A probabilistic formulation of bed load transport to include spatial variability of flow and surface grain size distributions

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Abstract Bed load fluxes are typically calculated as a function of the reach averaged boundary shear stress and a representative bed grain size distribution. In steep, rough channels, heterogeneous bed surface texture and macro-roughness elements cause significant local deviations from the mean shear stress but this variability is often omitted in bed load calculations. Here we present a probabilistic bed load transport formulation that explicitly includes local variations in the flow field and grain size distribution. The model is then tested in a 10% gradient stream, to evaluate its predictive capability and to explore relations between surface grain size, boundary shear stress, and sediment transportation. The boundary shear stress field, calculated using a quasi-3D hydraulic model, displayed substantial variability between patch classes, but the patch mean dimensionless shear stress varied inversely with patch median grain size. We developed an empirical relation between the applied shear stress on each patch class and the reach averaged shear stress and median grain size. Predicted sediment volumes using this relation in our bed load equation were as accurate as those using complete shear stress distributions and more accurate than current bed load transport equations. Our results suggest that when spatially variable grain size distributions (e.g., patches of sediment) are present they must be explicitly included in bed load transport calculations. Spatial variability in shear stress was relatively more important than grain size variations for sediment transport predictions.

1. Introduction

Mountain rivers with steep slopes (longitudinal gradients greater than 3%) differ from those with lower slopes through a number of characteristics. Such channels are located in headwater catchments, their drainage area is relatively small, and they typically have low discharges and low relative submergences ($h/D$ where $h$ is the average flow depth and $D$ is the characteristic grain size) [Comiti and Mao, 2012]. Their channel beds feature wide grain size distributions (GSD) from sand to rarely mobile boulders [Yager et al., 2012a] and episodic landslides and debris flows may alter this composition. Additionally, these riverbeds are often composed of patches of sediment, which consist of distinct areas of the bed with relatively narrow GSD and larger sorting compared to the reach [Laronne et al., 2000; Dietrich et al., 2005]. Patches of sediment affect the flow and boundary shear stress fields, the rate and composition of sediment fluxes [Paola and Seal, 1995; Ferguson, 2003] and can also have biological implications, for instance salmon may find their preferred spawning-sized gravel in zones of finer textures [Kondolf and Wolman, 1993; Buffington and Montgomery, 1999b; Buxton et al., 2015a, 2015b; Hassan et al., 2015].

Although sediment patches are common, their formation mechanism is still unclear. When bar morphology or channel obstructions exist, patches can form from topographically induced divergences in boundary shear stress [Dietrich and Whiting, 1989; Lisle et al., 1991; Nelson et al., 2010]. Imbalances in local transport capacity and sediment supply have also been identified as a cause of patch formation [Dietrich et al., 1989; Lisle et al., 1993, 2000; Nelson et al., 2009]. Predicting the formation and location of patches is challenging because of the lack of correlation between local hydraulic properties (e.g., velocity or shear stress) and local surface grain texture [Lisle et al., 2000; Nelson et al., 2010]. However, most of these correlations were conducted for a single discharge. Shear stress and median grain size could correlate if a range of discharges is considered, or if scales larger than individual grains (e.g., patches) are analyzed. A relation between shear...
stress and patch median grain size could also have important consequences for sediment transport predictions.

Sediment patches can lead to large spatial and temporal variations in bed load transport rates, as has been documented in field [Yager et al., 2012a; Segura and Pitlick, 2015] and laboratory [Nelson et al., 2010] measurements and by theoretical modeling [Lisle et al., 1993; Paola and Seal, 1995; Ferguson, 2003; Chen and Stone, 2008]. Large relatively immobile boulders, which are a common feature of steep mountain channels, also create a three-dimensional and discharge-dependent flow structure, in which the flow velocity and turbulence intensity can significantly vary [Comiti et al., 2007; Strom and Papanicolaou, 2007; Lacey and Roy, 2008] even in zones not immediately adjacent to the roughness elements, especially in wake zones [Shvidchenko and Pender, 2000; Shamloo et al., 2001; Papanicolaou and Kramer, 2005; Tritico and Hotchkiss, 2005; Ghilardi et al., 2014; Hajimirzaie et al., 2014; Tsakiris et al., 2014]. Studies on low gradient rivers have found that spatial-temporal variations of shear stresses are a key factor in determining the areas of the bed that are highly active during sediment transport events [Segura and Pitlick, 2015].

Most sediment transport equations are inaccurate in steep streams because they do not include the energy losses or increased total flow resistance caused by macro-roughness elements (e.g., boulders and steps), the limited upstream sediment supply, and the wide GSD that are typical of high gradient channels [Bathurst et al., 1987; Lenzi and D’agostino, 1999; Rickenmann, 2001; Yager et al., 2007, 2012b, 2012c; Mueller et al., 2008; Nitsche et al., 2011; Schneider et al., 2015]. Even equations developed for steep channels [e.g., Smart, 1984; Bathurst et al., 1987; Graf and Suszka, 1987; Aziz and Scott, 1989] and those that explicitly [Yager et al., 2007, 2012c] or implicitly [e.g., Rickenmann, 1997, 2005; Lenzi et al., 1999; Nitsche et al., 2011] account for macro-roughness elements are only accurate to an order of magnitude at best. No investigation has determined if including spatial variability in flow and GSD at a scale smaller than the reach (e.g., patch) can improve bed load transport predictions in steep channels. At the patch scale, local hiding/exposure effects and the relative mobility of different grain sizes may be better captured if the patch’s median grain size and local flow conditions are considered instead of those of the reach.

Probabilistic formulations of sediment transport [Einstein, 1950; Paintal, 1971; Lisle et al., 1998] can, theoretically, include the variability of flow and surface texture but their application is generally more difficult than deterministic equations. Probabilistic equations require detailed measurements of flow and bed properties that are not commonly available, or involve extrapolation far beyond the conditions (typically flume experiments) where they were developed. For instance the sediment continuity equations of Parker et al. [2000] require functions for the probability of erosion and deposition, which, to date, have not been generalized. The bed load transport equation of Sun and Donahue [2000] works well for nonuniform sediment under full motion, but requires the time when a particle is in motion, which has also not been generalized. The use of probabilistic formulations in real field applications is even more unusual. In gravel bed streams, under partial motion conditions, the equation of Sun and Donahue [2000] does not perform as well [Sun and Donahue, 2000]. However, the field results of Wu and Yang’s [2004] fractional transport model suggest that a probabilistic approach has enormous potential.

