modes of vibration and (2) that the natural frequencies of a tree growing on subalpine forested slope do not differ between fall-line and cross-slope direction. Both these findings have contributed to a deeper understanding on how to model the tree response when exposed to wind forces.

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