The objectives of this study are to: (1) investigate whether sediment transport predictions in steep streams are improved when spatial variability in the flow field and grain size, in the form of sediment patches, are explicitly included, (2) explore the correlation between reach and patch averaged shear stresses; and its application in bed load flux predictions, and (3) analyze individual contributions of different patch classes to the total transported sediment volume. To meet our objectives, we use bedload transport, bed conditions and shear stress distributions for a steep stream and develop a new bedload transport formulation.

2. Field Measurements, Sediment Transport Equations and Flow Modeling

Our field work was conducted at the Erlenbach, a steep (10% gradient) stream located in central Switzerland. This stream has been described in detail in several studies [i.e., Rickenmann, 1997; Turowski et al., 2009; Nitsche et al., 2011; Yager et al., 2012a, 2012b; Beer et al., 2015] and we only focus on characteristics needed for flow modeling and bed load measurements and predictions. Bedload transport rates have been continuously recorded since 1986 using a series of Piezoelectric Bedload Impact Sensors (PBIS) or geophone based bed load impact sensors, both hereinafter called bed load sensors [Rickenmann and McArdell, 2007; Rickenmann et al., 2012]. The installation, sensitivities and other operating characteristics of these sensors
have been described in detail in several studies [Rickenmann, 1997; Rickenmann and McArdell, 2007; Turowski and Rickenmann, 2011]. Our field site is a 40 m long and 4.7 m wide reach composed of a series of steps, formed by large, relatively immobile boulders, and intervening finer, more mobile sediment patches whose GSD ranges from gravel to cobble [Yager et al., 2012a]. The bed load sensors are located directly downstream of our study site, in a steep concrete ramp designed to easily transport all sediment until it reaches a retention basin.

Our field measurement plan was designed to: a) obtain the detailed bed topography, including large roughness elements and banks, b) define each patch class GSD, c) determine the location and area of different patch classes that are potentially active (submerged) during each discharge, and d) estimate reach averaged properties such as step protrusion [Yager et al., 2012b].

2.1. Channel Geometry

On 1 August 2010 an extreme event [Turowski et al., 2013] rearranged the bed configuration, mobilized boulder steps, increased the in-channel sediment supply [Yager et al., 2012b, 2012c], and changed the bed surface packing and armoring. We split our data to account for these changes and conducted measurements during June–July 2010 (hereinafter called 2010 data set) and during July–August 2011 and July 2012 (2011 data set).

We surveyed the bed with a high-resolution ground-based light detection and ranging system (terrestrial LiDAR, Leica ScanStation C10, average point spacing 2 cm) and a total station (Leica-system 1200). Terrestrial LiDAR captured the banks, large boulders, and bed surface above the water level and the total station was used to measure the steps, pools, submerged bed, and any other zones the terrestrial LiDAR was unable to capture (average point spacing of 5 cm). We combined both data sets using AutoCAD® Civil 3D 2014 to obtain a digital elevation model (DEM) that was interpolated, using a linear Kriging algorithm, into a rectangular grid with an average point spacing of 5 cm. The grid size of our numerical simulation (~10 cm) would not have benefited from a more detailed topography.

2.2. Patch Mapping and Grain Size

Patch boundaries were visually identified as gradual or sharp gradations in GSD and were mapped with the total station. Delineation of patch boundaries has a certain degree of subjectivity [Nelson et al., 2014] and one of the same operators was present in all field campaigns to ensure the same criteria was applied. Patches were grouped in different classes following the classification of Buffington and Montgomery [1999a], where a certain patch is named based on the relative frequency of the surface grain sizes (gravel (2–64 mm), cobble (64–256), and boulder (>256 mm)). Surface grain sizes that occupied less than 5% of the patch area were excluded from the classification. We conducted pebble-counts (counting 100 grains using the grid method on mostly dry patches) on each patch class and divided all patches into immobile (Boulder patches, steps) or relatively mobile sediment [see Yager et al., 2012b]. Nine patch classes distributed into 74 individual patches and 6 patch classes with 62 individual patches composed the 2010 and 2011 data set, respectively (Table 1; Figures 1 and 2). The difference between data sets was likely due to bed restructuring and instability after the extreme event of August 2010 [Turowski et al., 2013].

2.3. Flow Measurements

We installed 6 and 5 staff plates in different cross sections for the 2010 and 2011 data sets, respectively (Figure 1). We measured water surface elevation (WSE) at each plate throughout 5 and 6 flow events with a total of 8 and 35 measurements for each cross section for the 2010 and 2011 data, respectively. Because of the unsteadiness of the water surface, flow depth (h) at each staff plate was the average of the maximum and minimum h over a period of 60 s. We developed rating curves between h at each cross section and the flow discharge (Q), which is measured at 1 min intervals (stream gage operated by the Swiss Federal Research Institute, WSL [Nitsche et al., 2011]) at the downstream end of our reach. For all cross section the average coefficient (R²) of determination was 0.84 and 0.83 for the 2010 and 2011 data sets, respectively. We tried to simultaneously measure WSE and local flow velocity in each cross section, but even for moderate flows the conditions were too dangerous to wade. We therefore calculated the reach averaged velocity (U) using the method of Yager et al. [2012b] (see supporting information for equation and details), which was originally calibrated for our study reach in 2004. The calibration was prior to extreme events in 2007 and 2010 [Yager et al., 2012b] and to further ensure that our velocities were in the appropriate range, we
compared our calculated $U$ (from FaSTMECH, see section 2.4) to those from other resistance equations [Whittaker, 1986; Egashira and Ashida, 1991; Pagliara and Chiavaccini, 2006; Rickenmann and Recking, 2011] and to the cross-sectionally averaged velocity ($U_{xs} = Q/A_{xs}$, where the cross-sectional area ($A_{xs}$) was calculated using the measured cross sections and the $h−Q$ rating curves).

2.4. Flow Modeling and Shear Stress Distributions

We modified Parker’s [1990] equations to include the spatially variable boundary shear stress ($\tau_b$) and GSD. To obtain $\tau_b$, we used the quasi-3D hydrodynamic model, FaSTMECH, which is included in the iRIC software package V2.3 (www.i-ric.org) and was developed by the U.S. Geological Survey. The model has been described in detail elsewhere [e.g., Nelson and Smith, 1989; Nelson and McDonald, 1995; Lisle et al., 2000; Nelson et al., 2003; Mcdonald et al., 2005; Kinzel et al., 2009] and used in several studies [Clayton and Pitlick, 2007; Nelson et al., 2010; Conner and Tonina, 2014; Maturana et al., 2014; Mueller and Pitlick, 2014; Segura and Pitlick, 2015], so only its major characteristics are mentioned here. The model solves the vertically integrated conservation of mass and momentum equations in a curvilinear grid coordinate system. Approximated vertical velocity profiles are calculated from the two-dimensional (2-D) solution using an assumed eddy viscosity structure [Rattray and Mitsuda, 1974]. A zero-equation model for the lateral eddy viscosity, which assumes homogenous and isotropic turbulence, is used for turbulence closure [Miller and Cluer, 1998; Nelson et al., 2003; Barton et al., 2005]. The lateral eddy viscosity was constant and equal to 0.005 m$^2$/s for all our simulated discharges.

![Figure 1](image-url)
Discharges ranging from 0.10 to 3.5 m$^3$/s, in increments of 0.05 m$^3$/s, were simulated to include the entire range of flows during which sediment transport was calculated. Flows below 0.10 m$^3$/s were not simulated because they produce almost no bed load transport. We used a spatially constant drag coefficient ($C_d$), that was calibrated for each discharge using the $h-Q$ rating curves and the estimated $U$ from Yager et al. [2012b]. The use of a constant $C_d$ can result in similar local shear stresses [Lisle et al., 2000; Nelson et al., 2010] or better predictions of WSE and $U$ [Segura and Pitlick, 2015] (see supporting information for more details on $C_d$). For both data sets (2010 and 2011) $C_d$ (range of 0.12–0.26) varied inversely with depth, which is consistent with the results of Pasternack et al. [2006] and Jarrett [1984].

The boundary shear stress was calculated at every node ($\tau_{bn}$, where $n$ is a node in the 10 cm mesh, see supporting information section S3) using the model’s outputs for the vertically averaged streamwise ($u_n$) and cross-stream ($v_n$) velocities,

$$\tau_{bn} = \rho C_d (u_n^2 + v_n^2).$$

where $\rho$ is the water density. Equation (1) was directly computed in FaSTMECH. Two different approaches were used to describe $\tau_{bn}$ for a given patch class: a) patch class averaged $\tau_{bn}$ (hereinafter “Patch mean”), and b) patch class variable $\tau_{bn}$ distribution (hereinafter “Variable distribution”). We could have used individual patches instead of patch classes, but their areas in most cases were often too small to have enough $\tau_{bn}$ values (less than 15) to fit probability distributions. For each patch class, data set (2010 or 2011), and discharge we fit five different probability distributions (normal, lognormal, gamma, exponential, and generalized extreme value) to the modeled shear stresses. The normal, lognormal [Bridge and Bennett, 1992; Kleinhans and van Rijn, 2002], and gamma [Paola, 1996; Nicholas, 2000; Bertoldi et al., 2009; Recking, 2013; Segura and Pitlick, 2015] have been used in previous studies and the exponential and generalized extreme value were included to extend our analysis. Parameters in each distribution were estimated using the maximum likelihood estimation (MLE) method and we used the nonparametric test Chi-Square ($\chi^2$) to determine
whether or not the observed shear stresses came from the hypothesized continuous distribution at a 95% significance level. Of those distributions that met the $\chi^2$ test, we selected the one with the highest $R^2$ from the fit between predicted (from the distribution) and modeled (FaSTMECH) shear stresses. We did this for every discharge and every patch class. If a patch class had less than 15 shear stress values for a given discharge we did not calculate any distribution and bed load transport was determined using the patch class averaged shear stress (see section 2.5). A more detailed analysis of the methods, results and implications of using continuous probability distribution with our probabilistic equations (see section 2.5) is described in supporting information section S4.

### 2.5. Sediment Transport Equations

Our modified version of Parker’s [1990] equations explicitly includes the grain sizes of different patch classes, spatial distribution of shear stresses, and limited sediment supply. The bed was divided into $J$ patch classes that were further divided into $N$ grain size classes with characteristic diameters $D_{ij}$, where $i$ ranges from 1 to $N$ and $j$ ranges from 1 to $J$. The geometric mean size ($D_{sgj}$) and arithmetic standard deviation ($\sigma_{sgj}$) in $\phi$ units for each patch class were calculated as:

\[
\ln D_{sgj} = \sum_{i=1}^{N} F_{ij} \ln D_{ij},
\]

\[
\sigma_{sgj}^2 = \sum_{i=1}^{N} \left[ \ln \left( \frac{D_{ij}}{D_{sgj}} \right) (\ln 2)^{-1} \right]^2 F_{ij},
\]

where $F_{ij}$ is the volume fraction of the $i$th grain-size class in the $j$th patch class. The local applied dimensionless shear stress ($\tau_{sgj}$, hereinafter the superscript * denotes a dimensionless quantity) at the $j$th patch class was calculated as:

\[
\tau_{sgj}^* = \frac{\tau_{sij}}{\rho R_s g D_{sgj}},
\]

where $\tau_{sij}$ is a local applied shear stress at the $j$th patch class, $R_s$ is the dimensionless submerged specific gravity of sediment and $g$ the acceleration due to gravity. Hereinafter we refer to $\tau_{sgj}^*$ simply as $\tau_j^*$. For sediment transport, the only relevant locations were where the local dimensionless shear stress was higher than the dimensionless critical shear stress (assumed $\tau_c^* = 0.045$ [Buffington and Montgomery, 1997]); we only used locations in which $\tau_j^* \geq \tau_c^*$. Although the smaller fractions of fine sediment patches could be in motion for these low shear stresses, their contribution to the total transported volume was negligible and neglecting low $\tau_j^*$ allowed us to simplify our calculations. The probability density function ($P_{\tau_j^*}$; see Figure 3 for definitions) of $\tau_j^*$ follows the best-fit distribution that may change with the discharge. The lower, $\tau_j^{{\mu}}$, and upper, $\tau_j^{{\rho}}$, limits of the distribution were set as the 1st and 99th percentile of the best-fit distribution. Bedload transport rates were calculated at discrete intervals of $\tau_j^*$, where the width of each interval ($\Delta \tau_j^*$) is given by:

\[
\Delta \tau_j^* = \frac{\tau_j^{{\rho}} - \tau_j^{{\mu}}}{K},
\]

with $K = 25$ as the number of intervals. Since the patch mean approach does not require a discrete interval, $\tau_j^*$ is simply the average of all shear stresses for each discharge and patch class. Within a patch class the fraction of the bed area, $A_{jk}$, where a certain $\tau_j^*$ acts is:

\[
A_{jk} = \Delta \tau_j^* P_{\tau_j^*}.
\]

with $k$ ranging from 1 to $K$, therefore the $\tau_j^*$ acting on $A_{jk}$ is defined as $\tau_j^k$. The process for calculating $\tau_j^k$, $A_{jk}$ and $\tau_j^k$ is shown in Figure 3. The hiding function ($\phi_{ijk}$) for the $k$th subregion in the $j$th patch class is:

\[
\phi_{ijk} = \phi_{sgoj} \left( \frac{D_{ij}}{D_{sgj}} \right)^{-\beta},
\]

where $\phi_{sgoj}$ is:
The exponent $\beta$ is 0.16 for all patch classes and was calculated using tracer particles installed in our reach \cite{Yager2012} and $r_{sgo}$ is a dimensionless reference stress (0.0386, in the original Parker \cite{1990} equation). A sensitivity analysis of the bed load transport predictions to this hiding function is provided in supporting information S5). The empirical function $x_{jk}$ is defined as:

$$x_{jk} = \frac{1}{r_{sgo}} - \frac{x_0}{r_{sgo}} 2^{1}$$

where $r_{sgo}$ and $x_0$ are graphical functions from Parker \cite{1990}. The dimensionless bed load transport rate $W_{sijk}$ for each $i^{th}$ size class in for each $k^{th}$ subregion on the $j^{th}$ patch class is:

$$W_{sijk} = 0.00218G(\phi_{ijk})$$

The volumetric transport rate per unit width ($q_{ijk}$) for the $i^{th}$ size class in the $k^{th}$ subregion on the $j^{th}$ patch class:

$$q_{ijk} = 5474 \left(1 - \frac{0.853}{\phi_{ijk}}\right)^{4.5} \text{,} \quad \phi_{ijk} > 1.59$$

$$G(\phi_{ijk}) = \exp \left[14.2(\phi_{ijk} - 1) - 9.28(\phi_{ijk} - 1)^2\right], \quad 1 \leq \phi_{ijk} \leq 1.5,$$

$$\phi_{ijk} 14.2, \quad \phi_{ijk} < 1$$

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$$\phi_{ijk} 14.2, \quad \phi_{ijk} < 1$$
The volumetric transport rate per unit width for each $i^{th}$ size class on the $j^{th}$ patch class is:

$$q_{ij} = \frac{\left(\frac{L_s}{P}\right)^{1.5} F_{ij} W_{sijk}^*}{R_i g} A_{jk},$$

(12)

and the volumetric transport rate per unit width for the $j^{th}$ patch class is $q_j = \sum_{i=1}^{N} q_{ij}$. The total transport rate per unit width ($q_T$) is:

$$q_T = \left( \sum_{j=1}^{J} q_j \frac{A_j}{A_{Tw}} \right) \frac{A_m}{A_T} Z_m^*,$$

(14)

where $A_j$ is the wetted, or submerged, bed area of the $j^{th}$ patch class, $A_{Tw}$ is the total wetted bed area, $A_m$ is the area fraction occupied by relatively mobile grains (i.e., the bed excluding immobile steps) and $A_T$ is the total bed area. The ratio $A_j/A_{Tw}$ is a weighting factor that accounts for the individual contributions of each patch class to the total bed load. $Z_m^*$ is the ratio of the mobile sediment deposit thickness at the time of an individual flow event ($Z_t$) to that immediately after the last extreme event ($Z_{t0}$).

$$Z_m^* = \frac{Z_t}{Z_{t0}}.$$

(15)

If the mean immobile-grain diameter ($D_i$) does not vary with time ($t$, in units of months, see supporting information for details) then $Z_m^*$ can be approximated as:

$$Z_m^* = \frac{1}{0.85} (1 - 0.15^{0.21}).$$

(16)

The upstream sediment supply, which is typically limited in steep streams, is included by scaling the predicted total transport rate by the volumetric proportion of the bed covered by relatively mobile sediment, $(A_m/A_T)Z_m^*$. This scaling factor was based on the observations of Yager et al. [2007] in which the immobile grain protrusion varied with sediment supply. Later, Yager et al. [2012c] showed that step protrusion was a proxy for sediment supply at any given time given that it increased with greater time elapsed since an extreme event, which are associated with high sediment supplies [Turowski et al., 2009]. The ratio $A_m/A_T$ changed between the two data sets (0.70 and 0.74 for the 2010 and 2011 data sets, respectively), but it was assumed constant within each set.

### 2.6. Sediment Flux Measurements and Predictions

Our transport predictions depend on bed topography, patch class GSD, and assumed relative sediment supply, which in turn depend on the time elapsed since the last extreme event. The last recorded extreme events occurred on 20 June 2007 and 1 August 2010 [Turowski et al., 2009, 2013], and therefore we limit our analysis to events after 2007. The bed load sensors were not calibrated for low sediment yield events [Rickenmann and McArdell, 2007], and all events with less than 3 m$^3$ of transported sediment were excluded from our analysis (~37% of the events). In addition, events containing discharges larger than 9 m$^3$/s (the two extreme events) were also omitted for two reasons: (i) there are likely large uncertainties in extrapolating our WSE to those high discharges, and (ii) the WSE is likely outside our numerical domain boundaries (overbank flow). Measured transported volumes were corrected for porosity (assumed 40%) (details on measured transport in Rickenmann and McArdell [2007]; Rickenmann et al. [2012]; Yager et al. [2012b]). Although our simulations do not cover all possible individual discharges throughout each hydrograph (see section 2.4), they bracket the measured values. To obtain $q_{ij}$ and $A_j$ for any specific discharge within the measured hydrograph we used spline cubic interpolations between known values. We did not analyze if modifications to sediment transport equations other than the Parker [1990] equation would further improve predictions (but see Schneider et al. [2015] for accuracy of other equations).
3. Results

3.1. Hydrodynamic Model Simulations and Results

Water surface elevation RMSE (root mean squared error) for our simulated discharges was 0.06 m (standard deviation (std) 0.06 m; 10% of mean flow depth) and 0.04 m (std 0.04 m; 7% of mean flow depth) for the 2010 and 2011 data, respectively. The RMSE of the reach-averaged velocity from the hydrodynamic model versus the Yager et al. (2012b) method, for both data sets was 0.014 m/s. The equation of Ferguson [2007] that uses the coefficient from Rickenmann and Recking [2011], gave similar velocities, although slightly larger, to those predicted by the hydrodynamic model (differences of 5–9% and 2–8% for the 2010 and 2011 data sets). This similitude between velocities confirms the successful calibration of our hydrodynamic model (Figure 4). The equations of Whittaker [1986], Egashira and Ashida [1991], and Pagliara and Chiavaccini [2006] gave larger velocities than Rickenmann and Recking [2011], Yager et al. [2012b], and the hydrodynamic model.

The RMSE between the model’s cross-sectionally averaged velocity (hereinafter hydrodynamic model. [2006] gave larger velocities than Chiavaccini model (Figure 4). The equations of Whittaker [1986], Egashira and Ashida [1991], and Pagliara and Chiavaccini [2006] gave larger velocities than Rickenmann and Recking [2011], Yager et al. [2012b], and the hydrodynamic model.

The RMSE between the model’s cross-sectionally averaged velocity (hereinafter $U_{AV,\text{model}}$) and $U_{AV}$ for the 2010 and 2011 data sets were 0.081 and 0.076 m/s, respectively. The major differences between $U_{AV,\text{model}}$ and $U_{AV,s}$ were observed at high discharges (approximately over 1.5 m/s) and could be partly arise from uncertainties of (i) the stage-discharge and $h-Q$ relations, where fewer calibration measurements for higher discharges exist, (ii) the hydrodynamic model’s simplifications of governing equations, (iii) assuming a constant $C_s$, and (iv) the use of constant cross section geometries for $U_{AV,s}$ that may not have been perpendicular to the local flow at all discharges.

3.2. Patch and Reach Averaged Shear Stress

To study the correlation between ($\tau_{bn,s}$ grid scale) and local grain size we conducted an analysis similar to that of Lisle et al. [2000]. We did not have a detailed map of the local median grain size (i.e., at a subpatch scale) and used the patch class median grain size ($D_{50}$) instead. We obtained a very low coefficient of determination between $\tau_{bn,s}$ and $D_{50}$ ($R^2=0.23, \sigma = 0.05$, Figure 5a), which confirms the results of Lisle et al. [2000] and Nelson et al. [2010]. However, the patch class averaged shear stress ($\tau$) varied directly with $D_{50}$ for a given discharge (Figure 5b).

Although we use the patch-scale shear stress, the reach averaged dimensionless shear stress ($\tau^*$) is typically used for bed load transport calculations. We tested how $\tau^*$ varied using two different calculation methods (Figures 6a and 6b): (i) First, the mean dimensionless shear stress for each patch class ($\tau^*_j$) was a function of the mean shear stress of each patch class ($\tau_j$) normalized by the median grain size ($D_{50}$) of that patch class and then $\tau^*$ was the patch class area (only wetted portion) weighted average of all $\tau^*_j$ and is hereinafter called $\tau^*_{\text{var}D_{50}}$. (ii) Second, $\tau^*$ was calculated in almost the same way but is instead normalized using the reach-averaged surface median grain size ($D_{50}$), which is normally done in the literature. The entire area of each patch class was used instead of just the wetted area and this method is called $\tau^*_{\text{var}D_{50}}$. For any given discharge and for both data sets, $\tau^*_{\text{var}D_{50}}$ was lower than $\tau^*_{\text{var}D_{50}}$ (Figure 6a) because of the use of a single $D_{50}$ and patch area in $\tau^*_{\text{var}D_{50}}$. Although $\tau^*_{\text{var}D_{50}}$ was not used in our bed load transport calculations, it shows that the patch class-scale $D_{50}$ can significantly impact the reach-averaged shear stress.

We also calculated the shear stress acting on the potentially mobile sediment ($\tau^*_{mn}$), using the shear stress partitioning method of Yager et al. [2012b, 2012c], to compare to $\tau^*_{\text{var}D_{50}}$. This method accounts for immobile grain drag to indirectly reduce the shear stress on mobile patches whereas our hydrodynamic model includes the flow divergence caused by boulders to directly affect the shear stress on mobile patches. For most discharges and both data sets, $\tau^*_{\text{var}D_{50}}$ was lower than $\tau^*_{mn}$, with percent differences as high as 50% for large discharges (Figure 6b). However, within the discharge range where most sediment transport occurs (based on a magnitude frequency analysis, light gray area in Figure 6b, see Nitsche et al. [2011]), both methods predict similar dimensionless shear stresses for a given discharge, indicating that $\tau^*_{mn}$ can roughly capture the effects of boulders on shear stress. Our results are slightly different from Segura and Pitlick [2015] who demonstrated that the differences between mean shear stress from a 2-D flow model and the total shear stress (slope-depth product) decreased with increasing flow.

Our model captured significant spatial variability in shear stress that was not represented by the approach of Yager et al. [2012b] (Figure 6c). The coefficient of variation in $\tau^*_{\text{var}D_{50}}$ (CV), defined as the ratio of the standard deviation to the mean value, was fairly constant for both data sets.
3.3. Predictions of Sediment Flux

We used three different approaches (see section 2.4) to calculate bed load transport: (i) Patch mean; (ii) Variable distribution; and (iii) Yager et al. [2012c], which uses a single shear stress for a given discharge and does not include the effect of patches (Figure 7, method Shear stress – grain size relation within this figure is explained later in section 3.4). Predictions of sediment transport for each of the 43 tested events (36...
7 for 2010 and 2011, respectively) used the bed topography (and associated shear stresses) for that time period. If an event occurred before or after the extreme event of August 2010, we used the 2010 or 2011 data sets, respectively. Bedload volumes calculated using the "Patch mean" approach were always within one order magnitude of the measured values, had the lowest RMSE (m³), and did not systematically over or under predict the measured volumes (Figure 7). 53% of all events for the "Patch mean" approach had a ratio of the predicted to the measured transported volumes between 0.5 and 2 (factor of 2). The "Variable distribution" approach produced similar results for the bulk of the measured events, but the volume of one event was overpredicted by over an order magnitude (Figure 7). We chose the Yager et al. [2012c] method to compare to the performance of the "Patch mean" and "Variable distribution" approaches because: (a) it was already tested in exactly the same reach, and (b) it has relatively accurate predictions to compare against those of our equation.

The Yager et al. [2012b] approach had about double the RMSE (99 m³) than the "Patch mean" (40 m³) and "Variable distribution" (51 m³) approaches. For all three approaches most sediment transport events were overpredicted ("Patch mean" 58%; "Variable distribution": 65%; Yager et al. [2012b]: 63%) rather than underpredicted (Table 2).

3.4. Grain Size and Shear Stress Relation

Although bed load transport predictions using the "Patch mean" method are more accurate than those using Yager et al. [2012c], they rely on very detailed topographic information not commonly available for most rivers. A more broadly applicable equation that empirically includes the effects of patch classes is therefore desired. Given that patch class mean shear stress on the jth patch class ($\tau_j$) increases with patch median grain size ($D_{50j}$) (Figure 5b), we analyzed if this relationship could be used in our sediment transport equation. Since the reach-averaged shear stress ($\tau$) in steep rough channels can be easily estimated using flow resistance partitioning techniques [e.g., Comiti et al., 2007; Yager et al., 2007, 2012b; Nitsche et al., 2011; Rickenmann and Recking, 2011], we developed a function that relates $\tau_j$ to $D_{50j}$ and $\tau$. Two different approaches were used to define $\tau$ for all discharges: (i) $\tau_{var D_{50j}}$ as discussed in section 3.2 and (ii) total shear stress ($\tau_T$) defined $\rho g h S$ (where $S$ is the average bed slope) which is the most accessible flow parameter used in sediment transport predictions.

We used a power law to relate $\tau_j$ to $\tau$,

$$\tau_j = c_p \tau^{e_p}$$  \hspace{1cm} (17)

where the $R^2$ was 0.75 for both $\tau_{var D_{50j}}$ and $\tau_T$. The coefficient ($c_p$) and exponent ($e_p$) of equation (17) varied with the dimensionless median grain size $D_{50j}/D_{50}$ (Figures 8a and 8b). When using $\tau_{var D_{50j}}$ a power law and a logarithmic relation with respect to $D_{50j}/D_{50}$ 83% and 75% of the variability in $c_p$ and $e_p$ were explained, respectively (Figures 8a and 8b).
We analyzed the performance of equations (17)–(19) (hereinafter called $\tau_j$) by comparing their predicted $\tau_j$ to the actual hydrodynamic model results (Figure 8c). The average $R^2$ between the predicted and measured $\tau_j$ was 0.92 and 0.90 ($\alpha = 0.05$) for $\tau_{\text{var } D_{50}}$ and $\tau_T$, respectively. Therefore the $\tau_j - \tau_{D_{50}}$

\[ c_p = 5.31 \left( \frac{D_{50}}{D_{50}} \right)^{2.52}, \quad \tau = \tau_{\text{var } D_{50}} \]  

\[ c_p = 6.52 \left( \frac{D_{50}}{D_{50}} \right)^{2.46}, \quad \tau = \tau_T \]  

\[ e_p = -0.37 \ln \left( \frac{D_{50}}{D_{50}} \right) + 0.70, \quad \tau = \tau_{\text{var } D_{50}} \]  

\[ e_p = -0.33 \ln \left( \frac{D_{50}}{D_{50}} \right) + 0.61, \quad \tau = \tau_T \]

Figure 6. (a) Dimensionless reach-averaged shear stress dependence on the median grain size used for normalization ($\tau_{\text{var } D_{50}}$ and $\tau_{\text{cst } D_{50}}$ methods), shear stresses calculated from hydrodynamic model, (b) Reach averaged dimensionless shear stresses predicted by the hydrodynamic model ($\tau_{\text{var } D_{50}}$) for both data sets as functions of discharge compared to those predicted by the shear stress partitioning method of Yager et al. (2012b). The dark grey area represents four different step protrusions calculated at 0.5, 1, 2, and 5 years since an extreme event. Upper and lower limits correspond to 0.5 and 5 years respectively, whereas 1 and 2 years are labeled directly. The light grey area shows the discharges where 90% of the sediment transport occurred for our events, calculated using a magnitude frequency analysis based on the sediment transport events from June 2007 until September 2010 using a total of 4494 individual discharges. (c) The coefficient of variation of the dimensionless reach averaged shear stress ($\tau_{\text{var } D_{50}}$) as a function of discharge.

Figure 7. (a) The log of the ratio of the predicted to measured sediment volumes. Predicted bed load volumes by the Patch mean, Variable distribution and shear stress - grain size relation are compared to those of Yager et al. [2012c]. In the Figure Shear stress-grain size relation is our $\tau_j - \tau_{D_{50}}$ relation, see text for other definitions. The line of perfect agreement denotes the measured volume was predicted exactly. The top and bottom of each box are the 25th and 75th percentiles and the middle line inside the box is the median value. Lines extending out of the box correspond to the maximum and minimum predicted volume ratios. Arrows and corresponding text denote the percent of the sediment volume predictions that are greater or less than a factor of 2 (black) and greater than a factor of 10 (grey). The light gray shaded area represents predictions within a factor of 2 of the measured values. The dark grey shaded area represents over predictions by more than an order magnitude. The volume at the top of each box corresponds to the RMSE of the predicted bed load volume. (b) Comparison of the measured and predicted sediment transport volumes for the same equations. Some predicted sediment volumes by Yager et al. [2012c] fall outside the box (6 sediment transport events, higher than 250 m$^3$).
relations can be used to quantify shear stress variability between patch classes (Figure 8c). At low $\tau$ (100 Pa) fine patches have applied shear stresses that are significantly lower than the reach averaged value. As $\tau$ increases, $\frac{\tau_j}{\tau}$ approaches unity for most patch classes (500 Pa occurs at discharges of about 3.0 m$^3$/s in the Erlenbach), shear stress variability becomes relatively unimportant, and reach averaged shear stress may be adequate for sediment transport calculations.

We tested the $\frac{\tau_j}{\tau} = \frac{D_{50j}}{D_{50}}$ relations in sediment transport calculations using the same measured sediment transport events from section 3.3. All the predicted volumes were of comparable magnitude with those calculated using the “Patch mean” method (Figure 7). The RMSE was 47 m$^3$, which is slightly higher than that from the “Patch mean” approach (40 m$^3$, Table 2) and less than half of that from Yager et al. [2012c] (99 m$^3$). 53% of the predictions were within a factor of two. The results obtained using the $\frac{\tau_j}{\tau} = \frac{D_{50j}}{D_{50}}$ relations suggests that they are well suited for bed load predictions.

4. Discussion
4.1. Predictions of Boundary Shear Stress and Sediment Transport Equations

We demonstrated that bed load transport estimations can be improved, compared to those that use reach averaged properties, when local characteristics of the flow and the spatial distribution of grain sizes are considered. We presented three different approaches for spatially distributed shear stresses: i) “Patch mean”; ii) Table 2. Prediction Errors in Sediment Flux Calculations$^a$

<table>
<thead>
<tr>
<th></th>
<th>Patch Mean</th>
<th>Variable Distribution</th>
<th>Shear Stress Grain Size Relation</th>
<th>Yager et al. [2012c]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (m$^3$)</td>
<td>40</td>
<td>51</td>
<td>47</td>
<td>99</td>
</tr>
<tr>
<td>Within a factor of 2 (%)</td>
<td>53</td>
<td>51</td>
<td>53</td>
<td>51</td>
</tr>
<tr>
<td>Greater than 2 (%)</td>
<td>35</td>
<td>40</td>
<td>28</td>
<td>47</td>
</tr>
<tr>
<td>Less than 0.5 (%)</td>
<td>12</td>
<td>9</td>
<td>19</td>
<td>2</td>
</tr>
<tr>
<td>Overpredicted (%)</td>
<td>58</td>
<td>65</td>
<td>47</td>
<td>63</td>
</tr>
<tr>
<td>Underpredicted (%)</td>
<td>19</td>
<td>19</td>
<td>37</td>
<td>9</td>
</tr>
</tbody>
</table>

$^a$Within a factor of 2 denotes the percent of predicted sediment volumes that were within the range 0.5–2 of the measured values.
“Greater than 2” and “Less than 0.5” denote the percent of predictions that were greater or less than a factor of 2, respectively. “Overpredicted” and “Underpredicted” denote predictions that were greater than 1.25 or lower than 0.75 times the measured transported volumes. We considered predictions within a ratio of 1 ± 0.25 as “successfully predicted.” Shear stress – Grain size relation uses the $\frac{\tau_j}{\tau} = \frac{D_{50j}}{D_{50}}$ relations. 

relations can be used to quantify shear stress variability between patch classes (Figure 8c). At low $\tau$ (100 Pa) fine patches have applied shear stresses that are significantly lower than the reach averaged value. As $\tau$ increases, $\frac{\tau_j}{\tau}$ approaches unity for most patch classes (500 Pa occurs at discharges of about 3.0 m$^3$/s in the Erlenbach), shear stress variability becomes relatively unimportant, and reach averaged shear stress may be adequate for sediment transport calculations.

We tested the $\frac{\tau_j}{\tau} = \frac{D_{50j}}{D_{50}}$ (using the $\tau_{var} D_{50}$ version) relations in sediment transport calculations using the same measured sediment transport events from section 3.3. All the predicted volumes were of comparable magnitude with those calculated using the “Patch mean” method (Figure 7). The RMSE was 47 m$^3$, which is slightly higher than that from the “Patch mean” approach (40 m$^3$, Table 2) and less than half of that from Yager et al. [2012c] (99 m$^3$). 53% of the predictions were within a factor of two. The results obtained using the $\frac{\tau_j}{\tau} = \frac{D_{50j}}{D_{50}}$ relations suggests that they are well suited for bed load predictions.

4. Discussion
4.1. Predictions of Boundary Shear Stress and Sediment Transport Equations

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"Variable distribution"; and iii) $\tau_j - \tau - D_{50}$ relations. For the "Variable distribution" approach we found that no single probability distribution was able to represent the observed spatially distributed shear stresses in each patch and simulated discharge with statistical confidence (see supporting information section S4). This approach predicted the most sediment transport events outside a factor of 2 of the measured volumes because it uses the full range of shear stresses applied on each patch class and all locally high and low values are included (Figure 7). Locally high shear stresses normally are accompanied by scour and filling, which are not included in our calculations. It is not the objective of this study to test the accuracy of each probability distribution used in the "Variable distribution" approach.

The "Patch mean" approach was more accurate than the original method of Yager et al. [2012c] in terms of sediment volume RMSE ($m^3$) and reducing systematic over-prediction but generally had a similar percentage of events within a factor of two of the measured values. The "Patch mean" and "Variable distribution" did not dramatically improve bedload estimates over the method of Yager et al. [2012c]. It is important, therefore, to consider that the accuracy achieved with these new methods must be contrasted with the field measurements and the numerical modeling efforts, which can be an obstacle for some practical applications. The Yager et al. [2012c] equation likely worked well because it predicts the mean shear stress fairly accurately for most of the sediment transporting flows (Figure 6a). This mean shear stress could have worked well because the shear stresses on each patch class approach the reach-averaged stress with increasing discharge (Figure 8c.3). However, if we had tested the Yager et al. [2012c] equation for a wider range of discharges, it is likely that its performance compared to our equation would decline because it systematically over-predicts $\tau_m$ at higher discharges. A variable drag coefficient of the mobile sediment ($C_m$), instead of a constant one as assumed by Yager et al. [2012b], could result in a lower $\tau_m$ at high discharge. The $\tau_j - \tau - D_{50}$ relations are easier and more broadly applicable than the "Patch mean" and "Variable distribution" methods and can be used where data collection and/or numerical modeling are difficult to perform. The advantage of these equations is that they preserve the simplicity of reach-averaged relations while maintaining the accuracy of the spatially variable method (Figure 7). Only areas relevant for sediment transport, defined as $\tau_j \geq \tau_m$, were considered in our $\tau_j - \tau - D_{50}$ relations, similar relations and bedload prediction accuracies were obtained when the whole wetted area was considered (see supporting information for further details).

### 4.2. Shear Stress Variations With Median Grain Size

Local shear stress has previously been poorly correlated with local median grain size [Lisle et al., 2000; Nelson et al., 2010] and in our case it was also poorly correlated with patch median grain size (section 3.2, Figure 5a). However, the mean patch class shear stress varied directly with patch class median grain size and scaled fairly well with the reach-averaged shear stress (Figures 5b, 8a, and 8b). This implies that sediment patches may not only be a response to shear stress divergences [Nelson et al., 2010] but also to local stress magnitudes. Such a result is important for predicting the location and stability of sediment patches. In the particular case of salmon spawning, the ideal location for placement of relatively stable gravel could be potentially estimated from a shear stress map.

### 4.3. Relative Importance of Spatial Distribution of Shear Stresses and Grain Size

It is challenging to establish whether grain size or shear stress variations are more important in bedload predictions because of the nonlinear processes that govern sediment transport [Recking, 2013]. Flume...
experiments have shown that variations in local shear stress are relatively more important than those in grain size for bar formation [Nelson et al., 2010]. In steep streams, field observations suggest that local flow, and not spatial grain size variations, is the primary driver of local bed load transport variability [Yager et al., 2012a]. However, local grain-induced roughness may influence the initiation of sediment motion and therefore grain size variations could be important [Scheingross et al., 2013].

To analyze whether a spatial distribution of shear stresses or grain sizes is more important we considered three cases: 1) for case $GSD_{cst\ T}$, we used each patch class averaged shear stress (“Patch mean” method of section 2.4) and assumed that the GSD for all patches was the reach-averaged value. Note that although all patches classes had the same GSD they had different shear stresses applied over different bed areas. 2) Case $GSD_{cst\ var\ m}$ is the same except the mobile bed GSD was used (i.e., immobile grains were excluded), which may be more appropriate for bed load transport calculations. 3) In case $GSD_{var\ T}$, the shear stress acting on each patch class was the same for a given discharge and equal to that of the reach-averaged shear stress. The GSD was spatially variable and each patch class used its original measured grain sizes.

Sediment transport volumes in all three cases were less accurate than our “Patch mean” method (Figure 9) but were more accurate for the $GSD_{cst\ var\ m}$ cases than the $GSD_{var\ T}$ scenario. This suggests that the spatial variability of shear stress is relatively more important for sediment flux predictions than the spatial distribution of grain sizes, which has been also confirmed by the field study of Segura and Pitlick [2015]. The results of this experiment would need to be confirmed with sediment fluxes data from other streams but could help to decide how to allocate efforts to maximize sediment transport prediction accuracy with time or economic constrains.

**4.4. Individual Contributions of Patch Classes to Sediment Fluxes**

While some studies have found that, during low to moderate flow events, relatively fine patches are the only sources of bed sediment [Garcia et al., 1999; Vericat et al., 2008], others have observed motion on all patches [Dietrich et al., 2005; Yuill et al., 2010; Yager et al., 2012a]. In our study, for a given discharge, the individual contributions of each patch class to the total transported volume depended on the patch’s GSD, area (Figure 10) and applied stress. In general, patches with $D_{50} < D_{90}$ contributed at least 80% of the total transported sediment volume. B, gbC and cgB patches in the 2010 data set and B and bgC patches in the 2011 data set had the lowest contribution (Figures 10a and 10b). Even during low to moderate flow events coarse patches ($D_{50} > D_{90}$), whose contribution to the total transported volume was very small (Figures 10a and 10b), were still active and had some grain sizes in motion (Figures 10c and 10d).

In both data sets there were no patch classes that consistently contributed the largest fraction of the total transported volume. The contribution of each patch class to the total transported volume class was controlled by local topography, patch area and GSD. The topography determines flow routing throughout the channel, which directly affected the active area and applied shear stresses on each patch class. Patch class GSD influenced the relative mobility of different grain sizes and relative patch class area partly determines the proportional contribution of a patch class to the total sediment flux. The variability of different patch class contributions suggests that all patch classes must be considered in sediment flux predictions.
4.5. Transferability of the Grain Size and Shear Stress Relation

To study the potential transferability of the $s_j^2 = s^2 \frac{D_{50}}{j}$ relation we used the published data of Yager et al. [2012a, 2012b] during 2004, which includes areas, GSD for each patch class, and channel characteristics used to calculate reach average shear stresses. The data were assumed to be valid during 2002–2006 when a total of 15 sediment transport events occurred. The extreme event of 2007 completely reorganized boulder steps [Molnar et al., 2010] and all of the mobile patches, and therefore, we treat these data as if they were coming from a different stream to test the broader applicability of our equations. We used five different approaches using the 2004 data to estimate reach-averaged shear stress ($s$) for use in our relation. Three approaches use the total shear stress ($s_T$), which is easy to calculate in rivers where no detailed information is available. (1) For a given discharge we assumed that the reach-averaged shear stress was the same as what we calculated using the 2010 data and was equal to $\tau_{var \ D_{50}}$ (see section 3.2). (2) We do the same thing as in (1) but with the 2011 data. Then, for the other three approaches we used the total shear stress calculated using the equations (3) [Egashira and Ashida, 1991], (4) [Rickenmann and Recking, 2011], and (5) [Yager et al., 2012b]. Approaches (1) and (2) use equations (18a) and (19a) (Figures 8a and 8b), while approaches (3) to (5) use equations (18b) and (19b) to estimate the patch average shear stress. Note that we are not directly using the total shear stress for bed load transport predictions, we only use it to determine the shear stress on each patch class. Values used for sediment transport predictions in approaches (3) to (5) are summarized in supporting information Table S3. Details of the equations of Egashira and Ashida [1991] and Rickenmann and Recking [2011] can be found in the original publications and also in Nitsche et al. [2011]. For comparison purposes we include the original sediment volumes predicted by Yager et al. [2012c] for this time period.

The predicted sediment volumes from all approaches had similar RMSE, ranging from 33 to 52 m$^3$ (Figure 11, Table 3) and differences were largely caused by how the reach-averaged shear stress was specified. Predictions using the total shear stress (approaches 3–5), were roughly as accurate as those using the reach-averaged stress from the hydrodynamic model (approaches 1–2). Although the total shear stress does not include the effects of large roughness elements our relations for individual patch classes do.

The 2010, 2011, and Egashira and Ashida [1991] methods predicted sediment volumes that were within one order magnitude of the measured values. Most approaches using our sediment transport equation had a lower RMSE and more events predicted within a factor of two than those predicted by Yager et al. [2012c], which does not include the effects of patches. This improvement suggests that our method could be

Figure 10. The volume fraction that each patch class contributes to the total transported volume as functions of (a, b) the normalized patch median grain size and (c, d) the relative area that each patch class occupied at each discharge. 2010 data are shown in Figures 10a and 10c, and 2011 are shown in Figures 10b and 10d.
capable of predicting fluxes of better accuracy than other methods available, but we would recommend using this method with caution until it has been further tested.

5. Conclusions

Including the spatial variability of flow and GSD in sediment transport calculations improved predictions compared to equations that only used reach-averaged properties. Nonetheless, achieving this increase in accuracy requires intense numerical modeling and detailed field measurements that can limit its applicability in practical cases. However, predictions were improved mainly because patch class shear stress directly correlated with patch class median grain size, which allowed for a better representation of local sediment mobility and hiding effects. Simple empirical relations of reach and patch class averaged shear stress with median grain size were developed and tested with our sediment transport equation. When using these relations the simplicity of reach averaged equations is preserved while the accuracy of including spatial variability is achieved. The relation between shear stress and surface median grain size is also a first step toward a theory to explain and predict the formation and location of sediment patches. It indicates that, at the patch scale, surface grain size and shear stress are coupled and the patch characteristics may not just be controlled by the divergence of shear stress. For accurate sediment transport predictions all patch classes must be considered; no particular patch class was consistently the greatest contributor to the total transported sediment volume. Individual contributions of each patch class depended on both GSD and area occupied. Finally, the spatial variability in the flow was relatively more important for accurate sediment fluxes than the spatial variability of the GSD.

References


Figure 11. (a) Reach-averaged shear stress as a function of discharge. The shear stress for 2010 and 2011 used the $\tau_{50-D_{50}}$ method, the total shear stress was calculated using resistance equations of Egashira and Ashida [1991], Rickenmann and Recking [2011], and Yager et al. [2012b]. (b) The log of the ratio of the predicted to measured sediment volume for sediment transport events during 2002–2006 at the Erlenbach. The results of Yager et al. [2012c] were included only as a reference and the volumes predicted in the original study are shown here. All other predictions used our $\tau_{s_{j}-D_{50}}$ relations and sediment transport equation. See Figure 7 for an explanation of other figure properties.

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